



When the solar wind flows past Earth, it pushes on Earth's magnetic field and compresses it. There is a point in space called the magnetopause where the pressure of the solar wind balances the outward pressure of Earth's magnetic field. The distance from the Earth, R , (white arrow in drawing) where these two pressures are balanced is given by the equation:

$$R^6 = \frac{0.72}{8\pi DV^2}$$

In this equation, D is the density in grams per cubic centimeter (cc) of the gas (solar wind, etc) that collides with Earth's magnetic field, and V is the speed of this gas in centimeters per second. Let's do an example to see how this equation works!

The solar wind has a typical speed of 450 km/s or equivalently $V = 4.5 \times 10^7$ cm/s. To find the density of the solar wind in grams/cc we have to do a two-step calculation. The wind usually has a particle density of about 5 particles/cc, and since these particles are typically protons (each with a mass of 1.6×10^{-24} gm) the density is then $5 \times (1.6 \times 10^{-24}$ gm)/cc so that $D = 1.28 \times 10^{-23}$ gm/cc.

We substitute D and V into the equation and get $R^6 = 1105242.6$. so that $R = (1105242.6)^{1/6}$. To solve this, we use a calculator with a key labeled Y^x . First type '1105242.6' and hit the 'Enter' key. Then type '0.1666' (which equals 1/6) and press the Y^x key. In this case the answer will be '10.16' and it represents the value of R in multiples of the radius of Earth (6378 kilometers). Scientists simplify the mathematical calculation by using the radius of Earth as their unit of distance, but if you want to convert 10.16 Earth radii to kilometers, just multiply it by '6378 km' which is the radius of Earth to get 64,800 kilometers. That is the distance from the center of Earth to the magnetopause where the magnetic pressure is equal to the solar wind pressure for the selected speed and density. These will change significantly during a 'solar storm'.

Now lets apply this example to finding the magnetopause distance for some of the storms that have encountered Earth in the last five years. Complete the table below, rounding the answer to three significant figures:

Storm	Date	Day Of Year	Density (particle/cc)	Speed (km/s)	R (km)
1	11/20/2003	324	49.1	630	
2	10/29/2003	302	10.6	2125	
3	11/06/2001	310	15.5	670	
4	3/31/2001	90	70.6	783	
5	7/15/2000	197	4.5	958	

Question: The fastest speed for a solar storm 'cloud' is 1500 km/s. What must the density be in order that the magnetopause is pushed into the orbits of the geosynchronous communication satellites at 6.6 Re?

The information about these storms and other events can be obtained from the NASA ACE satellite by selecting data for H* density and V_x(GSE)

http://www.srl.caltech.edu/ACE/ASC/level2/lvl2DATA_MAG-SWEPAM.html

Storm	Date	Day Of Year	Density (particle/cc)	Speed (km/s)	R (km)
1	11/20/2003	324	49.1	630	42,700
2	10/29/2003	302	10.6	2125	37,000
3	11/06/2001	310	15.5	670	51,000
4	3/31/2001	90	70.6	783	37,600
5	7/15/2000	197	4.5	958	54,800

Question: The fastest speed for a solar storm 'cloud' is 3000 km/s. What must the density be in order that the magnetopause is pushed into the orbits of the geosynchronous communication satellites at 6.6 Re (42,000 km)?

Answer: Solve the equation for D to get:

$$D = \frac{0.72}{8 \pi R^6 V^2}$$

For 1500 km/s $V = 1.5 \times 10^8$ cm/s, and for $R = 6.6$, we have

$$D = 0.72 / (8 \times 3.14 \times 6.6^6 \times (1.5 \times 10^8)^2) = 1.52 \times 10^{-23} \text{ gm/cc}$$

Since a proton has a mass of 1.6×10^{-24} grams, this value for the density, D, is equal to $(1.52 \times 10^{-23} / 1.6 \times 10^{-24}) = 9.5$ protons/cc.

For Extra Credit, have students compute the density if the solar storm pushed the magnetopause to the orbit of the Space Station (about $R = 1.01$ RE).

Answer: $D = 3 \times 10^{-19}$ gm/cc or 187,000 protons/cc. A storm with this density has never been detected, and would be catastrophic!