



An artists illustration of Kepler-22b
(Credit NASA/Ames/JPL-Caltech)

NASA's Kepler mission has confirmed its first planet in the "habitable zone," the region where liquid water could exist on a planet's surface.

The newly confirmed planet, Kepler-22b, is the smallest yet found to orbit in the middle of the habitable zone of a star similar to our sun.

The planet is about 2.4 times the radius of Earth. Scientists don't yet know if Kepler-22b has a rocky, gaseous or liquid composition, but its discovery is a step closer to finding Earth-like planets.

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about $5,500 \text{ kg/meter}^3$). If the radius of Kepler-22b is $15,000 \text{ km}$, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass ($5.97 \times 10^{24} \text{ kg}$)?

Problem 2 - The acceleration of gravity on a planetary surface is given by the formula

$$a = \frac{GM}{R^2}$$

where M is in kilograms, R is in meters and G is the Newtonian Constant of Gravity with a value of $6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$. What is the surface acceleration of Kepler-22b A) In meters/sec²? B) In multiples of Earth's surface gravity 9.8 meters/sec^2 ?

Problem 3 - The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2 . If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b?

Problem 4 - The dimensions of a typical baseball park are determined by the farthest distance that an average batter can bat a home-run. This in turn depends on the acceleration of gravity, which is the force that pulls the ball back to the ground to shorten its travel distance. For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the minimum distance to the back-field fence from Home Plate along one of the two foul lines?

B) What are the dimensions of the baseball diamond?

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter³). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97x10²⁴ kg)?

Answer: A) First find the volume of the spherical planet in cubic meters, then multiply by the density of the planet to get the total mass.

$$R = 15,000 \text{ km} \times (1000 \text{ m}/1 \text{ km}) = 1.5 \times 10^7 \text{ meters.}$$

$$V = \frac{4}{3} \pi R^3 \\ = 1.33 \times 3.14 \times (1.5 \times 10^7 \text{ meters})^3 = 1.41 \times 10^{22} \text{ meters}^3$$

$$\text{Then } M = \text{density} \times \text{volume} \\ = 5,500 \text{ kg/m}^3 \times (1.41 \times 10^{22}) \\ = \mathbf{7.75 \times 10^{25} \text{ kg}}$$

$$\text{B) } M = 7.75 \times 10^{25} \text{ kg} / 5.97 \times 10^{24} \text{ kg} = \mathbf{12.9 \text{ Earths.}}$$

Problem 2 - A) In meters/sec²? B) In multiples of Earth's surface gravity 9.8 meters/sec²?

$$\text{Answer: A) } a = 6.67 \times 10^{-11} (7.75 \times 10^{25}) / (1.5 \times 10^7)^2 = \mathbf{23.0 \text{ meters/sec}^2}$$

$$\text{B) } 23.0 / 9.8 = \mathbf{2.3 \text{ times earth's surface gravity}}$$

Note: From the formula for M and a, we see that the acceleration varies directly with the radius change, which is a factor of 2.4 times Earth, so $a = 2.4a(\text{earth})$

Problem 3 – The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec². If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b?

Answer: By a simple proportion: $X/150 = 2.3/1.0$ so $x = 2.3 \times 150 = \mathbf{345 \text{ pounds.}}$

Problem 4 - For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the distance to the back-field fence from Home Plate along one of the two foul lines? Answer: $325 / 2.3 = \mathbf{141 \text{ feet.}}$

B) What are the dimensions of the baseball diamond? Answer: $90/2.3 = \mathbf{39 \text{ feet}}$ on a side.