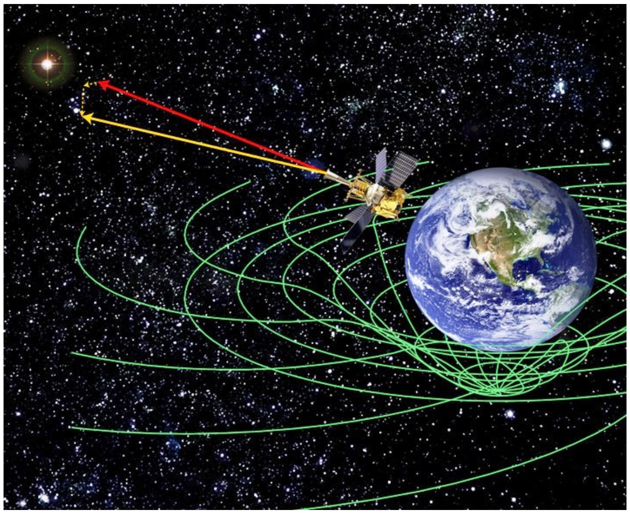

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Physicists at Stanford University have recently completed their analysis of data from the Gravity Probe-B (GP-B) satellite, launched in 2004, and have confirmed two predictions of Albert Einstein's relativistic theory of gravity called General Relativity.

The pointing direction of a high-precision gyroscope was measured for over 50 weeks as it orbited Earth. If Newton's theory of gravity were correct, the pointing direction should stay absolutely the same. If Einstein's theory was correct, it should point in a slightly different direction.

The effect is called 'frame dragging' and was first predicted in 1918 by Austrian physicists Josef Lense (1890-1985) and Hans Thirring (1888-1976) using Einstein's mathematical theory of gravity published in 1915. The rate at which the pointing angle will change is given by the formula for Ω, in degrees/sec, shown below:

$$\Omega = \frac{GJ}{2c^2a^3\left(1-e^2\right)^{3/2}}\left(\frac{360}{2\pi}\right)$$

c is the speed of light: $c = 300,000,000$ m/s

J is the angular momentum of Earth: $J = 5.861 \times 10^{33}\ \mathrm{m^2\ kg\ sec^{-1}}$

G is the Newtonian Gravitational constant $G = 6.67 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$

a is the semi-major axis of the satellite orbit

e is the eccentricity of the satellite orbit

Problem 1- The GP-B satellite orbits at a distance from Earth's center of a = 7020 km, in a circular orbit for which e=0. To two significant figures, what is the value for Omega in A) degrees per second? B) arcseconds per year? (Note 1 degree = 3600 arcseconds and 1 year = 3.1×10^7 seconds)

Problem 2 - The GP-B spacecraft took observations for 50 weeks. About what would be the accumulated angular shift by the end of this time to two significant figures?

Problem 1 - The GP-B satellite orbits at a distance from Earth's center of a = 7020 km, in a circular orbit for which e=0. To two significant figures, what is the value for Omega in A) degrees per second? B) arcseconds per year? (Note 1 degree = 3600 arcseconds and 1 year = 3.1×10^7 seconds)

$$\Omega = \frac{GJ}{2c^2a^3\left(1-e^2\right)^{3/2}}\left(\frac{360}{2\pi}\right)$$ in degrees/sec

$$\Omega = \frac{(6.67x10^{-11})(5.861x10^{33})(360)}{4(3.141)\left(3.0x10^{8}\right)^2\left(7.02x10^{6}\right)^3\left(1-0^2\right)^{3/2}}$$ **3.66 x 10^{-13}** degrees/sec

Answer

A) Ω = **3.6 x 10^{-13} degrees/sec**

B) Ω = **3.6 x 10^{-13}** degrees/sec x (3600 arcsec/1 degree) x (3.1x10^{7} sec/1 year)

= **0.04 arcseconds/year**

Problem 2 - The GP-B spacecraft took observations for 50 weeks. About what would be the accumulated angular shift by the end of this time to two significant figures?

Answer: 50 weeks is 50/52 = 0.96 years, so the total shift is just

$\Theta = \Omega$ x 0.96

= 0.04 arcseconds/year x (0.96 years)

= 0.038 arcseconds.

Note: This calculation is an approximation to the actual models used to represent the complex spacecraft motion and Earth's gravitational field. Because a more detailed model for Earth and the satellite's motion was used by the GP-B science team, the actual shift detected by the Gravity Probe-B satellite was 0.041 arcseconds, in agreement to within 1% with refined calculations from Einstein's theory.

The equation used in this problem, which predicts the rate of advance of the right ascension of the ascending node of the spacecraft's orbit due to the Lens-Thirring Effect, was obtained from the article:

"*Gravitation, Relativity and Precise Experimentation*" by C.W. Everitt, Proceedings of the First Marcel Grossmann Meeting on General Relativity, pp. 545-615, North Holland, 1977 (p. 567, Equation 22). See the archive of scientific papers at the GP-B website **http://einstein.stanford.edu/content/sci_papers/index.html**