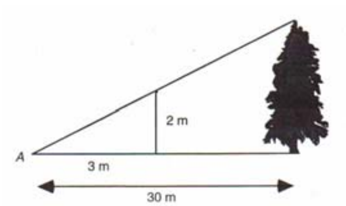
Using Proportions to Estimate the Height of a Cloud!

'A lonely cloud and its shadow!'
Courtesy: Henriette (2005)
The Cloud Appreciation Society

Sometimes, if you are lucky, you can see a single cloud and its shadow, perhaps while you were visiting the beach, standing in a meadow, or driving across the desert. By using the properties of similar triangles and a simple proportion, you can use this cloud and its shadow to figure out how high up the cloud is! You need a meter stick, and a bit of help from a friend to do this, though.

Let's see how this works for an example so that you can try this the next time you are at the beach...or the desert!

Similar Triangles and Proportions



The figure shows two similar right-triangles formed by the sides '3m' and '2m' and the sides '30m' and the unknown height of the tree. The basic geometric rule of similar triangles is that corresponding sides are in the same proportion. That means that the ratio formed by the sides '3m' and '30m' is the same numerical proportion as the sides '2m' and 'X' where X is the height of the tree. We can write this as:

$$\frac{30 \text{ meters}}{3 \text{ meters}} = \frac{X}{2 \text{ meters}}$$

and then solve this to get

X = 2 meters x (30/3)
= 20 meters.

Suppose that in this problem, instead of the 2 meter long stick we used the height of your thumb, about 1.5 inches (3.8 cm), and placed it at arms-length from your eye, which is 20 inches (51 cm). Suppose also that if you were at a distance of 269 meters from the base of the tree the height of the tree would exactly equal the height of your thumb. How tall would the tree be? Again we set-up the proportion:

$$\frac{269 \text{ meters}}{51 \text{ cm}} = \frac{X}{3.8 \text{ cm}}$$

and solve for X: X = 269 meters x (3.8 /51) = 20 meters.

So, using similar triangles and proportions is a very COOL way to figure out things about distant objects. In astronomy, you cannot even travel to these objects so using similar triangles and proportions is the only way to learn about their sizes and distances!

Now let's apply this proportion method to studying clouds!

Problem 1 – Measure the length of your out-stretched arm in centimeters. Now find a cloud near you that has a shadow close by where you are standing. Holding the meter stick at arm's length, how many centimeters is it from the base of the cloud down to the ground?

Problem 2 – Draw a scaled model right-triangle ABC, where side AB is the length of your arm in centimeters, and side BC is the vertical distance to the base of the cloud that you measured in Problem 1. Let's suppose that for this problem, AB = 20 inches and AC = 12 inches and that 1 inch = 2.5 cm.

Problem 3 – This next part is a bit tricky. As best you can, estimate how far it is from where you are standing to where the shadow of the cloud begins. You can also note some feature at this location like a tree or a rock formation, or a distant person sitting on a blanket! Count the number of paces it takes to get to this spot. Suppose that for this problem your pace was 2-feet long (0.7 meters) and you completed 3000 paces to get to the spot. How many meters did you travel?

Problem 4 – It is now time to use proportional reasoning! Use the similar triangle you created in Problem 1, with the distance you paced in Problem 3 to determine the actual height of the cloud above the distant point! What would be your answer for the example we used?

Common Core Math Standards:
CCSS.Math.Content.7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

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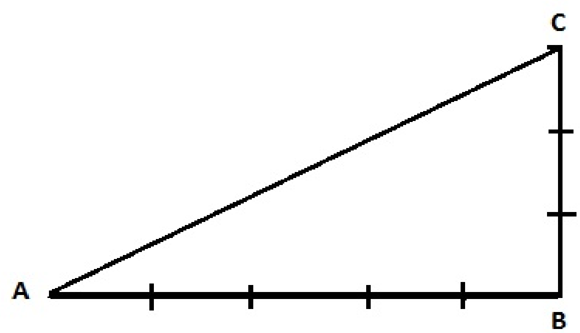
Answer: We want all measurements to be in centimeters in order to draw the scaled triangle. AB = 20 inches x (2.5 cm/1 inch) = 50 cm. AC = 12 inches x (2.5 cm/ 1 inch) = 30 cm. The drawing looks like this. Each division is 10 centimeters.



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Answer: In this example, 3000 paces x (0.7 meters/1 pace) = **2,100 meters**.

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Answer: Let X be the actual height of the cloud, then from the similar triangle BC/AB = X/2100 meters, and since BC=30cm and AB=50 cm we have 30/50 = X/2100 and so X = 2100 (30/50) = 1260 meters. **So the cloud is about 1260 meters above the ground!**