The Higgs Boson and the Mystery of Mass

Why is it that some elementary particles such as the photon and gluon do not carry any mass, while other elementary particles such as the electron, the quarks and the W and Z bosons carry different amounts of mass? In 1979, physicists Steven Weinberg, Sheldon Glashow and Abdus Salam received the Nobel Prize in Physics for explaining this mystery. To do so, they needed to add a new particle called the Higgs Boson to the known collection of fundamental particles. Depending on how strongly a particle such as electrons or neutrinos interacted with the Higgs Boson, it would 'pick up' varying amounts of mass. Photons didn't interact at all so they gained no mass, while the Top Quark interacted very strongly and gained a lot of mass.

The search is on for the Higgs Boson at the Large Hadron Collider, which began operation on November 23, 2009. The mass of the Higgs Boson is actually not constant, but depends on the amount of energy that is used to create it. This remarkable behavior can be described by the properties of the following equation:

$$V(x) = 2x^4 - (1-T^2)x^2 + \frac{1}{8}$$

This equation describes the potential energy, V, that is stored in the field that creates the Higgs Boson. The variable x is the mass of the Higgs Boson, and T is the collision energy being used to create this particle. The Higgs field represents a new 'hyper-weak' force in Nature that is stronger than gravity, but weaker than the electromagnetic force. This new field permeates space everywhere in the universe. The Higgs Boson is the particle that transmits the Higgs field just as the photon is the particle that transmits the electromagnetic field.

Problem 1 - What is the shape of the function V(x) over the domain [0,+1] for a collision energy of; A) T=0? B) T=0.5? C) T = 0.8 and D) T=1.0?

Problem 2 - The mass of the Higgs Boson is defined by the location of the minimum of V(x) over the domain [0, +1]. If the mass, M in GeV, of the Higgs Boson is defined by **M = 300x**, how does the predicted mass of the Higgs Boson change as the value of T increases from 0 to 1?

Problem 3 - The measured mass of the Top Quark is 170 GeV, and the Up Quark is 2 MeV and the electron is 0.5 MeV. According to the current models of the Higgs field, the masses are determined by the equations $Me = aM_H$, $Mt = bM_H$ and $Mu = cM_H$ where a, b and c are adjustable constants that have to be selected once the actual mass of the Higgs Boson, M_H, is established experimentally. If the current quark and electron masses are determined for T=0, what would be the predicted quark and electron masses for T = 0.5?

Problem 4 - The LHC will achieve collision energies of about E = 5,000 GeV. If T = E/300 GeV ,what will the function V(x) look like, and what will be the predicted mass of the Top Quark at these energies?

Problem 1 - Answer: The function can be programmed on an Excel spreadsheet or a graphing calculator. Select x intervals of 0.05 and a graphing window of x: [0,1] y:[0,0.3] to obtain the plot to the left below. The curves from top to bottom are for T = 1, 0.8, 0.5 and 0 respectively.

Problem 2 - Answer: The minima of the curves can be found using a graphing calculator display or by interpolating from the spreadsheet calculations. The x values for T = 1 ,0.8, 0.5 and 0 are approximately 0, 0.3, 0.45 and 0.5 so the predicted Higgs Boson masses from the formula M = 300x will be 0 Gev, 90 GeV, 135 GeV and 150 GeV respectively. Note that the observed mass will increase as the collision energy decreases, and consequently as the collision energy increases, the mass will decrease to zero!

Problem 3 - Answer: First we have to determine what the constants are for the condition T=0 where M_H = 150 GeV. For the electron, Me = 0.5 MeV = a x (150 GeV) and since 1 GeV = 1,000 MeV we get a = (0.5/150,000) = 3.3 x 10^{-6} then similarly, b = (170 GeV/150 GeV) = 1.13, and c = (2 MeV/150 GeV) = 1.3 x 10^{-5}. At T = 0.5, Problem 2 says that the Higgs mass is lowered to 135 GeV, so we have the mass of the electron Me = 3.3 x 10^{-6} x 135 GeV = **0.45 MeV**, the Top Quark is M = 1.13 x 135 GeV = **152 GeV** and the Up Quark is M = 1.3 x 10^{-5} x 135 GeV = **1.8 MeV**.

Problem 4 - The LHC will achieve collision energies of about E = 10,000 GeV. If T = E/300 GeV, what will the function V(x) look like, and what will be the predicted mass of the Top Quark at these energies? Answer: T = 10,000/300 = 33. The graph is shown to the lower right. The mass will be **near zero** because the curve is nearly a parabola with its vertex at x=0.

Note to Teacher: The function V(x) in this problem is meant to illustrate an important concept related to the way in which the Higgs Boson allows particles such as electrons and quarks to gain mass, rather than to remain massless particles. The answers to Problems 2-4 are not meant to be exact, but only to illustrate the basic mathematics. In actuality, V(X) and its changes with actual collision energies at the Large Hadron Collider are more complex than presented in these problems, and the masses of known particles are not expected to change my more than a few percent over the energy range for T being explored.