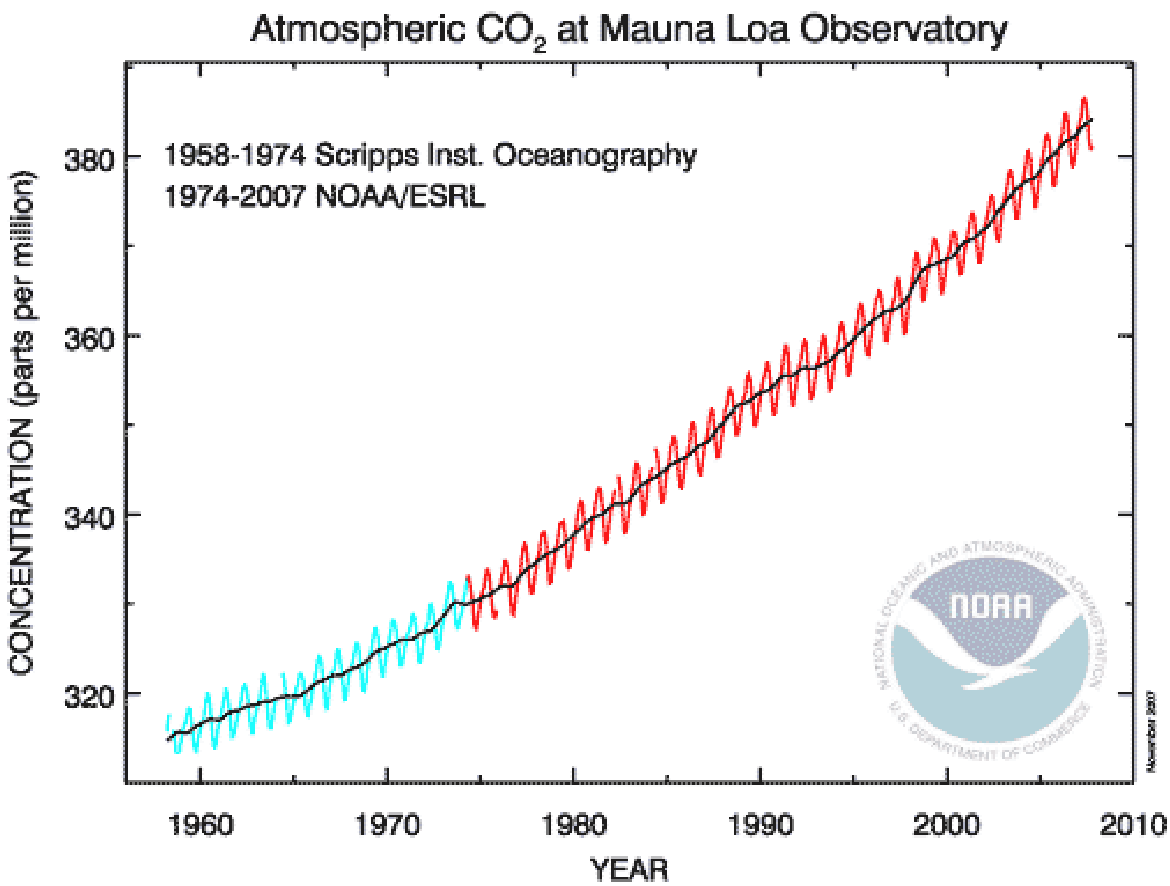

This is the Keeling Curve, derived by researchers at the Mauna Kea observatory from atmospheric carbon dioxide measurements made between 1958 - 2005. The accompanying data in Excel spreadsheet form for the period between 1982 and 2008 is provided at

http://spacemath.gsfc.nasa.gov/data/KeelingData.xls

Problem 1 - Based on the tabulated data, create a single mathematical model that accounts for, both the periodic seasonal changes, and the long-term trend.

Problem 2 - Convert your function, which describes the carbon dioxide volume concentration in parts per million (ppm), into an equivalent function that predicts the mass of atmospheric carbon dioxide if 383 ppm (by volume) of carbon dioxide corresponds to 3,000 gigatons.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm) and mass for the years: A) 2020? B)2050, C)2100?

Answer Key

Data from: C. D. Keeling, S. C. Piper, R. B. Bacastow, M. Wahlen, T. P. Whorf, M. Heimann, and H. A. Meijer, Exchanges of atmospheric CO_2 and $^{13}CO_2$ with the terrestrial biosphere and oceans from 1978 to 2000. I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, 88 pages, 2001. Excel data obtained from the Scripps CO2 Program website at http://scrippsco2.ucsd.edu/data/atmospheric_co2.html

Problem 1 - Answer: The general shape of the curve suggests a polynomial function of low-order, whose amplitude is modulated by the addition of a sinusoid. The two simplest functions that satisfy this constraint are a 'quadratic' and a 'cubic'… where 't' is the elapsed time in years since 1982

F1 = A sin (Bt + C) + (Dt2 + Et + F) and F2 = A sin(Bt + C) + (Dt3 + Et2 + Ft + G)

We have to solve for the two sets of constants A, B, C, D , E, F and for A, B, C, D, E, F, G. Using *Excel* and some iterations, as an example, the constants that produce the best fits appear to be: F1: (3.5, 6.24, -0.5, +0.0158, +1.27, 342.0) and F2: (3.5, 6.24, -0.5, +0.0012, -0.031, +1.75, +341.0). Hint: Compute the yearly averages and fit these, then subtract this polynomial from the actual data and fit what is left over (the residual) with a sin function.) The plots of these two fits are virtually identical. We will choose Fppm = F1 as the best candidate model because it is of lowest-order. The comparison with the data is shown in the graph below: red=model, black=monthly data. Students should be encouraged to obtain better fits.

Problem 2 - Answer: The model function gives the atmospheric carbon dioxide in ppm by volume. So take Fppm and multiply it by the conversion factor (3,000/383) = 7.83 gigatons/ppm to get the desired function, Fco2 for the carbon mass.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm), and mass for the years: A) 2020? B)2050, C)2100? Answer:

A) t = 2020-1982 = 38, so Fco2(38) = 7.83 x 410 ppm = 3,200 gigatons

B) t = 2050-1982 = 68, so Fco2(68) = 7.83 x 502 ppm = 3,900 gigatons

C) t = 2100-1982 = 118, so Fco2(118) = 7.83 x 718 ppm = 5,600 gigatons