



The graph to the left shows the 'Keeling Curve' which plots the increase in atmospheric carbon dioxide between 1958-2005. The average net annual rate is about 11 gigatons/year. The figure to the right shows a simplified view of the sources and sinks of the element carbon on Earth. Note that, for every 44 gigatons of the carbon dioxide molecule, there are 12 gigatons of the element carbon.

Problem 1 - What are the sources of carbon increases to the atmosphere in the above diagram? What are the sinks of carbon?

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon in gigatons.

Problem 3 - Integrate the equation in Problem 2, assuming that $C(2005) = 730$ gigatons, and derive A) the simple equation describing the total amount of carbon in the atmosphere as a function of time; B) the equation for the total amount of carbon dioxide.

Problem 4 - What does your model predict for the amount of carbon in the atmosphere in 2050 if the above source and sink rates remain the same?

Problem 5 - If the current carbon dioxide abundance is 384 ppm, what does your model predict for the carbon dioxide abundance in 2050?

Problem 6 - Does your answer for the net change in Problem 2 match up with the Keeling Curve data that indicates a net annual increase of carbon dioxide of +11 gigatons/year?

Problem 1 - Answer: The sources of the carbon (arrow pointed into the atmosphere in the figure) are Vegetation (+119.6 gigatons/yr), oceans (+88 gigatons/yr), human activity (+6.3 gigatons/yr) and changing land use (+1.7 gigatons/yr). The sinks remove carbon (the arrows pointed down in the figure) and include vegetation (-120 gigatons/yr), oceans (-90 gigatons/yr), and changing land use (-1.9 gigatons/yr).

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon dioxide, $C(t)$, in gigatons.

Answer:
$$\frac{dC(t)}{dt} = +119 + 88 + 6.3 + 1.7 - 120 - 90 - 1.9$$

so
$$\frac{dC(t)}{dt} = +3.1$$

Problem 3 - Answer: $C(t) = 3.1 t + a$ where a is the constant of integration.
Since $C(2005) = 730$

$$730 = 3.1 (2005) + a \quad \text{and so } a = -5500$$

$$C(t) = 3.1 t - 5500. \quad \text{for the total element carbon.}$$

Since 44 gigatons of carbon dioxide contain 12 gigatons of carbon, the equation for the CO₂ increase is $44/12 = 3.7x C(t)$ so that $CO_2(t) = 11.5 t - 20,300$

Problem 4 - What does your model predict for the amount of carbon dioxide in the atmosphere in 2050 if the above source and sink rates remain the same?

Answer: $C(2050) = 3.1 (2050) - 5500$
 $C(2050) = 860$ gigatons of carbon.

Problem 5 - If the current carbon dioxide abundance is 384 ppm, what does your model predict for the abundance in 2050? Answer: Since we are interested in the carbon dioxide increase we use the equation

$$CO_2(t) = 11.4 t - 20,300$$

For $t = 2050$ we get $CO_2(2050) = +3070$ gigatons of CO₂.

Since 384 ppm corresponds to 730 gigatons of CO₂, by a simple proportion, 3275 gigatons corresponds to $384 \text{ ppm} \times (3070/730) = 1,600$ ppm of CO₂.

Problem 5 - Does your answer for the net change in Problem 2 match up with the Keeling Curve data that indicates a net annual increase of carbon dioxide 11 gigatons/year?

Answer: Yes. The net change in Problem 2 was +3.1 gigatons/year of the element carbon. Since for every 44 gigatons of carbon dioxide there are 12 gigatons of the element carbon, we have $+3.1 \times (44/12) = +11.4$ gigatons of carbon dioxide increase, which is similar to the annual average of the increase implied by the Keeling Curve. This means that our simple model, based on the specified rates, provides a consistent picture of the atmospheric changes. Note: This simple, linear, model is only valid over the time span of the Keeling Curve.