The two solid rocket boosters (SRBs) for the Ares-V launch vehicle will each generate a total thrust of 3.8 million pounds (16.9 megaNewtons). The SRBs from tip to ground are 193 feet long and have a diameter of 12.2 feet. The actual fuel occupies a cylindrical volume about 160 feet (60 meters) long and 11.5 feet (3.7 meters) in diameter. They will 'burn' for a total of 126 seconds. You might think that the fuel burns from the bottom of the cylinder to the top, the way that a candle consumes its wax. The fuel in an SRB is actually designed to burn from the central axis of the cylinder to the outside casing! Let's see why this is a much better way to launch a rocket!

**Problem 1** - Thrust is created by burning the exposed surface area of the fuel in the SRB. To launch the 3.4 million pound Ares-V rocket, they have a combined burn rate of 8,740 kg of fuel each second. The density of the fuel is 1770 kg/m$^3$. If the exposed fuel area is just the circular cross section of the cylinder (see red area in the figure to left), and the burn depth is 0.025 meters each second;

A) What is the total burn rate?

B) Is this enough to launch Ares-V?

**Problem 2** - Suppose, instead, that a cylindrical core (red circle) with a diameter of 0.6 meters is cut out along the axis of the booster from top to bottom. The figure to the left shows the red areas where the fuel is burning.

A) What is the surface area of the exposed fuel in the core region?

B) If the burn depth is 0.025 meters each second, what is the mass rate in kg/sec?

C) Is this enough to launch Ares-V?
Problem 1 - Thrust is created by burning the exposed surface area of the fuel in the SRB. To launch the 3.4 million pound Ares-V rocket, they have to have a combined burn rate of 8,740 kg of fuel each second. The density of the fuel is $1770 \text{ kg/m}^3$. If the exposed fuel area is just the circular cross section of the cylinder (see red area in the figure to left), and the burn depth is 0.025 meters each second,

A) What is the total burn rate? Answer: A) Area = $\pi (3.7/2)^2 = 10.7 \text{ m}^2$. Volume change of disk = $10.7 \text{ m}^2 \times 0.025 \text{ m/sec} = 0.27 \text{ m}^3/\text{sec}$. Mass = $1770 \times 0.27 = 480 \text{ kg/sec}$. For both SRBs this yields a total of $960 \text{ kg/sec}$.

B) Is this enough to launch Ares-V? Answer: This is not a fast enough fuel burn rate to ensure the proper thrust, so an SRB cannot provide enough thrust by burning only the fuel exposed by the cylindrical cross-section at the tail-end of the booster.

Problem 2 - Suppose, instead, that a cylindrical core with a diameter of 0.6 meters is cut out along the axis of the booster from top to bottom.

A) What is the surface area of the exposed fuel in the core region? Answer: Burn Area = $2\pi Rh = 2 \times 3.14 \times (0.6 \text{ meters}/2) \times 60 \text{ meters} = 113 \text{ m}^2$.

B) If the burn rate is 0.025 meters each second, what is the mass rate in kg/sec? Answer: $113 \text{ m}^2 \times 0.025 \text{ m/sec} \times 1770 \text{ kg/m}^3 = 5,000 \text{ kg/sec}$. For both SRBs this equals $10,000 \text{ kg/sec}$.

C) Ares-V needs 8,740 kg/sec to launch so the 10,000 kg/sec produced by burning along the cylindrical core surface provides more than enough thrust!

Note to Teacher: Problem 2 used the 'tubular' method. Sketch this cross section on the board and ask your students to predict what the thrust profile will look like as the fuel burns outward from the core to the outer layer of the cylinder. Repeat this exercise for other kinds of fuel shapes and cross-sections.