Computing the Orbit of a Comet

This historic image of the nucleus of Halley's Comet by the spacecraft Giotto in 1986 reveals the gases leaving the icy body to form the tail of the comet.

Once astronomers discover a new comet, a series of measurements of its location allows them to calculate the orbit of the comet and predict when it will be closest to the Sun and Earth.

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years?

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

Answer: The standard formula for an ellipse is $x^2/a^2 + y^2/b^2 = 1$ so we can re-write this as $b^2x^2 + a^2y^2 = a^2b^2$.

Then for Point 1 we have
$10^2b^2 + 4^2a^2 = (ab)^2$ so $\mathbf{100b^2 + 16a^2 = (ab)^2}$. Similarly for Point 2 we have
$14^2b^2 + 3^2a^2 = (ab)^2$ so $\mathbf{196b^2 + 9a^2 = (ab)^2}$

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.
Answer:
$100b^2 + 16a^2 = (ab)^2$
$196b^2 + 9a^2 = (ab)^2$
Difference the pair to get $7a^2 = 96b^2$ so $a^2 = (96/7)b^2$.
Substitute this into the first equation to eliminate b^2 to get
$(700/96) + 16 = (7/96)a^2$ or $a^2 = 2236/7$ and so **a = 17.8 AU**.
Then substitute this value for a into the first equation to get
$5069 = 217\ b^2$ and so **b = 4.8 AU**.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years? Answer: $P = a^{3/2}$ so for a = 17.8 AU we have **P = 75.1 years**.

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the Sun in this orbit in kilometers?
Answer: From the definition for c as $c = (a^2 - b^2)^{1/2}$ we have c = 17.1 AU and so the perihelion distance is just d = 17.8 – 17.1 = 0.7 AU. Since 1 AU = 150 million km, it comes to within **105 million km** of the Sun. This is near the orbit of Venus.