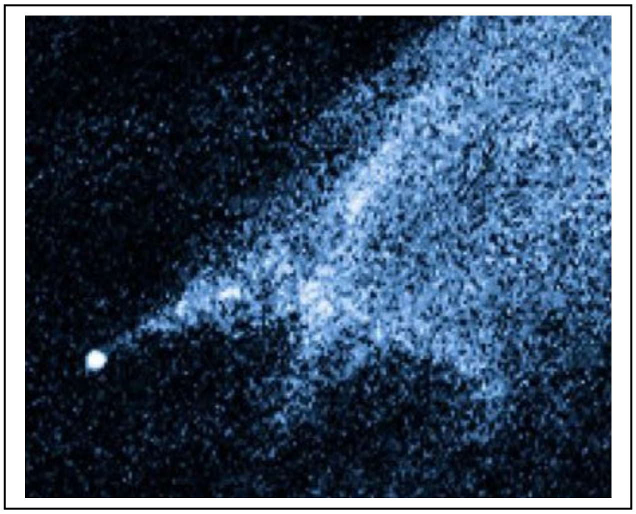
Hubble Spies Colliding Asteroids!



It doesn't look like much, but this picture taken by the Hubble Space Telescope on January 25, 2010 shows all that remains of two asteroids that collided! The object, called P/2010 A2, was discovered in the asteroid belt 290 million kilometers from the sun, by the Lincoln Near-Earth Asteroid Research sky survey on January 6, 2010.

Hubble shows the main nucleus of P/2010 A2, about 150 meters in diameter, lies outside its own halo of dust. This led scientists to the interpretation that it is the result of a collision.

How often do asteroids collide in the asteroid belt? The collision time can be estimated by using the formula:

$$T = \frac{1}{NAV}$$

Where N is the number of bodies per cubic kilometer, v is the speed of the bodies relative to each other in kilometers/sec and A is the cross-sectional area of the body in square-kilometers. The answer, T, will be in the average number of seconds between collisions.

Estimating A: Assume that the bodies are spherical and 100 meters in diameter .What will be A, the area of a cross-section through the body?

Estimating the asteroid speed V: At the orbit of the asteroids, they travel once around the sun in about 3 years. What is the average speed of the asteroid in kilometers/sec at a distance of 290 million kilometers?

Estimating the density of asteroids N: - This quantity is the number of asteroids in the asteroid belt, divided by the volume of the belt in cubic kilometers. A) Assume that the asteroid belt is a thin disk 1 million kilometers thick, with an inner radius of 1.6 AU and an outer radius of 2.5 AU. If 1 AU = 150 million kilometers, want is the volume? B) Based on telescopic observations, an estimate for the number of asteroids in the belt that are larger than 100 meters across is about 30 billion. From this information, and your volume estimate, what is the average density of asteroids in the asteroid belt?

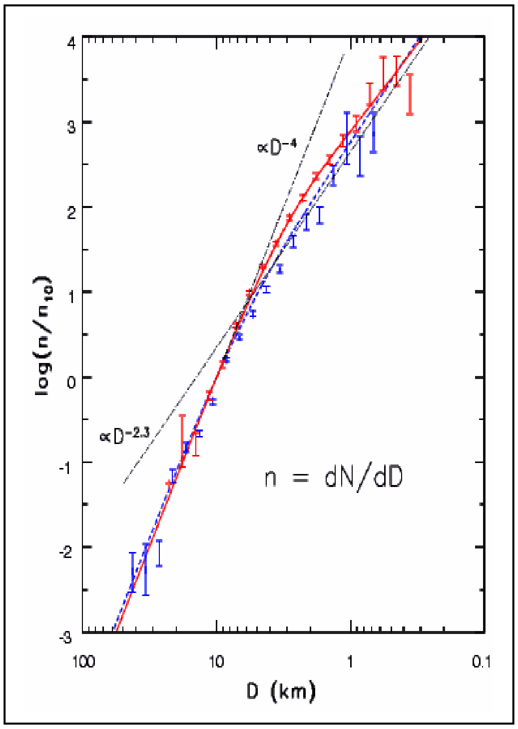
Estimating the collision time T: From the formula, A) what do your estimates for N, V and A imply for the average time between collisions in years? B) What are the uncertainties in your estimate?

Estimating A: Answer: The cross-section of a sphere is a circle, so $A = \pi R^2$ or for R = 100 meters, we have $A = 3.14 (0.05)^2 = 8\times10^{-3}$ km^2.

Estimating the asteroid speed V: Answer: The circumference of the orbit is C = 2 π (290 million km) = 1.8×10^9 kilometers, and the time in seconds is 3.0 years x (3.1×10^7 seconds/1 year) = 9.3×10^7 seconds, so the average speed V= C/T = 19 kilometers/sec. But what we want is the relative speed. If all the asteroids were going around in their orbits at a speed of 19 km/sec, their relative speeds would be zero. Example, although two cars are on the freeway traveling at 65 mph, their relative speeds are zero since they are not passing or falling behind each other.

Estimating the density of asteroids N: - Answer: R(inner) = 2.4×10^8 km .R(outer)= 3.4×10^8 km, so the volume of the disk is V = disk area x thickness = [$\pi(3.4\times10^8)^2 - \pi(2.4 \times 10^8)^2$] x 10^6 so the volume is **1.8×10^{23} km^3**. Then the average asteroid density is N= 3×10^{10} asteroids/1.8×10^{23} km^3 = **1.7×10^{-13} asteroids/km^3.**

Estimating the collision time T: Answer: A) T = 1/NAV so T = 1/($1.7 \times 10^{-13} \times 8\times10^{-3} \times 19$) = 3.8×10^{13} seconds. Since 1 year = 3.1×10^7 seconds, we have about **1,200,000 years between collisions**. B) Students can explore a number of places where large uncertainties might occur such as: 1) the thickness of the asteroid belt; 2) the number of asteroids; 3) their actual relative speeds. 4) The non-uniform distribution of the asteroids…not smoothly distributed all over the volume of the asteroid belt, but may be in clumps or rings that occupy a smaller actual volume. Encourage them to come up with their own estimates and see how that affects the average time between collisions. For advanced students see the problem below.



Extra Credit with Calculus: Data from the Sloan Digital Sky Survey suggests that the number of asteroids in specific size ranges follow a piecewise power-law distribution:

$$N(D) = \begin{cases} 1.8x10^9 D^{-2.3} & \text{for D <70 km} \\ 2.4x10^{12} D^{-4.0} & \text{for D>70 km} \end{cases}$$

Integrate N(D) from 100 meters to infinity to determine the number of asteroids larger than 100 meters.

Answer: $n = \int_{0.1}^{70} N(D)dD + \int_{70}^{\infty} N(D)dD$

= $2.7\times10^{10} + 2.32\times10^6 = 2.7\times10^{10}$ asteroids.

Note: the figure is scaled to the number of 10-km asteroids (n_{10}) which we take to be 10,000.