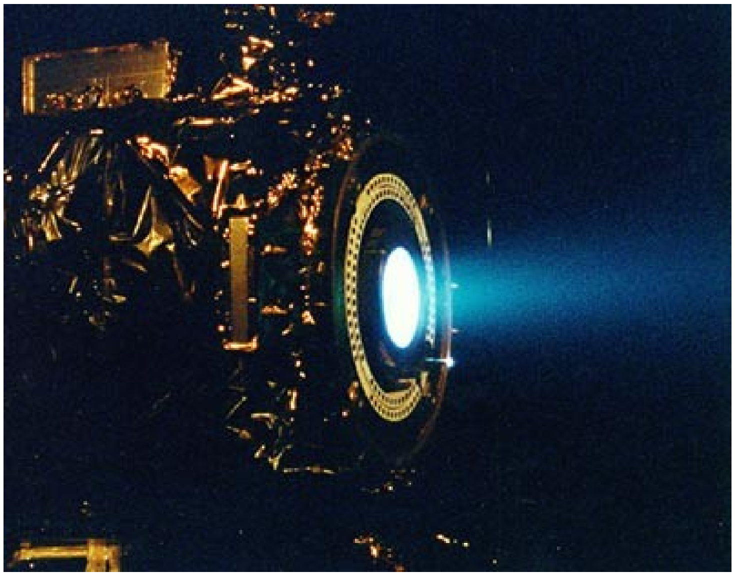
The Mathematics of Ion Rocket Engines



Believe it or not, NASA has been using ion engines for decades, and most commercial satellites use them too!

This image of a xenon ion engine, being tested at NASA's Jet Propulsion Laboratory, shows the faint blue glow of charged atoms being emitted from the engine. It was used in both the Deep Space 1 and Dawn satellites in their historic journeys to explore asteroids. The operating principle is simple.

Heavy atoms such as cesium, xenon are ionized, accelerated through a high-voltage grid, and ejected out the back of the thruster. The momentum of the ejected heavy atoms, when multiplied by the trillions of atoms in the beam, produces a steady, constant thrust that can be maintained for years at a time. Because the speed of the atoms is thousands of times higher than the exhaust from chemical rocket engines, very little mass is needed in the beam of atoms to generate a large thrust over time. For the Dawn spacecraft, launched in 2007, the 'fuel' mass is only 425 kilograms, but ejected steadily for 8 years, the 1,200 kilogram satellite will reach a speed of over 36,000 km/hour (22,300 miles/hour). This is equal to 315 million kilometers/year or the distance to the sun and back from Earth! Here is some of the mathematics, in a highly simplified form, that will take you through the basic ideas behind these exciting rocket technologies!

Problem 1 - Charged particles gain speed in an electric field - The kinetic energy of a particle is given by K.E. = 1/2 mv^2. The energy a charged particle gains from falling through a potential difference of V volts is given by E = qV. The NSTAR ion engine developed for the Deep Space 1 satellite uses xenon atoms with a mass of 2.2×10^{-25} kg, and a charge of q = 1.6×10^{-19} coulombs. What will be the speed of the atom, in kilometers/hour, if the voltage grid of the ion engine is 1,300 volts?

Problem 2 - The smaller the grid separation, the higher the acceleration - The NSTAR engine has a grid separation of 0.7 mm. From your answer to Problem 1, A) what is the average acceleration of the ions as the leave the grid? B) What is the force they experience, in Newtons?

Problem 3 - The thrust depends on particle flow rate - How many particles have to be ejected in the time it takes to cross the grid, to create a thrust of 0.90 Newtons? (Express the answer in particles per second).

Problem 4 - Charged particle flows produce electrical currents - If each particle carries exactly one unit of charge, and 1 Ampere = 6.25 x 1018 particles/sec, what is the current needed in the beam to give the thrust in Problem 3?

Problem 5 - Currents require power to maintain them - What is the beam power, in watts, defined by Power = Voltage x Amperage?

Answer Key

Problem 1 - Equate K.E = E and solve for v to get:

$$v^2 = 2 \text{ qV/m} = 2\,(1.6 \times 10^{-19})(1{,}300) / (2.2 \times 10^{-25}) = 2.0 \times 10^9$$

so v = 44.2 km/sec or 159,000 km/hr

Problem 2 - Answer: A) At an average speed of 44.2 km/sec, the atoms take about T = 0.7 mm/ (22.1 km/sec) = 3.1×10^{-8} seconds to travel the distance. This is an average acceleration of 22,100 m/sec / 3.1×10^{-8} sec = 7.1×10^{11} meters/sec^2. B) The force F = (2.2×10^{-25}) $(7.1 \times 10^{11}$ meters/sec$^2)$ = 1.6×10^{-13} Newtons/particle.

Problem 3 - 0.090 Nt = N x 1.6×10^{-13} Newtons/particle so N = 5.6×10^{11} particles. The time to cross the grid is 3.1×10^{-8} seconds so the particle flow has to be 5.6×10^{11} particles / 3.1×10^{-8} seconds = 1.8×10^{19} particles/second.

Problem 4 - 1.8×10^{19} particles/second / $(6.25 \times 10^{18}$ particles/sec//Ampere) = 2.9 Amperes!

Problem 5 - P = 1,300 Volts x 2.9 Amperes = 3,800 watts.