Gas clouds in interstellar space are acted upon by external pressure and their own gravity, and would otherwise collapse, but if they are hot enough, they can remain stable for a long time. That seems to be the case for objects called Bok Globules.

This photo of Thackeray’s Globule (IC-2944) taken by the Hubble Space Telescope may be a stable dark cloud containing 10 times the mass of our sun at a temperature of less than 100 K.

A gas sphere with a radius, R, a mass, M, and a temperature, T, is subject to an external pressure, P so that

\[
P = \frac{3 M k T}{4 \pi R^3 \mu m} - \frac{3 G M^2}{20 \pi R^4}
\]

where \(k\), \(G\) and \(\mu\) are constants.

**Problem 1** - At a given external temperature, \(P\), the cloud will be in equilibrium and not collapse if the sum of the two internal pressure terms on the right-side of the equation add up to the same value as the external pressure.

At what minimum radius will the cloud start to collapse for a given mass and temperature because the internal pressure no longer equals the external pressure? (Hint: determine \(dP/dR\) and find minimum.)
**Problem 1** - The problem states that the mass and temperature are held constant, so the only free variable is $R$. For complicated equations, it is always a good idea to group all constants together and define new constants. You can later replace the new constants by the old ones. Let's define $A = \left(\frac{3MkT}{4\pi\mu m}\right)$ and $B = \left(\frac{3GM^2}{20\pi}\right)$, then the equation becomes $P = AR^{-3} - BR^{-4}$. To find the extremum, we calculate $dP/dR$ and set this equal to zero. This gives us $dP/dR = A(-3)R^{-4} - B(-4)R^{-5} = 0$. This leads to $R = 4B/3A$ which upon substituting back for the definitions of $A$ and $B$ gives us

$$R_c = \frac{12G\mu m}{45kT}$$