The farther a particle falls towards a black hole, the faster it travels, and the more kinetic energy it has. Kinetic energy is mathematically defined as 

\[ \text{K.E.} = \frac{1}{2} m V^2 \]

where \( m \) is the mass of the particle and \( V \) is its speed.

Suppose all this energy is converted into heat energy by friction as the particle falls, and that this added energy causes nearby gases to heat up. How hot will the gas get? The equivalent amount of thermal energy, \( \text{T.E.} \), carried by a single particle is

\[ \text{T.E.} = \frac{3}{2} kT \]

where Boltzmann's Constant \( k = 1.38 \times 10^{-23} \) Joules/deg. If we set \( \text{K.E} = \text{T.E} \) we get

\[ T = \frac{mV^2}{3k} \]

If all the particles in a gas carried this same kinetic energy, then we would say the gas has a temperature of \( T \) in kelvins. We also know that the potential energy of the particle is given by

\[ \text{P.E.} = \frac{GMm}{R} \]

So if we set \( \text{P.E} = \text{T.E} \) we also get the temperature

\[ T = \frac{2GMm}{3kR} \]

**Problem 1** - The formula \( T = \frac{2GMm}{(3kR)} \) gives the approximate temperature of hydrogen gas (\( m = 1.6 \times 10^{-27} \) kg) in an accretion disk around a black hole. To two significant figures, what is the temperature for the material at the distance of Earth's orbit for a solar-mass black hole? (\( R = 1.47 \times 10^{11} \) m, \( M = 1.9 \times 10^{30} \) kg, for the constant of gravity \( G = 6.67 \times 10^{-11} \) Nt m²/kg²)?

**Problem 2** - How hot would the disk be at the distance of Neptune (\( R = 4.4 \times 10^{12} \) meters)?

**Problem 3** - X-rays are the most common forms of energy produced at temperatures above 100,000 K. Visible light is produced at temperatures above 2,000 K. Infrared radiation is commonly produced for temperatures below 500 K. What would you expect to see if you studied the accretion disk around a solar-mass sized black hole?
Answer Key:

**Problem 1** - The formula \( T = \frac{2}{3} \frac{GM}{kr} \) gives the approximate temperature of hydrogen gas \((m = 1.6 \times 10^{-27} \text{ kg})\) in an accretion disk around a black hole. To two significant figures, what is the temperature for a solar-mass black hole disk near the orbit of Earth? \((R = 1.47 \times 10^{11} \text{ m}, M = 1.9 \times 10^{30} \text{ kg}, \text{for } G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)\)?

Answer: \[ T = \frac{2}{3} \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} \times 1.6 \times 10^{-27} / (1.38 \times 10^{-23} \times 1.47 \times 10^{11}) \]
\[ = 65,000 \text{ K. to 2 significant figures} \]

**Problem 2** - How hot would the disk be at the distance of Neptune \((R = 4.4 \times 10^{12} \text{ meters})\)?

Answer: \[ T = \frac{2}{3} \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} \times 1.6 \times 10^{-27} / (1.38 \times 10^{-23} \times 4.4 \times 10^{12}) \]
\[ = 2,200 \text{ K.} \]

**Problem 3** - X-rays are the most common forms of energy produced at temperatures above 100,000 K. Visible light is produced at temperatures above 2,000 K. What would you expect to see if you studied the accretion disk around a solar-mass sized black hole?

Answer: The inner disk region would be an intense source of x-rays and visible light, because the gas is mostly at temperatures above 65,000 K. In the outer disk, the gas is much cooler and emits mostly visible or infrared light.