The least expensive form of radiation shielding is a planetary atmosphere, but just how efficient is it? The walls of the International Space Station and the Space Shuttle provide substantial astronaut protection from space radiation, and have an equivalent thickness of 10 grams/cm² of aluminum, which has a density of 2.7 gm/cm³. Compare this shielding to the spacesuits worn by Apollo astronauts of only 0.1 gm/cm². The atmosphere of Earth is a column of air with density of 0.0012 gm/cm³, that is 100 kilometers tall. How much shielding does this provide at different altitudes above the ground?

In the previous problem 'Atmospheric Shielding from Radiation II' we estimated the atmospheric shielding of Earth and Mars and compared the potential radiation dosages on the planetary surface. In this problem, we will create a more accurate estimate by using a realistic model for the atmospheres of these planets. Assume R (Earth) = 6,378 kilometers, R (Mars = 3,374 km)

The formula for S is given by

\[ S(R,h,z,\theta) = \left( (R + h)^2 \sin^2 \theta + 2R(z - h) + z^2 - h^2 \right)^{1/2} - (R + h) \sin \theta \]

**Problem 1:** What is the form of the function S for h=0?

The density of a planetary atmosphere is defined by the exponential function

\[ N(z) = N(0) e^{-z/H} \]

where H is the scale-height of the gas. For the composition of Earth’s atmosphere, temperature, and surface gravity, H = 8.5 km. For Mars, H = 11.1 km. The sea-level density for Earth, N(0) = 0.0012 gm/cm³, while for Mars, N(0) = 0.00020 g/cm³. The amount of surface shielding for radiation arriving from a direction, θ, is given by evaluating the integral below:

\[ D = \int_{0}^{\infty} N(z) ds \]

**Problem 2:**

A) Determine the form for S for the case of \( \theta = 90 \) which gives the minimum planetary shielding at the surface for radiation entering from directly overhead.

B) Evaluate the integral for Earth and for Mars.

C) Assuming that the radiation environments of Mars and Earth are otherwise similar, about how many times more would your radiation dosage be on the surface of Mars compared to Earth?

D) How does the atmospheric shielding of Earth compare to the shielding provided by the International Space Station or the Space Shuttle?
The formula for \( S \) is given by
\[
S(R, h, z, \theta) = \left( (R + h)^2 \sin^2 \theta + 2R(z - h) + z^2 - h^2 \right)^{1/2} - (R + h) \sin \theta
\]

**Problem 1:** What is the form of the function \( S \) for \( h=0 \)?
\[
S(R, z, \theta) = \left( R^2 \sin^2 \theta + 2Rz + z^2 \right)^{1/2} - R \sin \theta
\]

**Problem 2:** The density of a planetary atmosphere is defined by the exponential function \( N(z) = N(0) e^{-z/H} \) where \( H \) is the scale-height of the gas. For the composition of Earth’s atmosphere, temperature, and surface gravity, \( H = 8.5 \text{ km} \). For Mars, \( H = 11.1 \text{ km} \). The sea-level density for Earth, \( N(0) = 0.0012 \text{ gm/cm}^3 \), while for Mars, \( N(0) = 0.00020 \text{ gm/cm}^3 \).

A) From the definition of \( S \) in Problem 1, determine the form for \( S \) for the case of \( \theta = 90 \) which gives the minimum planetary shielding at the surface for radiation entering from directly overhead.
Answer: \( \sin(90) = 1 \) so 
\[
S = \left( R^2 + 2Rz + z^2 \right)^{1/2} - R = (R+z) - R \quad \text{so} \quad S = z \quad !!!
\]

B) Evaluate the integral for Earth and for Mars.

From A) \( s = z \) so by substituting \( s \) for \( z \), the integral becomes ………

Using the variable substitution \( x = z/H \), you can put the integrand in a standard form ………

Evaluating the integral…….

\[
D = N(0) H \left( e^{-0} - e^{-\left(\text{infinity}\right)} \right)
\]

The answer is that
\[
D = N(0) H
\]

For Earth: \( D = 1.2 \text{ kg/m}^3 \times 8.5 \text{ km} = 1,020 \text{ gm/cm}^2 \)
For Mars: \( D = 0.020 \text{ kg/m}^3 \times 11.1 \text{ km} = 22 \text{ gm/cm}^2 \)

C) Assuming that the radiation environments of Mars and Earth are otherwise similar, about how many times more would your radiation dosage be on the surface of Mars compared to Earth?
Answer: Note: This means that, because your maximum radiation dosage comes from radiation reaching you from the vertical direction (less shielding), on Mars, you will be receiving about \( 1,020 \text{ gm/cm}^2 / 22 \text{ gm/cm}^2 \) or 46 times as much radiation on the ground as you would get on Earth. On Earth, your annual cosmic ray dosage is about 27 mRem /year, so on Mars the dosage could be 46 x 0.027 Rem/year = 1.2 Rem/year.

D) How does the atmospheric shielding of Earth compare to the shielding provided by the International Space Station or the Space Shuttle?
Answer: The ISS shielding is about \( 10 \text{ gm/cm}^2 \), but the atmospheric shielding on the ground for Earth is \( 1,020 \text{ gm/cm}^2 \) which is 100 times greater!

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