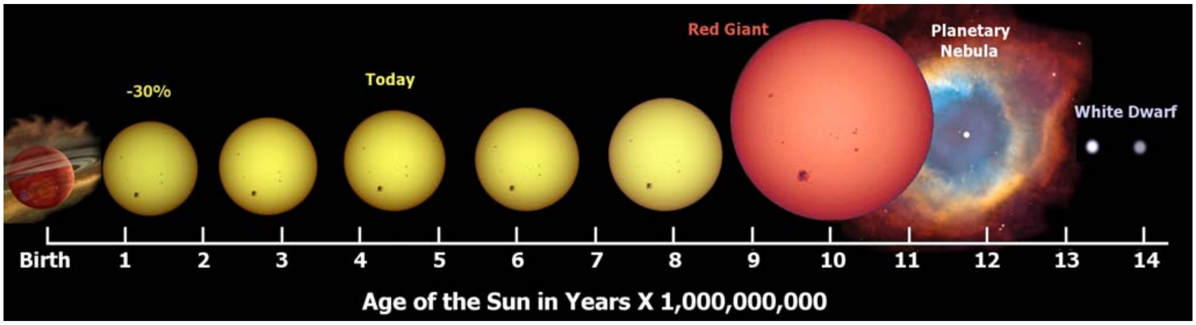


Our sun was formed 4.6 billion years ago. Since then it has been steadily increasing its brightness. This normal change is understood by astronomers who have created detailed mathematical models of the sun's complex interior. They have considered the nuclear physics that causes its heating and energy, gravitational forces that compress its dense core, and how the balance between these processes change in time. The diagram above shows the major stages in our sun's evolution from birth to end-of-life after 14 billion years. A simple formula describes how the power of our sun changes over time:

$$L = \frac{L_0}{1 + \frac{2}{5}(1 - x)}$$

where $x = t/t_0$ t_0 = 4.6 billion yrs, L_0 = 1.0 for the luminosity of the sun today.

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years

Problem 2 – By what percentage will L increase when it is 2 billion years older than it is today?

Problem 3 – A simple formula for the temperature, in kelvins, of Earth is given by:

$$T = 284[(1 - A)L]^{\frac{1}{4}}$$

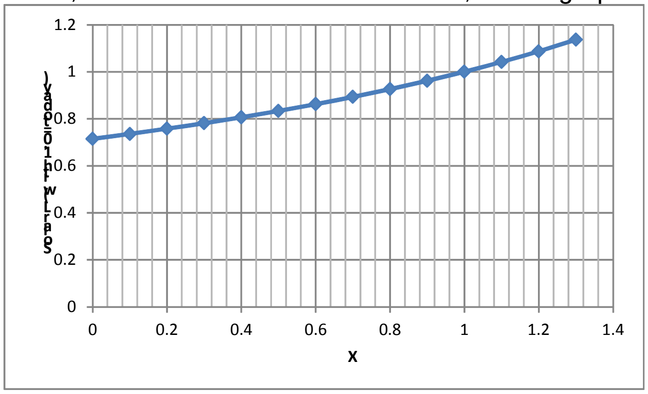
where L is the solar luminosity (today L=1.0), and A is the surface albedo, which is a number between 0 and 1, where asphalt is A=0 and A=1.0 is a perfect mirror A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Problem 4 - Combine the two formulae above to define a new formula that gives Earth's temperature in kelvins only as a function of time, t, and albedo, A.

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 150° F (339 kelvins)? (Note: it is currently 60° F)

$$L = \frac{L_0}{1 + \frac{2}{5}(1-x)}$$

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years.
Answer: t = 0 means X=0, t=6 billion means x = 6/4.6 = 1.3, so the graph domain is [0,1.3]



Problem 2 - By what percentage will L increase when it is 2 billion years older than it is today?

Answer: X = (4.6 + 2.0)/4.6 = 1.43, then L = 1 / (1+0.4(1-1.43)) so L = 1.20 this is **20% brighter** than today.

Problem 3 – A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Answer: A) L = 1.0 today so T = $284((1-0.4)\times1.0)^{1/4}$ = **250 kelvins** (or -23^{o} Celsius)
B) L = 1.05 so T = $284(0.6\times1.05)^{1/4}$ = **253 kelvins** (or -20^{o} Celsius)

Problem 4 - Combine the formulae for L(x) and T to define a new formula, T(x,A) that gives Earth's temperature only as a function of time, x, and albedo, A, and assumes that L_0=1.0 today.

$$T(x, A) = 425\left(\frac{1-A}{7-2x}\right)^{\frac{1}{4}}$$

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 100^{o} F (310 kelvins)? (Note: it is currently 60^{o} F)

Answer: $310 = 425\,(0.4/(7-2x))^{1/4}$
$0.4\,(425/310)^4 = 7-2x$
1.4 = 7 – 2x
2x = 5.6 so x = 2.8
and the age of the sun will be t = 2.8 x 4.6 billion = **12.9 billion years**.
This occurs 12.9 – 4.6 = 8.3 billion years in the future.