



A computer model developed by NASA scientists at the Goddard Institute for Space Science shows that without carbon dioxide, the terrestrial greenhouse would collapse and plunge Earth into an icebound state. Today, the average temperature is +15°C. Within 50 years the average temperature would drop to -21°C without the warming provided by atmospheric carbon dioxide. The delicate link between the planet's temperature and carbon dioxide has also been proved by geologic records of CO₂ levels during ice ages and interglacial periods. The temperature difference between

an ice age period and an interglacial period is only 5°C. During previous ice ages, CO₂ levels were near 180 parts per million (ppm). During the warm interglacial periods the levels were near 280 ppm. Today we are living in an interglacial period that started 12,000 years ago and may last another 40,000 years. Scientists continue to worry that, as CO₂ levels approach 400 ppm, we are in uncharted territory with no historical precedent as far back as 1 million years.

Although there is no known process that would instantly remove all CO₂ from the atmosphere, this computer model is important for another reason. It helps us predict how warm a planet would be if it had no greenhouse gases, even though it is close to its star.

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1 - \sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where $T < 0^\circ\text{C}$?

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of $A = 1.0$ is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of $A=0$ is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of $A=0.7$ and ocean water has $A=0.2$. After how many years will its albedo increase to 0.6 according to the computer models?

<http://www.giss.nasa.gov/research/news/20101014/>
 How Carbon Dioxide Controls Earth's Temperature
 October 14, 2010

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where $T < 0^\circ\text{C}$?

Answer: From the formula, we need $A/4\pi R^2 = \frac{1}{2}$
 so $\frac{1}{2} = 1 - \sin\theta$ and so $\theta = 30^\circ$ latitude

From the model, the zone where $T = -1^\circ\text{C}$ to $+1^\circ\text{C}$ reaches a latitude of 30° occurs after a time of **5 years**.

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of $A = 1.0$ is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of $A=0$ is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of $A=0.7$ and ocean water has $A=0.2$. After how many years will its albedo increase to 0.6 according to the computer models?

Answer: The average albedo is found by averaging the albedo of the area covered by the ice caps ($A=0.7$) with the albedo of the area covered by the ocean ($a=0.2$).

The area covered by ice caps = $4\pi R^2(1-\sin\theta)$
 The remaining area covered by water = $4\pi R^2 - 4\pi R^2(1-\sin\theta) = 4\pi R^2(\sin\theta)$

$$0.6 (4\pi R^2) = 0.7 (4\pi R^2)(1-\sin\theta) + 0.2(4\pi R^2)[\sin\theta]$$

Simplifying: $0.6 = 0.7(1-\sin\theta) + 0.2\sin\theta$
 $0.6 = 0.7 - 0.7\sin\theta + 0.2\sin\theta$
 $-0.1 = -0.5\sin\theta$
 $\sin\theta = 0.2$

So $\theta = 12^\circ$

So, when the zone $-1^\circ\text{C} < T < +1^\circ\text{C}$ reaches a latitude of $+12^\circ$, the albedo of Earth will have increased from its current value of 0.4 to a much higher reflectivity of 0.6. According to the model graph, this happens after 10 years.

Note: Although CO_2 loss is an important cooling process, the rapid increase in albedo is a significant cause for cooling and an important 'feed back' in the process. As the planet cools, more ice appears and the albedo increases, causing more cooling and more ice to appear...