The trillions of particles in Saturn’s rings orbit the planet like individual satellites. Although the rings look like they are frozen in time, in fact, the rings orbit the planet at thousands of kilometers per hour! The speed of each ring particle is given by the formula:

\[ V = \frac{29.4}{\sqrt{R}} \text{ km/s} \]

where \( R \) is the distance from the center of Saturn to the ring in multiples of the radius of Saturn (\( R = 1 \) corresponds to a distance of 60,300 km).

**Problem 1** – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

**Problem 2** – The Cassini Division contains nearly no particles and is the most prominent ‘gap’ in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

**Problem 3** – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

**Problem 4** – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

**Problem 5** – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Space Math  
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Problem 1 – The inner edge of the C Ring is located 7,000 km above the surface of Saturn, while the outer edge of the A Ring is located 140,300 km from the center of Saturn. How fast are the C Ring particles traveling around Saturn compared to the A Ring particles?

Answer: \[ R = \frac{(60300 \text{ km} + 7,000 \text{ km})}{60300 \text{ km}} = 1.12, \quad \text{so } V = 23.6 \text{ km/sec} \]
\[ R = \frac{140300 \text{ km}}{60300 \text{ km}} = 2.33, \quad \text{so } V = 16.3 \text{ km/sec}. \]

Note: The International Space Station orbits Earth at a speed of 7.7 km/s.

Problem 2 – The Cassini Division contains nearly no particles and is the most prominent ‘gap’ in the ring system easily seen from earth. It extends from 117,580 km to 122,170 km from the center of Saturn. What is the speed difference between the inner and outer edge of this gap?

Answer: \[ R = \frac{117580}{60300} = 1.95 \quad \text{V} = 17.83 \text{ km/s} \]
\[ R = \frac{122170}{60300} = 2.02 \quad \text{V} = 17.52 \text{ km/s} \]

The outer edge particles travel about 17.83 – 17.52 = 0.31 km/sec slower than the inner edge particles.

Problem 3 – If the particles travel in circular orbit, what is the formula giving the orbit period for each ring particle in hours?

Answer: Orbit circumference = \( 2 \pi r \) km, but \( r = 60300 \text{ R} \) so \( C = 2 (3.141) \times 60300 \text{ R} \), \( C = 379,000 \text{ R} \) km, where \( R \) is in Saturn radius units. Since the orbit speed is \( V = \frac{24.9}{R^{\frac{1}{2}}} \), then Time = \( \frac{C}{V} = \frac{15220 \text{ R}^{\frac{3}{2}}}{\text{seconds}}. \) Since 1 hour = 3600 seconds, we have \( T = 4.22 R^{\frac{3}{2}} \text{ hours}. \)

Problem 4 – What are the orbit times for particles near the inner and outer edge of the Cassini Division?

Answer: \( R = 1.95 \) so \( T = 11.49 \text{ hours} \).
\( R = 2.02 \) so \( T = 12.11 \text{ hours} \).

Problem 5 – The satellite Mimas orbits Saturn every 22.5 hours. How does this orbit period compare to the period of particles at the inner edge of the Cassini Division?

Answer: \( 22.5/11.49 = 1.94 \) which is nearly 2.0. This means that every time Mimas orbits once, the particles in the Cassini Division orbit about twice around Saturn. This is an example of an orbit resonance. Because the ring particles encounter a push from Mimas’s gravitational field at the same location every two orbits, they will be ejected. This is an explanation for why the Cassini Division has so few ring particles.