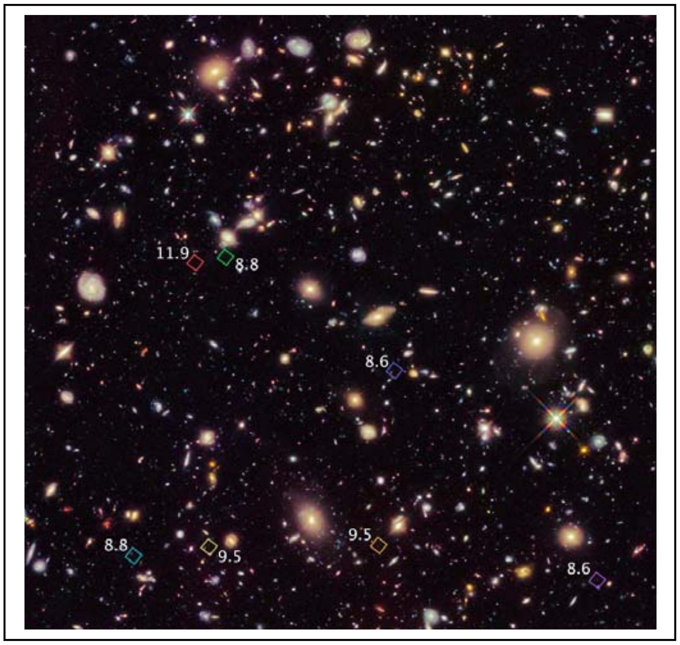


Using the Ultra Deep Field Image created by NASA's Hubble Space Telescope, astronomers have uncovered a previously unseen population of seven primitive galaxies that formed more than 13 billion years ago.

The age of the universe is 13.7 billion years, so we are seeing these galaxies as they were when the universe was only 4 percent of its present age. This is like an 80 year old human seeing a picture of themself when they were only 3 years old!

Among the predictions of Big Bang cosmology is a mathematical relationship T(Z) between the redshift of a galaxy denoted by the variable Z, and the time since the light was emitted by that galaxy, T, so that it can arrive at Earth today, some 13.7 billion years after the Big Bang occurred. During this time, the space within the universe has expanded, and Z indicates the amount of stretching of space that has occurred between the time when the light was emitted and the current era.

Using the exact solution for T(Z) from Big Bang theory, an astronomer wants to create a faster way to compute T(Z) that he can use on his hand calculator. He derives an approximation to the function T(Z) for Z between 2 and 15 given by

$$T(Z) = 0.000154z^5 - 0.0072Z^4 + 0.1301Z^3 - 1.143Z^2 + 5.0141Z + 3.7677$$

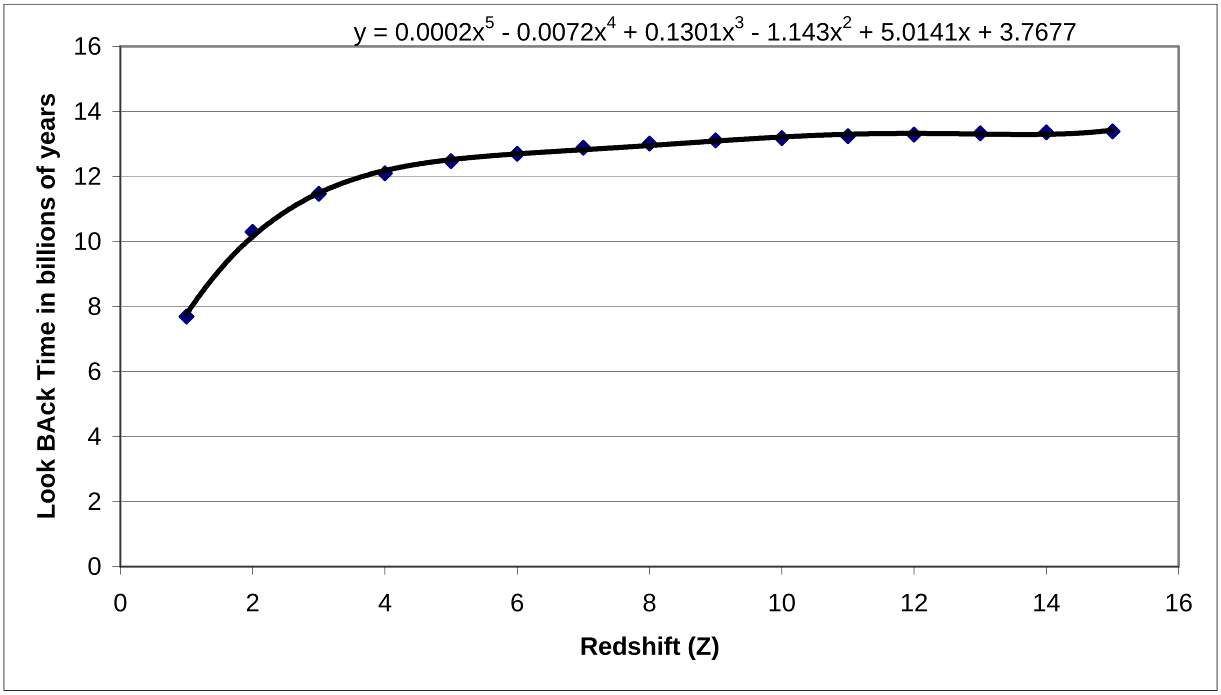
Problem 1 - Graph this function over the redshift range Z:[2.0, 15.0].

Problem 2 - Most of the galaxy redshift measurements the astronomer will make will be made over the much smaller range from 9.0 to 12.0. What is the slope of T(Z) over the range Z:[9.0, 12.0]?

Problem 3 - What is the linear equation that matches the redshift formula over Z:[9.0, 12.0]?

Problem 4 - If the astronomer uses the 'linear approximation' to T(Z) over Z:[9.0, 12.0] by what percentage will his estimates for the look-back time T differ from the original equation over the range Z:[5.0 ,15.0]?

Problem 1 - Graph this function over the redshift range Z:[2.0, 15.0].



Problem 2 - Most of the galaxy redshift measurements will be made over the much smaller range from 9.0 to 12.0. What is the slope of T(Z) over the range Z:[9.0, 12.0]?

Answer: change in Z = 12-9 = 3.0, T(12) = 13.18 billion years T(9)= 13.00 billion years, Slope M = 13.18-13.00/3.0 = 0.06.

Problem 3 - What is the linear equation that matches the redshift formula over Z:[9.0, 12.0]?

Answer: T = mZ + b, T(9) = 13.00 and T(12) = 13.18. Use the two-point formula: y-y1 - (x-x1) (y2-y1)/(x2-x1) then T - 13.00 = (z-9) (13.18-13.00)/(12-9) so T = 13.00 +(0.06(z-9) and so **T(Z) = 12.46 + 0.06Z**.

Problem 4 - If the astronomer uses the new 'linear approximation' to T(Z) over Z:[9.0, 12.0] by what percentage will his estimates for the look-back time T differ from the original equation over the range Z:[5.0 ,15.0]?

Z	Actual	Predicted	Difference	Percent
5	12.47	12.76	- 0.29	2.3%
15	13.39	13.36	+0.03	0.2%

So using the new, faster to compute, linear approximation only gives answers that differ by no more than 3% from the original, slower to compute, fifth-order polynomial approximation for T(Z).