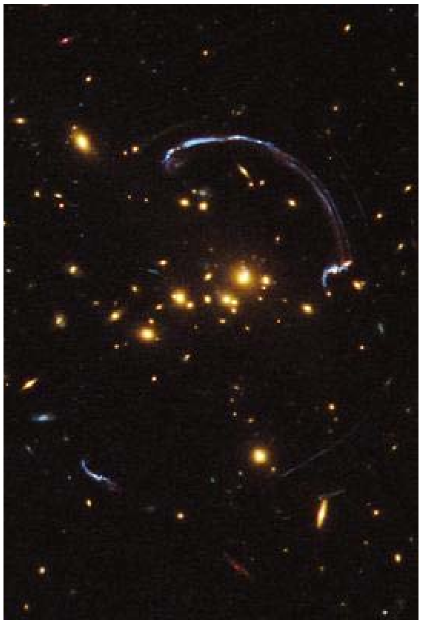
Using a Gravity Lens to 'Weigh' a Cluster of Galaxies



The Hubble Space Telescope recently photographed a distant galaxy being 'lensed' by the gravity field of a foreground cluster of galaxies called RCS2-032727-132623 located 5.4 billion light years away in the southern constellation Cetus the Whale. The cluster lies directly between the more distant galaxy and Earth. Its gravity causes the image of the distant galaxy to be distorted into several arcs (blue).

Thanks to the magnification provided by this 'gravity lens' astronomers can study the blue galaxy located 9.7 billion light years from Earth – details that would have been impossible to see otherwise.

Astronomers can also use the geometry of the arc and its radius to determine the total mass of the cluster of galaxies, including any dark matter that it might contain!

The diameter of a gravity lens ring is related to the mass of the object producing the gravitational field by the formula:

$$M = \frac{\theta^2 c^2 (dD)}{4G(d-D)}$$

where θ is the angular diameter in radians of the ring, D is the distance in meters from Earth to the cluster of galaxies, d is the distance in meters between Earth and the galaxy whose image is being lensed, c is the speed of light (3×10^8 m/s) and G is the constant of gravity (6.67×10^{-11} m^3 kg^{-1} sec^{-2})

Problem 1 - For convenience, astronomers often re-write equations in the common units of astronomy where angles, θ, are measured in arcseconds (1 radian = 206265 arcseconds), distances (d and D) are measured in light years and masses (M) are given in multiples of the sun's mass). If 1 light year = 9.5×10^{15} meters and Msun = 2.0×10^{30} kg, what is the value of the constant, C, in the new equation

$$M = C\left[\frac{\theta^2 (dD)}{d-D}\right]$$

Problem 2 - The diameter of the gravitational lens ring in RCS2-032727-132623 is 38 arcseconds. What is the mass of the galaxy cluster?

Hubble Zooms in on a Magnified Galaxy
http://www.nasa.gov/mission_pages/hubble/science/magnified-galaxy.html
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Problem 1 - For convenience, astronomers often re-write equations in the common units of astronomy where distances (d and D) are measured in light years and masses (M) are given in multiples of the sun's mass). If 1 light year = 9.5×10^{15} meters and Msun = 2.0×10^{30} kg and 1 radian = $2.0\text{x}10^{5}$ arcseconds, constant, C, in the new equation.

Answer: First insert the indicated units into the formula to get the mass, M, in kilograms. Note 1 arcsecond = $1/2.0\text{x}10^{5} = 5\text{x}10^{-6}$ radians so

$$M = \frac{(5.0x10^{-6})^2(3.0x10^{8})^2(9.5x10^{15})^2}{4(6.67x10^{-11})(9.5x10^{15})} \quad = \quad 8.0 \times 10^{31}$$

Then convert M to solar mass units by dividing by $2.0\text{x}10^{30}$ to get C = 40

Problem 2 - The diameter of the gravitational lens ring in RCS2-032727-132623 is 38 arcseconds. What is the mass of the galaxy cluster?

Answer; From the text, d = 9.7 billion light years, D = 5.4 billion light years and θ = 38 arcseconds, so

M = (37.7) $(38)^2$ (9.7 billion)(5.4 billion) / (9.7-5.4) billion

M = **6.5×10^{14} solar masses**!

Note: Another independent way to measure the mass of the cluster is by using the formula for the speed of an object in a circular orbit:

$$V^2 = \frac{GM}{R}$$

The average speed of the galaxies in the cluster is V = 988 km/sec and the average radius, R, of the cluster is 9.5 million light years

So M = $(9.5 \times 10^{15})(9.5 \times 10^{6})(988{,}000)^2 / (6.67\text{x}10^{-11})$
M = 1.3×10^{45} kg

Or M = 6.6×10^{14} solar masses