An important concept in cosmology is that the 'empty space' between stars and galaxies is not really empty at all! Today, the amount of invisible energy hidden in space is just enough to be detected as Dark Energy, as astronomers measure the expansion speed of the universe. Soon after the Big Bang, this Dark Energy caused the universe to expand by huge amounts in less than a second. Cosmologists call this early period of the Big Bang Era, Cosmic Inflation.

Physicists have developed a number of theories to quantify how Cosmic Inflation occurred. All the theories link this phenomenon to changes in the way that the forces of nature operated at the very high temperatures and energies that existed soon after the Big Bang. The basis for these explanations is a mathematical equation of the form shown below, which connects the energy of empty space to the existence of a new force in nature. This force, called the Hyper-weak Force, is transmitted between a new field hidden in empty space, and the known particles such as electrons, quarks and the particles that transmit the other three forces: electromagnetism and the strong and weak nuclear forces.

An interesting property of this new field, whose energy is represented by the function V(x), is that the shape of this function changes as the temperature of the universe changes. The result is that the way that this field, represented by the variable x, interacts with the other elementary particles in nature, changes. As this change from very high temperatures (T=1) to very low temperatures (T=0) occurs, the universe undergoes Cosmic Inflation!

$$V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}$$

Problem 1 - What are the domain and range of the function V(x)?

Problem 2 - What is the axis of symmetry of V(x)?

Problem 3 - Is V(x) an even or an odd function?

Problem 4 - For T=0, what are the critical points of the function in the domain [-2, +2]?

Problem 5 - Over the domain [0,+2] where are the local minima and maxima located for T=0?

Problem 6 - Using a graphing calculator or an Excel spreadsheet, graph V(x) for the values T=0, 0.5, 0.8 and 1.0 over the domain [0,+1]. Tabulate the x-value of the local minimum as a function of T. In terms of its x location, what do you think happens to the end behavior of the minimum of V(x) in this domain as T increases?

Problem 7 - What is the vacuum energy difference V = V(0) - V(1/2) during the Cosmic Inflation Era?

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?

Problem 1 - **Answer: Domain [- infinity, + infinity], Range [0,+infinity]**

Problem 2 - **Answer: The y-axis: x=0**

Problem 3 - **Answer: It is an even function.**

Problem 4 - **Answer: X = 0, X = -1/2 and x = +1/2**

Problem 5 - **Answer: The local maximum is at x=0; the local minimum is at x = +1/2**

Problem 6 - **Answer: See the graph below where the curves represent from top to bottom, T = 1.0, 0.8, 0.5 and 0.0. The tabulated minima are as follows:**

T	X
0.0	0.5
0.5	0.45
0.8	0.30
1.0	0.0

The end behavior, in the limit where T becomes very large, is that V(x) becomes a parabola with a vertex at (0, +1/8)

Problem 7 - What is the vacuum energy difference V = V(0) - V(1/2) during the Cosmic Inflation Era? **Answer: V(0) = 1/8 V(1/2) = 0 so V = 1/8.**

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?
Answer: V = 1/8 x 10^{35} Joules/meter^3 = **1.2 x 10^{34} Joules/meter^3**.

Note to Teacher: This enormous energy was available in every cubic meter of space that existed soon after the Big Bang, and the time it took the universe to change from the V(0) to V(1/2) state lasted only about 10^{-35} seconds. This was enough time for the universe to grow by a factor of 10^{35} times in its size during the Cosmic Inflation Era.