Cayrel’s Star (See ESO photo) is a faint star located 13,000 light years away in the constellation Cetus the Whale. In 2001 a group of astronomers led by Roger Cayrel made the first direct measurement of the age of this star by using the radioactive decay of Uranium-238, which decays to lead with a half-life of 4.47 billion years.

A detailed spectroscopic study revealed numerous, clear lines of uranium in the star’s spectrum. From this they could determine the abundances of the elements, and use their ratios to figure out an age for this star.

The measurements showed that the uranium abundances are about 1/7 that of our own sun. The age our sun is known to be 4.6 billion years.

**Problem 1** – The half-life formula states that the initial abundance of a radioactive element, \( N_0 \), with a half-life of \( t_{1/2} \) is related to its current abundance \( N(T) \) by

\[
N(T) = N_0 e^{-0.69(T / t_{1/2})}
\]

In the case of our sun, what fraction of the Uranium-238 isotope remains in the sun today given an age of 4.6 billion years?

**Problem 2** – If the abundance of Uranium-238 in Cayrel’s Star is 1/7 of our sun, what is the age of Cayrel’s Star?

**Problem 3** – Astronomers were able to detect the element Thorium-232 in the atmosphere of Cayrel’s Star. It has a half-life of 14.1 billion years. If the amount of thorium-232 in our sun today is \( 4.4 \times 10^{19} \) kg, how much was there in the sun when it was first formed?

Problem 1 – The half-life formula states that the initial abundance of a radioactive element, No, with a half-life of t1/2 is related to its current abundance N(t) by

\[ N(t) = N_0 e^{-\frac{0.69 t}{t_{1/2}}} \]

In the case of our sun, what fraction of the Uranium-238 isotope remains in the sun today given an age of 4.6 billion years?

Answer:

\[ N(4.6) = N_0 e^{-0.69 \times \frac{4.6}{4.7}} \]

so \( N/n_0 = 0.49 \) and so there is half as many atoms of uranium-238 today as there was when the sun was formed.

Problem 2 – If the abundance of Uranium-238 in Cayrel’s Star is 1/7 of our sun, what is the age of Cayrel’s Star?

Answer:

\[ 1/7 = e^{-0.69 \frac{T}{4.47}} \]

solving for \( T \) we get \( \ln(1/7) = -0.69(T/4.47) \) then \( T = 12.6 \text{ billion years} \).

Problem 3 – Astronomers were able to detect the element Thorium-232 in the atmosphere of Cayrel’s Star. It has a half-life of 14.1 billion years. If the amount of thorium-232 in our sun today is \( 4.4 \times 10^{19} \) kg, how much was there in the sun when it was first formed?

Answer: The decay fraction is just \( e^{-0.69 \times 14.1} = 0.80 \). So the initial amount was \( N(4.6)/0.8 = n_0 \) and so \( n_0 = 5.5 \times 10^{19} \) kg.

Note: The abundance of Thorium-232 in our sun today is estimated to be 0.0335 parts per million of silicon atoms, which are 650 ppm of the mass of the sun, which in turn has a total mass of \( 2.0 \times 10^{30} \) kg. So,

\[ \text{Amount of thorium} = 2.0 \times 10^{30} \text{ kg} \times 650 \times 10^{-6} \text{ (silicon/Msun)} \times 0.0335 \times 10^{-6} \text{ (thorium/silicon)} = 4.4 \times 10^{19} \text{ kg.} \]