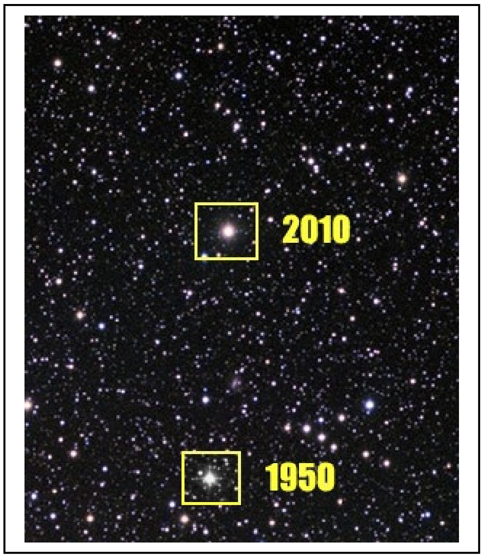

Stars travel along different orbits through the Milky Way. Near our sun, stars are going mostly in the same direction, but from time to time they pass close together. Barnard's Star is currently in the constellation Ophiuchus, but travels across the sky so quickly that it traverses the diameter of the full moon every 180 years.

With the sun at the center of a Cartesian coordinate grid, Barnard's Star can be represented as a point located at (2.0, 5.6) where the units are in light years. But because it is moving through space at a speed of 143 km/sec, its future position relative to the sun changes quickly in time.

The parametric equations for the X and Y location of Barnard's Star can be approximated as follows:

$$X(T) = 2.0 + 0.09\,T \qquad Y(T) = 5.67 - 0.25\,T$$

where T is in thousands of years from the present time, and all units are in light years.

Problem 1 – What is the distance to Barnard's Star at the present time?

Problem 2 – By using the differential calculus, what is the equation of the line y = Mx + B that is represented by the parametric functions?

Problem 3 – By using the Pythagorean distance formula and using the differential calculus to find the minimum distance, at what time, T will Barnard's Star be closest to the sun along this trajectory?

Problem 1 – What is the distance to Barnard's Star at the present time?

Answer: d = ($2.0^{\mathbf{2}} + 5.67^{\mathbf{2}})^{\mathbf{1/2}}$ = 6.0 light years.

Problem 2 – What is the equation of the line Y = Mx + B that is represented by the parametric functions?

$$M = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$ so M = -0.25/0.09 and so M = -2.8

Since a point on this line is at (2.0, 5.6) we have y – 5.6 = M(x-2.0)
Then y = 5.6 -2.8x +5.6
Therefore **y = 11.2 – 2.8x**

Problem 3 – At what time, T, will Barnard's Star be closest to the sun along this trajectory?

$D(T)^2 = (2.0 + 0.09T)^2 + (5.67 – 0.25T)^2$

Take the derivative to get $2D\frac{dD}{dt}$ = 2 (2.0 + 0.09T)(0.09) + 2(5.67-0.25T)(-0.25)

This simplifies to $D\frac{dD}{dt}$ = 0.18 + 0.0081T -1.42 + 0.0625T = -1.24 + 0.0706T

Set this equal to zero to get 0 = -1.24 + 0.0706T then **T = 17.6**.

So in about 18,000 years from now, Barnard's Star will be at its closest to the sun. Its distance at this time will be $d^{\mathbf{2}} = (2.0+0.09*17.6)^{\mathbf{2}} + (5.67 – 0.25*17.6)^{\mathbf{2}}$ = 14.5 so d= **3.8 light years**.