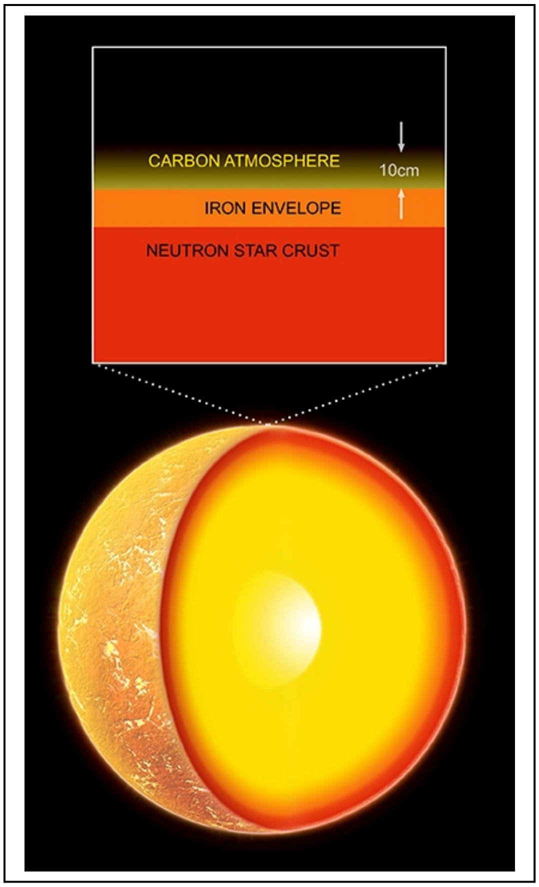

The atmosphere of Earth has a thickness of over 100 kilometers, however the Chandra X-Ray Observatory recently detected an atmosphere on the neutron star Cassiopeia-A that measured only 10 centimeters thick. How can a small planet have a deeper atmosphere than an entire stars-worth of matter?

The size of an atmosphere depends on a balance between the force of gravity pulling it towards the center of the planet, and the pressure of the atmosphere due to its temperature and density, pushing in the opposite direction. A pair of simple equations then defines how the density of the atmosphere has to rearrange itself with height above the surface so that gravity and pressure are always in balance. The equations look like this:

$$n(z) = n_0 e^{-\frac{z}{H}} \quad \text{where} \quad H = \frac{kT}{mg}$$

The exponential equation says that as you get farther from the surface, the density of the gas, N, drops very fast. The quantity, H in meters, is called the 'scale height' and its value is defined by the atmosphere's temperature, T, in Kelvins, and the acceleration of gravity at the surface, g, in multiples of Earth's acceleration (9.8 meters/sec^2). It also depends on the average mass, m, of the particles in the atmosphere. A light atmosphere made from hydrogen (m=1) will produce a value for H that is much larger than one made from pure oxygen (m=16). In this equation, k is Boltzman's Constant and equals 1.38 x 10^{-23} Joules/degree.

Problem 1 – Graph the density of Earth's atmosphere using H = 8 kilometers and n = 0.0013 grams/cc.

Problem 2 – The surface acceleration of the neutron star is 100 billion times that of Earth, the temperature of the gas is 3 million Kelvins compared to Earth's of 300 Kelvins, and the neutron star atmosphere is composed of carbon (A=12) rather than Earth's mixture of nitrogen and oxygen (A=28). From the formula for H, and the way in which it scales with m, T and g, what would you predict as the scale height for the neutron star atmosphere if for Earth, H = 8 kilometers?

Problem 3 – How far from the surface would you have to travel in order for the density of the atmosphere to fall by 1 million times for: A) Earth and B) the neutron star?

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Answer: This is an exercise in scaling and proportionality using equations and their variables. H is proportional to T and inversely proportional to the product of m and g, so that means that if we double only T, H increases by a factor of 2; if we double only g, H decreases by a factor of 2; and if we double only the mass of the atmosphere particles, H decreases by a factor of 2. Combining these proportionalities and starting with the value of H=8 kilometers for Earth, we have that T increases by 3 million/300 = 10,000 times; m decreases by 12/28 = 0.4 and g increases by 100 billion g/1g = 100 billion times. The new scale height is then H = 8 kilometers x (10,000)/(0.4 x 100 billion) **= 0.2 centimeters**.

Problem 3 – How far from the surface would you have to travel in order for the density of the atmosphere to fall by 1 million times for: A) the Earth and B) the neutron star?

Answer: We use the equation for the atmosphere density N = n0 e-x/H and determine for what value of the height, x, that $e^{-x/H}$ = 1/1000000. We can solve the equation as follows:

Take reciprocals of both sides:	1,000,000 = e(x/h)
Take the log, base-e, of both sides	13.8 = x/H
Solve for x	13.8 H = x

A) For Earth, H = 8 kilometers, so 13.8 (8 kilometers) = x
x = **110 kilometers!**

B) For the neutron star, H = 0.13 centimeters so 13.8 (0.2 cm) = x
x = **2.8 centimeters.**

Note to teacher: At the densities of the neutron star atmosphere (3.5 grams/cc) the carbon is not a gas but acts more like a crystalline solid so that the thickness, H, will not be completely determined by the equilibrium equations we have been using for Earth's atmosphere.