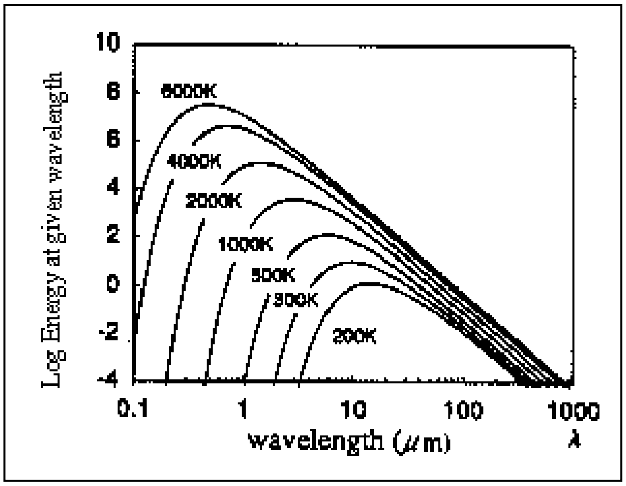
Star Light…Star Bright



Astronomers can 'take the temperature' of a star by measuring the star's brightness through two filters that pass radiation in the 'blue' and 'visual' regions of the visible spectrum.

From the ratio of these brightnesses, a simple cubic relationship yields the temperature of the star, in Kelvin degrees as follows:

$$T(x) = 9391 – 8350\,x + 5300\,x^2 – 1541\,x^3$$

This formula works for star temperatures between 9,000 and 3,500 Kelvin.

Problem 1 - From the indicated temperature range, what is the domain of this function?

Problem 2 – The sun has a temperature of 5770 K. What is the corresponding value for x?

Problem 3 – To save computation time, an astronomer uses the approximation for T(x) based on a quadratic formula given by $F(x) = 1844x^2 – 6410x +9175$. What is the formula that gives the percentage error, P(x), between F(x) and T(x)?

Problem 4 – A what temperature, T(x), does P(x) defined in Problem 3 have its maximum absolute percentage error in the domain of x: [0,1.4], and what is this value?

Problem 1 - From the indicated temperature range, what is the domain of this function?
Answer: We have to solve T(x) = 9,000 and T(x) = 3,500. This can be done using a graphing calculator by programming T(X) into an Excel spreadsheet. Acceptable answers for [9000,3500] should be near **[0.06, 1.50]**

Problem 2 – The sun has a temperature of 5770 K. What is the corresponding value for x? Answer: T(x) = 5770, so **x = 0.67**

Problem 3 – To save computation time, an astronomer uses the approximation for T(x) based on a quadratic formula given by $F(x) = 1844x^2 - 6410x + 9175$. What is the formula that gives the percentage error, P(x) between F(x) and T(x)?

Answer: P(x) = 100% x (estimate – actual) / actual
= 100 (F(x)-T(x)) / T(x)

$$= \frac{100(1844x^2 - 6410x + 9175 - (9391 - 8350\,x + 5300\,x^2 - 1541\,x^3))}{9391 - 8350\,x + 5300\,x^2 - 1541\,x^3}$$

$$= \frac{100\,[1541\,x^3 - 3456\,x^2 + 1940\,x - 216]}{9391 - 8350\,x + 5300\,x^2 - 1541\,x^3}$$

Problem 4 – A what temperature, T(x), does P(x) defined in Problem 3 have its maximum absolute percentage error in the domain of x: [0,1.4], and what is this value?

Answer: Students can use a graphing calculator to display this function, or use Excel to program and plot it, to manually determine the maximum. The graph shows a local minimum of -4.6% at x=1.1, and a local maximum of +1.55% at x=0.4. The absolute maximum error is then 4.6% at x=1.1. The function T(X) evaluated at x=1.1 yields **T = 4,600 K**.