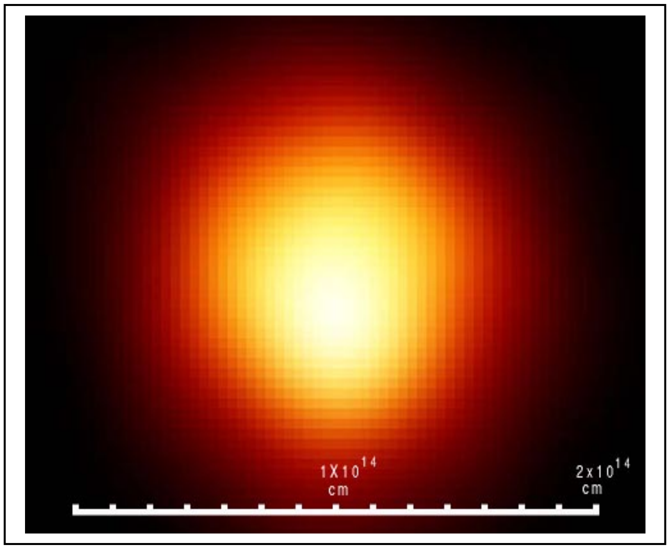

The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun!

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4 \pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzman radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in degrees Kelvin. The formula then becomes,

$$L = 4 \pi R^2 \sigma T^4$$

Problem 1 - The Sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

Problem 2 - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

Problem 3 - The nearby star, Sirius, has a temperature of 9,200 K and a radius of 1.76 times our Sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our Sun?

Calculus*:*

Problem 4 - Compute the total derivative of L(R,T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Answer Key

Problem 1 - We use $L = 4 (3.141) R^2 (5.67 \times 10^{-5}) T^4$ to get L (ergs/sec) = $0.00071\ R(cm)^2\ T(degreesK)^4$ then,

A) L(ergs/sec) = $0.00071 \times (696{,}000\ km \times 10^5\ cm/km)^2 (5700)^4 =$ **3.6×10^{33} ergs/sec**

B) L(watts) = 3.6×10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6×10^{25} watts**.

Problem 2 - A) The radius of Antares is $700 \times 696{,}000\ km = 4.9 \times 10^8$ km.
L(ergs/sec) = $0.00071 \times (4.9 \times 10^8\ km \times 10^5\ cm/km)^2 (3500)^4 =$ **2.5×10^{38} ergs/sec**
B) L(Antares) = (2.5×10^{38} ergs/sec) / (3.6×10^{33} ergs/sec) = **71,000 L(sun).**

Problem 3 - Sirius-A radius = $1.76 \times 696{,}000\ km = 1.2 \times 10^6$ km
L(Sirius-A) = $0.00071 \times (1.2 \times 10^6\ km \times 10^5\ cm/km)^2 (9200)^4 =$ **7.3×10^{34} ergs/sec**
L = (7.3×10^{34} ergs/sec) / (3.6×10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = $0.00071 \times (4900\ km \times 10^5\ cm/km)^2 (27{,}400)^4 =$ **9.5×10^{31} ergs/sec**
L(Sirius-B) = 9.5×10^{31} ergs/sec / 3.6×10^{33} ergs/sec = **0.026 L(sun).**

Advanced Math:

Problem 4 (Note: In the discussion below, the symbol *d* represents a partial derivative)

$$dL(R,T) = \frac{dL(R,T)}{dR}\, dR + \frac{dL(R,T)}{dT}\, dT$$

$$dL = [\, 4 \pi\ (2)\ R \sigma T^4\,]\ dR + [\, 4 \pi\ (4)\ R2 \sigma T^3\,]\ dT$$

$$dL = 8 \pi\ R \sigma T^4\ dR + 16 \pi\ R^2 \sigma T^3\ dT$$

To get percentage changes, divide both sides by $L = 4 \pi R^2 \sigma T^4$

$$\frac{dL}{L} = \frac{8 \pi\ R \sigma T^4}{4 \pi\ R^2 \sigma T^4}\, dR + \frac{16 \pi\ R^2 \sigma T^3}{4 \pi\ R^2 \sigma T^4}\, dT$$

Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40 The star's luminosity will increase by 40%.**

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0. This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 71,000 L(sun) or 2.5×10^{38} ergs/sec. If I increase its radius by 10% from 4.9×10^8 km to 5.4×10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = $0.00071 \times (5.4 \times 10^8\ km \times 10^5\ cm/km)^2 (3325)^4 =$ **2.5×10^{38} ergs/sec**