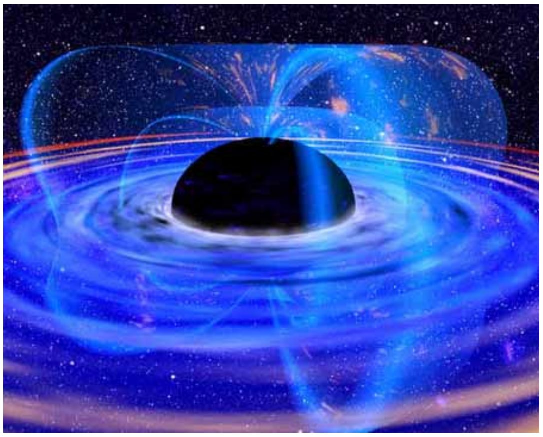
Black Holes....III



Artists illustration of a black hole with an orbiting disk of gas and dust. Friction in the disk causes matter to steadily flow inwards until it reaches the black hole event horizon. Magnetic forces in the disk cause matter to flow in complex jets and plumes. Time dilation causes delays in events taking place near the black hole compared to what distant observers will record.

Time dilation near a black hole is a lot more extreme than what the GPS satellite network experiences in orbit around Earth (See Problem 29).

$$T = t\sqrt{1 - \frac{2GM}{Rc^2}}$$

T = the time measured by someone located on a planet (seconds)

t = the time measured by someone located in space (seconds)

M = the mass of the planet (grams)

R = the distance to the far-away observer from the planet (cm)

And the natural constants are:
$G = 6.67 \times 10^{-8}$
$C = 3 \times 10^{10}$

Problem 1 - In the time dilation formula above, evaluate the quantity $2\,G\,M\,/c^2$ for a black hole with a mass of one solar mass (1.9×10^{33} grams), and convert the answer to kilometers.

Problem 2 - Re-write the formula in a more tidy form using your answer to Problem 1.

Problem 3 - In the far future, a scientific outpost has been placed in orbit around this solar-mass black hole at a distance of 10 kilometers. What will the time dilation factor be at this location?

Problem 4 - A series of clock ticks were sent out by the satellite once each hour .What will be the time interval between the clock ticks by the time they reach a distant observer?

Problem 5 - If one tick arrived at 1:00 PM at the distant observer, when will the next clock tick arrive?

Problem 6 - A radio signal was sent by the black hole outpost to a distant observer. At the frequency of the signal, when transmitted from the outpost, the individual wavelengths take 0.000001 seconds to complete one cycle. From your answer to Problem 3, how much longer will they take by the time they arrive at the distant observer?

Answer Key:

Problem 1 - In the time dilation formula above, evaluate the quantity 2 G M /c^2 for a black hole with a mass of one solar mass (1.9 x 10^{33} grams), and convert the answer to kilometers.
Answer - 2 x 6.67 x 10^{-8} x 1.9 x 10^{33} / (3 x $10^{10})^2$ = 281,600 centimeters or 2.82 kilometers.

Problem 2 - Re-write the formula in a more tidy form using your answer to Problem 1.

Answer -

$$T = t \ (1 - 2.82/R)^{1/2}$$

where R is in units of kilometers.

Problem 3 - In the far future, a scientific outpost has been placed in orbit around this solar-mass black hole at a distance of 10 kilometers. What will the time dilation factor be at this location?

Answer - $(1 - 2.82/10)^{1/2} = (0.718)^{1/2} = 0.847$

Problem 4 - A series of clock ticks were sent out by the satellite once each hour .What will be the time interval between the clock ticks by the time they reach a distant observer?

Answer - Time interval = 3600 / 0.847 = 4,250 seconds.

Problem 5 - If one tick arrived at 1:00 PM at the distant observer, when will the next clock tick arrive?

Answer - 1:00 PM + 4250 seconds = 1:00 PM + 1 Hour + (4250-3600) = 2:00 PM + 650 seconds = 2:10:50 PM

Problem 6 - A radio signal was sent by the black hole outpost to a distant observer. At the frequency of the signal, when transmitted from the outpost, the individual wavelengths take 0.000001 seconds to complete one cycle. From your answer to Problem 3, how much longer will they take by the time they arrive at the distant observer?

Answer - 0.000001 seconds / 0.847 = 0.00000118 seconds.