Investigating the Elliptical Orbit of Juno around Jupiter

The Juno spacecraft, launched August 5, 2011, will arrive at Jupiter in early-July, 2016. Upon arrival, it will be placed in an elliptical polar orbit, where it will remain until the end of the mission after 33 orbits. The spacecraft will then dive into Jupiter’s atmosphere and burn-up.

Problem 1 - For convenience, distances around Jupiter are given in multiples of Jupiter’s radius, which is 71,600 km. This unit is called RJ, so that, for example, a distance of 3.0 RJ corresponds to 3 x (71,600 km) or 214,800 km. If the formula for Juno’s elliptical orbit is given by \( 38x^2 + 400y^2 = 15,200 \) where \( x \) and \( y \) are in units of RJ, what is the equation for Juno’s orbit written in Standard Form for an ellipse?

Problem 2 - The periJovium, \( Rp \), is the closest distance from the orbit to the center of Jupiter, while the apoJovium, \( Ra \), is the farthest distance. Use the following properties of an ellipse to determine, \( Rp \), \( Ra \) and the eccentricity, \( e \), of Juno’s orbit, where \( f \) is the shortest distance from the focus to the ellipse and

\[
\begin{align*}
    b &= a \left(1-e^2\right)^{1/2} \\
    Ra &= a + f \\
    Rp &= a-f \\
    f &= ae
\end{align*}
\]

Problem 3 - According to Kepler’s Third Law, the period, \( P \), in days for a body in an orbit around Jupiter is related to its semi-major axis, \( a \), in RJ given by

\[
a^3 = 66.5P^2
\]

A) To the nearest day, what is the orbit period of Juno in days?
B) How long will the spacecraft remain in orbit before atmospheric entry?

Space Math
http://spacemath.gsfc.nasa.gov
Problem 1 - For convenience, distances around Jupiter are given in multiples of Jupiter’s radius, which is 71,600 km. This unit is called RJ, so that, for example, a distance of 3.0 RJ corresponds to 3 x (71,600 km) or 214,800 km. If the formula for Juno’s elliptical orbit is given by $38x^2 + 400y^2 = 15,200$ where $x$ and $y$ are in units of RJ, what is the equation for Juno’s orbit written in Standard Form for an ellipse?

Answer: Divide both sides by 15,200 and simplify to the Standard Form:

$$\frac{38}{15200}x^2 + \frac{400}{15200}y^2 = 1$$

so in Standard Form:

$$\frac{x^2}{20^2} + \frac{y^2}{6.2^2} = 1$$

The semi-major axis is $a = 20$ RJ and is along the $x$-axis. The semi-minor axis is $b = 6.2$ RJ and is along the $y$-axis.

Problem 2 - The periJovium, $R_p$, is the closest distance from the orbit to the center of Jupiter, while the apoJovium, $R_a$, is the farthest distance. Use the following properties of an ellipse to determine, $R_p$, $R_a$ and the eccentricity, $e$, of Juno’s orbit, where $f$ is the shortest distance from the focus to the ellipse and

$$b = a \left(1 - e^2\right)^{1/2}$$

$$R_a = a + f$$

$$R_p = a - f$$

$$f = ae$$

Answer: From the first equation, $a$ and $b$ are known so solve for $e$ to get $e = 0.95$. Use the fourth equation to solve for $f$ to get $f = 19.0$ RJ. Then use the second and third equations to find the periJovium and apoJovium as $R_a = 1.0$ RJ and $R_p = 39$ RJ.

Problem 3 - According to Kepler’s Third Law, the period, $P$, in days for a body in an orbit around Jupiter is related to its semi-major axis, $a$, in RJ given by

$$a^3 = 66.5P^2$$

A) To the nearest day, what is the orbit period of Juno in days?

B) How long will the spacecraft remain in orbit before atmospheric entry?

Answer: A) Since $a = 20$ RJ, so solving for $P$ we get $P = 10.96$ days or 11 days. B) The information states that after 33 orbits it will be directed to enter the Jovian atmosphere to burn up. The mission lifetime is then $T = 33 \times 11$ days = 330 days.

Note: Due to the intense ‘van Allen’ radiation belts around Jupiter, the spacecraft’s electrical systems will not survive intact for more than a year so no useful data will be returned for a prolonged stay. Meanwhile, Atmospheric entry will allow the instruments to measure the Jovian atmosphere before they start to fail.

Space Math http://spacemath.gsfc.nasa.gov