The speed of the Juno spacecraft in its elliptical orbit around the sun is given by two equations. The first one specifies the spacecraft location in its orbit given by

$$r(\theta) = \frac{5}{3 - 2 \cos \theta}$$

The second equation is the speed approximately given for the Juno spacecraft by

$$s = 40 \sqrt{\frac{2}{r} - \frac{1}{3}}$$

where $s$ is in kilometers/sec, $r$ is Juno's distance from the sun in AUs, and $\theta$ is the orbital angle.

**Problem 1** – Combining the two equations, and evaluating the constant to the nearest km/s, what is the function specifying the spacecraft speed defined solely in terms of the angle parameter $s(\theta)$?

**Problem 2** - About how far from the sun will Juno be in its orbit for A) $\theta=0$? B) $\theta=180$?

**Problem 3** - About what will Juno's orbital speed be when it arrives at Jupiter?

**Problem 4** - For what value of $\theta$ will A) the spacecraft speed be about 25 km/s, and B) how far will it be from the sun at that time?
**Problem 1** – Combining the two equations, and evaluating the constant to the nearest km/s, what is the function specifying the spacecraft speed defined solely in terms of the angle parameter $s(\theta)$?

\[
s = 40 \sqrt{\frac{2}{(5 / (3 - 2 \cos \theta))}} - \frac{1}{3}
\]

\[
s = 40 \sqrt{\frac{6(3 - 2 \cos \theta) - 5}{15}}
\]

\[
s = \frac{8 \sqrt{15}}{3} \sqrt{13 - 12 \cos \theta}
\]

\[
s = 10 \sqrt{13 - 12 \cos \theta}
\]

Note: $8(15)^{1/2}/3 = 10.33$ which to the nearest km/sec becomes 10

**Problem 2** - About how far from the sun will Juno be in its orbit for A) $\theta=0$? B) $\theta=180$?

Answer: A) *5 Astronomical Units*. B) *1 Astronomical Unit*.

**Problem 3** - About what will Juno’s orbital speed be when it arrives at Jupiter?

Answer: $s = 40 \sqrt{\left(\frac{2}{5} - \frac{1}{3}\right)}$ so $s = 10$ km/sec.

**Problem 4** - For what value of $\theta$ A) will the spacecraft speed be about 25 km/s, and B) how far will it be from the sun at that time?

Answer: A)

\[
25 = 40 \sqrt{\left(\frac{2}{r} - \frac{1}{3}\right)}
\]

so solving for $r$ we get $r = 2.76$ Astronomical Units.

B) $2.76 = \frac{5}{3 - 2 \cos \theta}$ so $\cos \theta = 0.594$ and so $\theta = 54$ degrees.