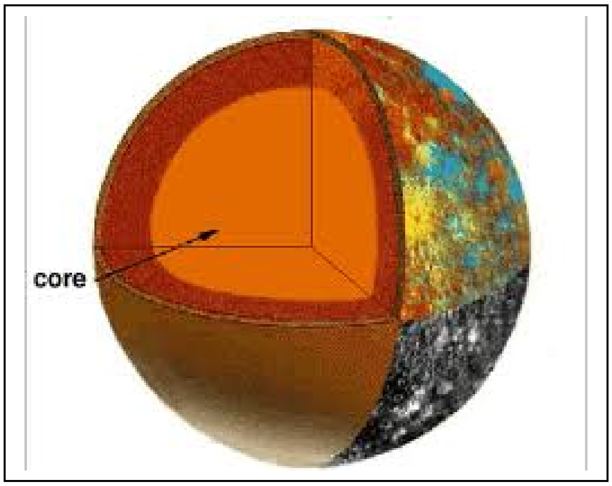

This MESSENGER spacecraft is in orbit around the planet Mercury, with an elliptical orbit period of 8 hours at an average distance of 7715 kilometers. At its closest distance it is only 320 km above the surface of Mercury, which allows the spacecraft to obtain high-resolution images of the mysterious surface of Mercury.

A detailed study of the spacecrafts speed and altitude also lets astronomers study the gravity field of Mercury and deduce something about its interior.

We can create a simple model of the interior of Mercury by dividing it into a spherical core region, and an overlying shell of matter that reaches to the observed surface of the planet. Here is how we do this!

Problem 1 - The mass of Mercury is 3.31×10^{23} kilograms. If the planet is a perfect sphere with a radius of 2,425 kilometers, what is the average density of the planet Mercury in kg/meter^3 defined as density = mass/volume?

Problem 2 - Astronomers believe that the crust of the planet has an average density of 3,000 kg/m^3 and the iron-rich core has a density of 7,800 kg/m^3.

A) What is the formula that gives the total mass of the core, if the core has a radius of Rc in meters?

B) What is the formula for the outer shell of Mercury if the density equals the density of the crust, and its inner radius is Rc and its outer radius is the actual radius of the planet of 2,425 kilometers?

Problem 3 - If the sum of the core and shell masses must equal the mass of the planet, what is the value for Rc, the radius of the core in kilometers, that leads to a solution for this simple model?

Problem 1 - The mass of Mercury is 3.31 x 10^{23} kilograms. If the planet is a perfect sphere with a radius of 2,425 kilometers, what is the average density of the planet Mercury in kg/meter^3 defined as density = mass/volume?

Answer: Volume = $\frac{4}{3}\pi R^3$

so Volume = $\frac{4}{3}(3.141)(2,425,000)^3$ = 5.97 x 10^{19} meters^3

Density = 3.31 x 10^{23} kilograms / 5.97 x 10^{19} meters^3 = **5542 kg/m^3**.

Problem 2 - Astronomers believe that the crust of the planet has an average density of 3,000 kg/m^3 and the iron-rich core has a density of 7,800 kg/m^3. A) What is the formula that gives the total mass of the core, if the core has a radius of Rc in kilometers? B) What is the formula for the outer shell of Mercury if the density equals the density of the crust, and its inner radius is Rc and its outer radius is the actual radius of the planet of 2,425 kilometers?

Answer A) M(core) = $\frac{4}{3}\pi(1000Rc)^3(7800)$ = 3.26x10^{13}Rc^3 kilograms

B) Volume of the spherical shell = $\frac{4}{3}\pi(2,425,000)^3 - \frac{4}{3}\pi(1000Rc)^3$ cubic meters

Then M(shell) = 3,000 x V(shell)

= $2,200\left(\frac{4}{3}\pi(2,425,000)^3 - \frac{4}{3}\pi(1000Rc)^3\right)$ kilograms

=$1.3x10^{23} - 9.21x10^{12}Rc^3$ kilograms

Problem 3 - If the sum of the core and shell masses must equal the mass of the planet, what is the value for Rc, the radius of the core in kilometers, that leads to a solution for this simple model?

Answer: 3.31 x 10^{23} = 3.26x10^{13}Rc^3 + 1.3x10^{23} - 9.21x10^{12} Rc^3

So 2.01x10^{23} = 2.34x10^{13} Rc^3

And so Rc = **2,046 kilometers!**

So the dense iron core of Mercury occupies 100%x(2046/2425) = 84% of the radius of Mercury! By comparison, Earth's core is only 30% of its radius.