



Saturn's third-largest moon Dione can be seen through the haze of its largest moon, Titan, in this view of the two posing before the planet and its rings from NASA's Cassini spacecraft. This view looks toward Titan (about 5000 kilometers across) with Saturn in the background, and the smaller moon Dione as it is about to be eclipsed by Titan. The images were obtained with the Cassini spacecraft narrow-angle camera on May 21, 2011 at a distance of approximately 1.4 million miles (2.3 million kilometers) from Titan.

**Problem 1** – From the image above, what is the ratio of the apparent diameter of Dione to the diameter of Titan rounded to the nearest, simple fraction (example:  $0.34 = 1/3$ )?

**Problem 2** - Suppose that Dione were exactly the same diameter, in kilometers, as Titan. From the vantage point of Cassini, about what would be the distance to Dione in kilometers?

**Problem 3** – The actual diameter of Dione is known to be about 1000 kilometers. What is its actual distance from Cassini at the time this image was taken? Explain your reasoning.

. Press Release: **NASA's Cassini Delivers Holiday Treats From Saturn**

December 22, 2011

[http://www.nasa.gov/mission\\_pages/cassini/whycassini/cassini20111222.html](http://www.nasa.gov/mission_pages/cassini/whycassini/cassini20111222.html)

**Problem 1** – From the image above, what is the ratio of the apparent diameter of Dione to the diameter of Titan rounded to the nearest, simple fraction (example:  $0.34 = 1/3$ )?

Answer: Use a millimeter ruler to measure the apparent diameters of the two moons. Titan is about 60 millimeters and Dione is about 9 millimeters in diameter. The ratio is  $9/60$  or  $3/20$ . Students should use  **$1/7$**  as 'simplist fraction' approximation.

**Problem 2** - Suppose the Dione were exactly the same diameter, in kilometers, as Titan. From the vantage point of Cassini, about what would be the distance to Dione in kilometers?

Answer: To appear as large as it seems to Cassini, Titan is at a distance of 2.3 million kilometers. If Dione were the same physical diameter as Titan ( 5000 kilometers) it would have to be 7 times farther from Cassini in order for the apparent diameters to be in the ration  $1/7$ . The distance to Dione is therefore  $7 \times 2.3$  million km or **16 million km**.

**Problem 3** – The actual diameter of Dione is known to be about 1000 kilometers. What is its actual distance from Cassini at the time this image was taken? Explain your reasoning.

Answer: If Titan and Dione were of comparable physical size, Dione would have to be 16 million km from Cassini in order for Dione to appear to be  $1/7$  the diameter of Titan.

But, Dione's physical diameter compared to Titan is 1000 km compared to 5000 km for Titan, to their actual diameter ratio when seen at the same distance from Cassini is  $1000/5000 = 1/5$ . If Dione and Titan were sitting next to each other, the ratio of their physical diameters alone would make the disk of Dione appear to be  $1/5$  the diameter of Titan or 12 millimeters instead of the 9 millimeters we measured.

To make the apparent ratio equal the slightly smaller ratio of  $1/7$  at a distance of 2.3 million km, we have to move Dione slightly further away from Cassini than Titan by an amount equal to the proportion:

$$\frac{(1/7)}{2.3\text{million}} = \frac{(1/5)}{X} \quad \text{so } x = 2.3 \text{ million} \times (1/5)/(1/7) = 2.3 \text{ million} \times 7/5 = \mathbf{3.2 \text{ million km}}$$

Note: even though Titan and Dione appear close together in the picture, they are actually over (3.2-2.3) million = 900,000 kilometers apart...about 3 times the distance from Earth to the Moon!