



The Cassini Division is easily seen from Earth with a small telescope, and splits the rings of Saturn into two major groups. A little detective work shows that there may be a good reason for this gap that involves Saturn's nearby moon, Mimas.

Mimas orbits Saturn once every 22 hours, and would-be particles in the Cassini Division would orbit once every 11-12 hours, so that the ratio of the orbit periods is close to 2 to 1. This creates a resonance condition where the gravity of Mimas perturbs the Cassini particles and eventually ejects them.

Imagine a pendulum swinging. If you lightly tap the pendulum when it reaches the top of its swing, and do this every other swing, eventually the small taps add up to increasing the height of the pendulum.

Problem 1 – The mass of Mimas is 4.0×10^{19} kilograms, and the distance to the center of the Cassini Division from Mimas is 67,000 kilometers. Use Newton's Law of Gravity to calculate the acceleration of a Cassini Division particle due to the gravity of Mimas if

$$\text{Acceleration} = G \frac{M}{R^2} \text{ in meters/sec}^2$$

where $G = 6.67 \times 10^{-11}$, M is the mass of Mimas in kilograms and R is the distance in meters.

Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

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$$\begin{aligned} \text{Answer: Acceleration} &= 6.67 \times 10^{-11} (4.0 \times 10^{19}) / (6.7 \times 10^7 \text{ meters})^2 \\ &= \mathbf{5.9 \times 10^{-7} \text{ meters/sec}^2} \end{aligned}$$

Problem 2 – The encounter time with Mimas is about 2 hours every orbit for the Cassini particles. If speed = acceleration x time, what is the speed increase of the particles after each 12-hour orbit?

$$\begin{aligned} \text{Answer: } 1 \text{ hour} &= 3600 \text{ seconds, so } 2 \text{ hours} = 7200 \text{ seconds and} \\ \text{speed} &= 5.9 \times 10^{-7} \text{ m/sec}^2 \times 7200 \text{ sec} \\ &= 4.2 \times 10^{-3} \text{ meters/sec per orbit.} \end{aligned}$$

Problem 3 – If a particle is ejected from the Cassini Division once its speed reaches 1 km/sec, how many years will it take for this to happen?

$$\begin{aligned} \text{Answer: } (1000 \text{ m/s}) / (0.0042 \text{ m/s}) &= 238000 \text{ orbits. Since } 1 \text{ orbit} = 12 \text{ hours, we have } 12 \times \\ 238,000 &= 2,856,000 \text{ hours or about } \mathbf{326 \text{ years}}. \end{aligned}$$