A prediction of Albert Einstein's relativistic theory of gravity says that the pointing direction of a spinning gyroscope orbiting a massive body should slowly change over time. For Earth, this amount equals degrees/year, and this was recently confirmed by NASA's Gravity Probe-B satellite in 2011.

Einstein's theory predicts much larger shifts if the satellite orbits close to our sun, or to a dense body such as a neutron star.

The effect is called 'frame dragging' and was first predicted in 1918 by Austrian physicists Josef Lense (1890-1985) and Hans Thirring (1888-1976) using Einstein's mathematical theory of gravity published in 1915. The rate, in degrees per second, at which the gyroscope pointing angle will change is given by the formula for $\Omega$, in degrees/sec, shown below:

$$\Omega = \frac{Rac}{r^3 + a^2 r + Ra^2} \left(\frac{360}{2\pi}\right)$$

where $R = \frac{2GM}{c^2}$ and $a = \frac{2Rs^2}{5c} \left(\frac{2\pi}{T}\right)$

and where $c$ is the speed of light (300,000,000 m/s), $R_s$ is the radius of the massive body in meters, $M$ is its mass in kilograms, $T$ is the satellite orbit period in seconds, and $G$ is the Newtonian Gravitational constant $6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$. For the GP-B satellite orbiting near Earth at an altitude of 700 km, the measured value for $\Omega$ is about $1.2 \times 10^{-5}$ degrees/year.

**Problem 1** - In the future, physicists might like to verify this effect near the sun by placing a satellite in a circular orbit at a distance of 10 million kilometers ($r = 10^{10}$ meters). If the radius of the sun is $R_s = 6.96 \times 10^8$ meters, and its rotation period is $T = 24.5$ days, and the mass of the sun is $M = 2.0 \times 10^{30}$ kg. To two significant figures, what is the value for the Lens-Thirring rate, $\Omega$, in degrees/year? (Note: 1 degree = 3600 arcseconds)

**Problem 2** - A neutron star is the compressed nuclear core of a massive star after it has become a supernova. Suppose the mass of a neutron star is equal to our sun, its radius is 12 kilometers, a gyroscope orbits the neutron star at a distance from its center of $r = 6,000$ kilometers, and its orbit period is $T = 8$ seconds. To two significant figures, what is $\Omega$ for such a dense, compact system in degrees/year?
**Problem 1** - In the future, physicists would like to verify this effect near the sun by placing a satellite in a circular orbit at a distance of 10 million kilometers ($r = 10^{10}$ meters). The radius of the sun is $R_s = 6.96 \times 10^8$ meters, and its rotation period is $T = 24.5$ days, and the mass of the sun is $M = 2.0 \times 10^{30}$ kg. To two significant figures, what is the value for the Lens-Thirring rate, $\Omega$, in degrees/year?

$$ R = \frac{2(6.67 \times 10^{-11})(2.0 \times 10^{30})}{(300,000,000)^2} = 2,964 \text{ m} \quad a = \frac{2(6.96 \times 10^8)^2}{5(300,000,000)} \left(\frac{2(3.141)}{24.5(24)3600}\right) = 1,883 \text{ m} $$

then

$$ \Omega = \frac{(2964)(1883)(3 \times 10^8)}{(10^{10})^3 + 1883^2(10^{10}) + (2964)(1883)^2 \left(\frac{360}{2(3.14)}\right)} = 9.60 \times 10^{-14} \text{ degrees/sec} $$

$$ \Omega = 9.6 \times 10^{-14} \text{ deg/sec} \times (365 \text{d}/1\text{yr}) \times (24 \text{h}/1\text{day}) \times (3600 \text{ s}/1 \text{hr}) = 3.0 \times 10^{-7} \text{ deg/yr} $$

Note, for GP-B the effect near Earth was $1.2 \times 10^{-5}$ degrees/year because GP-B was orbiting closer to the mass of Earth than our hypothetical satellite around the sun.

**Problem 2** - A neutron star is the compressed nuclear core of a massive star after it has become a supernova. Suppose the mass of a neutron star is equal to our sun, its radius is 12 kilometers, a gyroscope orbits the neutron star at a distance from its center of $r = 6,000$ kilometers, and its orbit period is $T = 8$ seconds. To two significant figures, what is $\Omega$ for such a dense, compact system in degrees/year?

$$ R = \frac{2(6.67 \times 10^{-11})(2.0 \times 10^{30})}{(300,000,000)^2} = 2,964 \text{ meters} \quad a = \frac{2(12,000)^2}{5(300,000,000)} \left(\frac{2(3.141)}{8.0}\right) = 0.15 \text{ meters} $$

then

$$ \Omega = \frac{(2964)(0.15)(3 \times 10^8)}{(6.0 \times 10^6)^3 + (0.15)^2(6.0 \times 10^6) + (4150)(0.15)^2 \left(\frac{360}{2(3.14)}\right)} $$

$$ \Omega = \frac{(1.33 \times 10^{11})}{(2.16 \times 10^{20}) + (1.35 \times 10^3) + (93.4) \left(\frac{360}{6.242}\right)} = 3.65 \times 10^{-8} \text{ degrees/sec} $$

$$ \Omega = 3.65 \times 10^{-8} \text{ deg/sec} \times (365 \text{d}/1\text{yr}) \times (24 \text{h}/1\text{day}) \times (3600 \text{ s}/1 \text{hr}) = 1.1 \text{ deg/yr} $$

Note this is nearly 100,000 times the corresponding Lens-Thirring rate near Earth.

For a detailed discussion of the derivation of the formula for $\Omega$ in the equatorial plane of a spinning body, see Wikipedia:

http://en.wikipedia.org/wiki/Frame-dragging

Space Math http://spacemath.gsfc.nasa.gov