According to geophysicist Richard Gross at NASA's Jet Propulsion Laboratory in Pasadena, California, the March 11, 2011 Japan Quake caused the rotation of Earth to speed up by about 1.8 millionths of a second (that's 0.0000018 seconds). This doesn't sound like much, but in today's high-tech world where time is regularly measured in billionths of a second, this is a huge change!

The reason this happens is similar to a spinning ice skater pulling in her arms during a spin. The mass in her hands brought closer to her spin axis causes her to spin faster in order to conserve momentum. Earthquakes may move mass, like the island of Japan, slightly closer to the center of Earth and so it will also spin up to conserve momentum. Japan was moved around by nearly 4 meters, which changed the way matter was distributed in Earth's crust (the ice skater's hands) in a significant way. How does this work? Here is a simple model.

Problem 1 - The formula for the angular momentum of a uniform sphere is just

$$J = \frac{2}{5} M r^2 \omega$$

If the mass, M, is conserved and the angular momentum, J, is conserved, what is the formula for the initial radius, r, and angular velocity, ω, compared to the final radius and angular velocity?

Problem 2 - If the Earth rotates 2π radians in exactly 24 hours, with a radius of 6378.00 km, what will its final angular velocity be in radians/sec, after it has shrunk by 1.00 kilometer?

Problem 3 - What will be the difference in its rotation period in seconds?

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Answer: $r_i^{\,2}\omega_i = r_f^{\,2}\omega_f$

Problem 2 - If the Earth rotates 2π radians in exactly 24 hours, with a radius of 6378 km, what will its final angular velocity be after its has shrunk by 1 kilometer?

R_i = 6378 km ω_i = 6.28318/86400sec = 7.27220x10^{-5} radians/sec

R_f = 6377 km, then solve for w_f

$$\frac{(6378)^2 (7.27220x10^{-5})}{(6377)^2} = \omega_f$$

ω_f = 7.274481 x 10^{-5} radians/sec.

Problem 3 - What will be the difference in its rotation period?

Answer: The difference will be

$$P = \frac{2\pi}{\omega_f} - \frac{2\pi}{\omega_i}$$

$$P = \frac{6.2831853}{\left(7.274481 \text{ x } 10^{-5}\right)} - \frac{6.2831853}{\left(7.27220\text{x}10^{-5}\right)}$$

P = 86372.970 seconds - 86400.000 seconds
P = -27.0 seconds.

So the 'toy' Earth will have a day that is 27 seconds shorter.

Note: The Japan quake is a more complex type of crustal motion, but the basic principle is the same.