The neat thing about ballistic problems (flying baseballs or rockets) is that their motion in the vertical dimension is independent of their motion in the horizontal dimension. This means we can write one equation that describes the movement in time along the x axis, and a second equation that describes the movement in time along the y axis. In function notation, we write these as y(t) and y(t) where t is the independent variable representing time.

To draw the curve representing the trajectory, we have a choice to make. We can either create a table for X and Y at various instants in time, or we can simply eliminate the independent variable, t, and plot the curve y(x).

Problem 1 - The Ares 1X underwent powered flight while its first stage rocket engines were operating, but after it reached the highest point in its trajectory (apogee) the Ares 1X capsule coasted back to Earth for a water landing. The parametric equations that defined its horizontal downrange location (x) and its altitude (y) in meters are given by

$$x(t) = 64,000 + 1800t$$
$$y(t) = 45,000 - 4.9t^2$$

Using the method of substitution, create the new function y(x) by eliminating the variable t.

Problem 2 - Determine how far downrange from launch pad 39A at Cape Canaveral the capsule landed, (y(x)=0), giving your answer in both meters and kilometers to two significant figures.

Problem 3 - Why is it sometimes easier not to work with the parametric form of the motion of a rocket or projectile?

Problem 1 - Answer: The parametric equations that defined its horizontal downrange location (x) and its altitude (y) in meters are given by

$$x(t) = 64,000 + 1800t$$
$$y(t) = 45,000 - 4.9t^2$$

We want to eliminate the variable, t, from y(t) and we do this by solving the equation x(t) for t and substituting this into the equation for y(t) to get y(t(x)) or just y(x).

$$t = \frac{x - 64,000}{1800}$$

$$y = 45,000 - 4.9\left[\frac{x - 64,000}{1800}\right]^2$$

$$y = 38,800 + 0.19x - 0.0000015x^2$$

Problem 2 - Determine how far downrange from the launch pad the capsule landed, (y(x)=0), giving your answer in both meters and kilometers to two significant figures. Answer: Using the Quadratic Formula, find the two roots of the equation y(x)=0, and select the root with the largest positive value.

$$x_{1,2} = \frac{-0.19 \pm \sqrt{0.036 - 4(-0.0000015)38800}}{2(-0.0000015)} \text{ meters}$$

so x1 = -109,000 meters or -109 kilometers
x2 = +237,000 meters or 237 kilometers.

The second root, x2, is the answer that is physically consistent with the given information. Students may wonder why the mathematical model gives a second answer of -109 kilometers. This is because the parabolic model was only designed to accurately represent the physical circumstances of the coasting phase of the capsule's descent from its apogee at a distance of 64 kilometers from the launch pad. Any extrapolations to times and positions earlier than the moment of apogee are 'unphysical'.

Problem 3 - Why is it sometimes easier not to work with the parametric form of the motion of a rocket or projectile? Answer: In order to determine the trajectory in space, you need to make twice as many calculations for the parametric form than for the functional form y(x) since each point is defined by (x(t), y(t)) vs (x, y(x)).