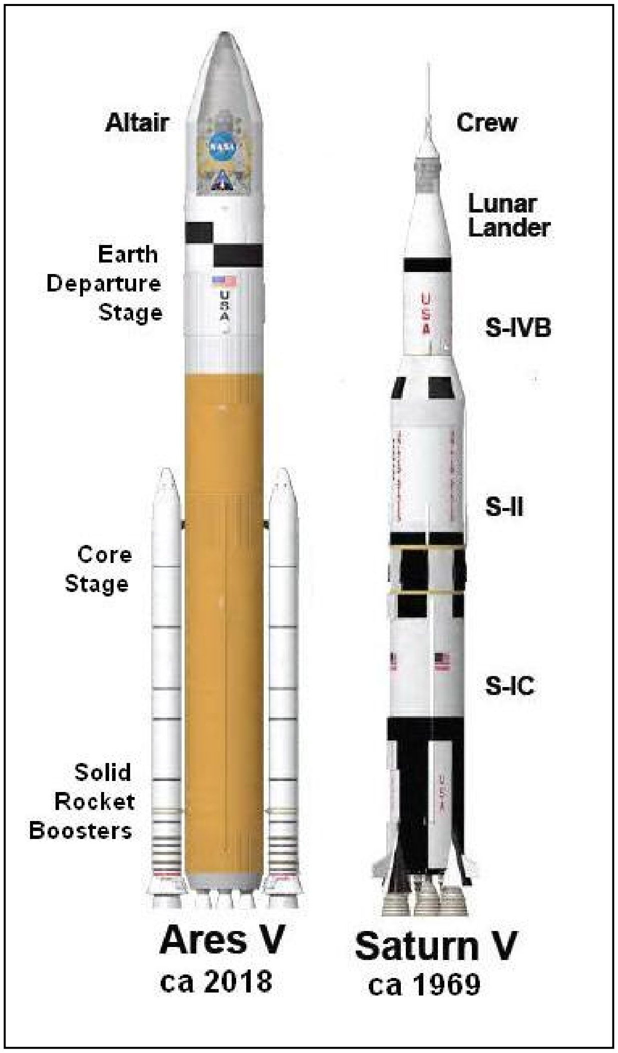
The Ares-V Cargo Rocket



The Ares-V rocket, now being developed by NASA, will weigh 3,700 tons at lift-off, and be able to ferry 75 tons of supplies, equipment and up to 4 astronauts to the moon. As a multi-purpose launch vehicle, it will also be able to launch complex, and very heavy, scientific payloads to Mars and beyond. To do this, the rockets on the Core Stage and Solid Rocket Boosters (SRBs) deliver a combined thrust of 47 million Newtons (11 million pounds). For the rocket, let's define:

T(t) = thrust at time-t
m(t) = mass at time-t
a(t) = acceleration at time-t

so that:

$$a(t) = \frac{T(t)}{m(t)}$$

The launch takes 200 seconds. Suppose that over the time interval [0,200], T(t) and m(t) are approximately given as follows:

$$T(x) = 8x^3 - 16x^2 - x^4 + 47$$

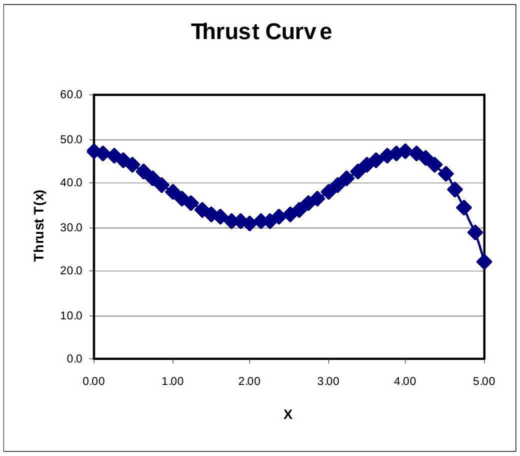
$$m(x) = 35 - x^2$$

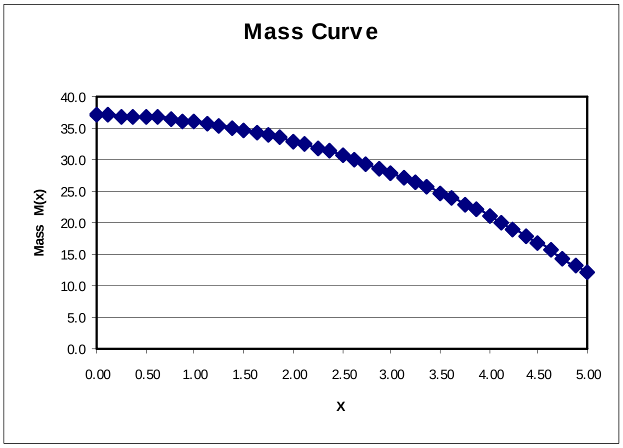
where t = 40 x

Problem 1 - Graph the thrust curve T(x), and the mass curve m(x) and find all minima, maxima inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 2 - Graph the acceleration curve a(x) and find all maxima, minima, inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? How many seconds will this be after launch? You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

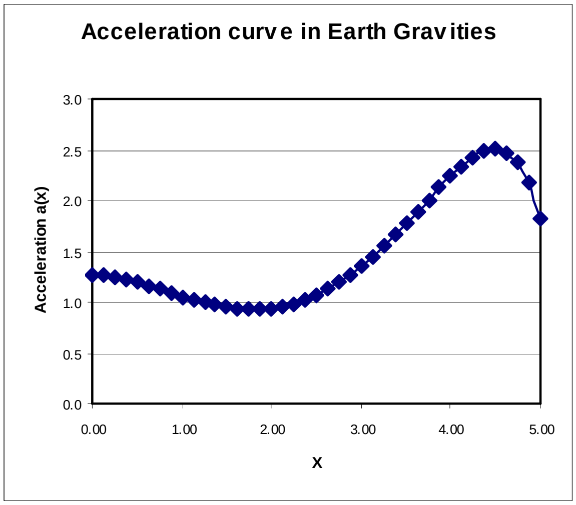




Problem 1 - The above graphs show T(x) and m(x) graphed with Excel. Similar graphs will be rendered using a graphing calculator. For the thrust curve, T(x), the relative maxima are at (0, 47) and (4,47). The relative minimum is at (2,31).

Problem 2 - For the mass curve, M(x), the absolute maximum is at (0,37).

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? You may use a graphing calculator, or Excel spreadsheet, or differential calculus. How many seconds will this be after launch?



Answer: The curve reaches its maximum acceleration near (4.5,2.5). Because t = 40 X, this occurs about 40 x 4.5 = 180 seconds after launch. ***Note to teacher:*** *The units for acceleration are in Earth Gravities (1 G = 9.8 meters/sec^2) so astronauts will feel approximately 2.5 times their normal weight at this point in the curve.* U sing the Quotient Rule in differential calculus, and setting da(x)/dx = 0 we get:

$$0 = \frac{2x^5 - 8x^4 - 140x^3 + 840x^2 - 1026x}{(35 - x^2)^2}$$

Although after factoring out 'x' from the numerator we see that X=0 is a trivial solution, the locations of the remaining two extrema near x=1.5-2.0 and x=4.0-5.0 have to be found by solving a 4th-order equation. A graphing calculator can be used to find these two points at x ~ 1.8 and x ~ 4.56 where da/dt ~ 0. Since t = 40x, we see that the <u>absolute maximum</u> acceleration occurs near t = 4.56 x 40 = 182 seconds after launch.