The Mass of the Moon

On July 19, 1969 the Apollo-11 Command Service Module and LEM entered lunar orbit. The orbit period was 2.0 hours, at a distance of 1,737 km from the lunar center.

Believe it or not, you can use these two pieces of information to determine the mass of the moon. Here's how it's done!

Problem 1 - Assume that Apollo-11 went into a circular orbit, and that the inward gravitational acceleration by the Moon on the capsule, Fg, exactly balances the outward centrifugal acceleration, Fc. Solve Fc = Fg for the mass of the Moon, M, in terms of V, R and the constant of gravity, G, given that:

$$F_g = \frac{G\,M\,m}{R^2} \qquad F_c = \frac{m\,V^2}{R}$$

Problem 2 - By using the fact that for circular motion, $V = 2\pi R / T$, re-express your answer to Problem 1 in terms of R, T and M.

Problem 3 - Given that $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-2}$, R = 1,737 km and T = 2 hours, calculate the mass of the Moon in kilograms!

Problem 4 - The mass of Earth is 5.97×10^{24} kg. What is the ratio of the Moon's mass, derived in Problem 3, to Earth's mass?

Problem 1 - From Fg = Fc, and a little algebra to simplify and cancel terms, you get

$$M = \frac{R V^2}{G}$$

Problem 2 – Substitute 2 π R/T for V and with a little algebra you get:

$$M = \frac{4\pi^2 R^3}{G T^2}$$

Problem 3 - First convert all units to meters and seconds: R = 1.737 x 10^6 meters and T = 7,200 seconds. Then substitute values into the above equation:

$M = 4 \times (3.14)^2 \times (1.737 \times 10^6)^3 / (6.67 \times 10^{-11} \times (7200)^2))$

$M = (39.44 \times 5.24 \times 10^{18}) / (3.46 \times 10^{-3})$

M = **5.97 x 10^{22} kg**

More accurate measurements, allowing for the influence of Earth's gravity and careful timing of orbital periods, actually yield 7.4 x 10^{22} kg.

Problem 4 - The ratio of the masses is 5.97 x 10^{22} kg / 5.97 x 10^{24} kg which equals **1/100.** The actual mass ratio is 1 / 80.