



The space between the planets is filled with fragments of asteroids, comets and material left over from the formation of the planets. These rocks and debris rain down upon exposed surfaces at speeds up to 30 km/sec.

The figure on the left summarizes the impact frequency of various sizes of particles in space. Note the graph is plotted in the Log-Log format due to the enormous range of masses and rates being described.

Understanding the data graph:

Problem 1 - A meteorite with a density of 3 grams/cm³ has a diameter of 4 centimeters (about 1 1/2 inches).

- A) What is the mass of this meteorite assuming it is a sphere?
- B) From the graph, where on the horizontal axis are objects of this mass located?
- C) What is the number of impacts per year you would expect over an area of 10,000 square kilometers?

Problem 2 – The function that best models the data in the graph is given by

$$N(m) = 0.025 m^{-0.9}$$

where N(m) is the number of impacts per square kilometer per year for objects with a mass of m grams. Using integral calculus, what is the total mass in tons of impacting objects each year, over the surface of Earth, in the mass range from 1 gram to 10²⁰ grams? (Use π = 3.14 and assume a spherical Earth with a radius of 6,378 km).

Problem 1 - Answers: A) Mass = Density x Volume. Radius of sphere = 2 cm, so $M = 3.0 \times (4/3) (3.14) (2)^3 = \mathbf{100 \text{ grams}}$. B) The horizontal axis is in units of Log(grams) so $\text{Log}(100) = 2$, and this is the location **half-way between 0' and '4' on the axis**. C) From 'x=2', a vertical line intercepts the data at about 'y=-3.5' on the vertical axis. This represents $\text{Log}(N) = -3.5$ so that $N = 0.00032 \text{ impacts/km}^2/\text{year}$. Over an area of 10000 km^2 , there would be an estimated $0.00032 \text{ impacts/km}^2/\text{year} \times (10000 \text{ km}^2) = \mathbf{3.2 \text{ impacts per year}}$.

Problem 2 –What is the total mass in tons of impacting objects each year, over the surface of Earth, in the mass range from 1 gram to 10^{20} grams? (Use $\pi = 3.14$ and assume a spherical Earth with a radius of 6,378 km).

Answer: The total mass is the area under the curve: $\text{Mass} = N(m) dm$

$$M = \int_1^{10^{20}} 0.025m^{-0.9} dm$$

$$= 0.025(-0.9)[(1)^{0.1} - (10^{20})^{0.1}]$$

$$= 0.025(0.9)(100^2)$$

$$= \mathbf{2.25 \text{ grams/km}^2/\text{year}}$$

$$\text{Area of Earth} = 4\pi(6378)^2 = 5.1 \times 10^8 \text{ km}^2$$

So the total meteoritic mass per year is

$$2.25 \text{ grams/km}^2 \times 5.1 \times 10^8 \text{ km}^2 = 1.15 \times 10^9 \text{ grams}$$

or $1.15 \times 10^6 \text{ kg}$

or **1,150 tons**.

Note: Popular estimates range from 20,000 to 100,000 tons/year. The amount is sensitive to both the logarithmic function used to model the power-law data, and the integration limits!