Investigating Opacity and Extinction

Opacity is a term used to describe the difficulty for light to travel through a medium. A rain cloud with high opacity appears almost black as it hangs in the sky above your head. On the other hand, frosted glass lets some light through and has low opacity.

Extinction is a term that describes how much the intensity of light has been reduced as it passes through a medium.

The terms opacity and extinction are often used interchangeably, but mathematically, scientists who work with light define them differently. One basic equation that relates them together is:

\[ I = I_0 e^{-\tau} \]

\( I_0 \) is the initial intensity of the light as it strikes the front surface of the medium. \( I \) is the intensity of the light after it has left the medium, and \( \tau \) is the opacity of the medium that the light has passed through. A high opacity (opaque) medium is one for which \( \tau \) is large, and this causes the light leaving the medium, \( I \), to be reduced in intensity, which we call extinction.

When light passes through two different materials, one after the other, the final intensity is just

\[ I = (I_0 e^{-A})e^{-B} \quad \text{or} \quad I = I_0 e^{-(A+B)} \]

where \( A \) is the opacity (\( \tau \)) of the first medium and \( B \) is the opacity (\( \tau \)) of the second medium.

**Problem 1** – Show that for three different mediums, the total opacity of the materials is given by the formula

\[ \tau = \tau_A + \tau_B + \tau_C \]

**Problem 2** - A photographer is given three different filters with opacities of \( \tau_A = 5.2 \), \( \tau_B = 1.3 \) and \( \tau_C = 0.5 \). He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

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**Problem 1** – Show that for three different mediums, the total opacity of the materials is given by the formula

$$\tau = \tau_a + \tau_b + \tau_c$$

Answer: From our example, and extended for three medii:

$$I = ((I_0 \ e^{-\tau_A})e^{-\tau_B})e^{-\tau_C}) \text{ or } I = I_0 e^{-(\tau_A+\tau_B+\tau_C)}$$

But the total opacity is just

$$I = I_0 e^{-\tau},$$

so since \(A = \tau_A, B=\tau_B\) and \(C=\tau_C\) we have

$$I0e^{\tau} = I0e^{(\tau_a + \tau_b + \tau_c)}$$

And so

$$\tau = \tau_a + \tau_b + \tau_c$$

**Problem 2** - A photographer is given three different filters with opacities of \(\tau_A = 5.2, \tau_B = 1.3\) and \(\tau_C = 0.5\). He thinks that by placing the most opaque filter last that the light will be slightly brighter when it enters the camera. Do you think that this will work?

Answer: This will not work because the final opacity is the sum of the opacities of the three filters, and the order in which you add the filters does not matter in the sum. You will still get a final opacity of \(5.2+1.3+0.5 = 7.0\) and a drop in brightness by a factor of

$$I = I_0 \ e^{-7} \text{ or } 0.00091$$