Asteroid Gaspra about 15 km across. Image taken by the NASA Galileo spacecraft.

Planets are built in several stages. The first of these involves small, interstellar dust grains that collide and stick together to form centimeter-sized bodies. This can take millions of years. The second stage involves the formation of kilometer-sized asteroids from the centimeter-sized rocks. A simple model of this process can tell us about how long it takes to 'grow' an asteroid from rock-sized bodies.

Problem 1 – Assume that the forming asteroid is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the asteroid as a function of time, M(t)?

Problem 2 – The asteroid grows by absorbing incoming rocks that have an average mass of 5.0 grams and a density of N rocks per cubic centimeter in the cloud. The rocks collide with the surface of the forming asteroid at a speed of V cm/sec, what is the equation that gives the rate of growth of the asteroid's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc?

Problem 6 – The asteroid begins at t=0 with a mass of m=5 grams. The cloud density N = 1.0×10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer?

Problem 1 – Answer: Because mass = density x volume, we have
M = 4/3 π R^3 ρ and so **M(t) = 4/3 π ρ $R(t)^3$**

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of rocks land on the surface area of the forming asteroid per unit time, dt. The amount is proportional to the surface area of the asteroid, since the more surface area the asteroid has, the more rocks will be absorbed. Also, the rate at which rock mass is brought to the surface of the asteroid is proportional to the product of the rock density in the solar nebula, times the speed of the rocks landing on the surface of the asteroid. This leads to m x N x V where m is in grams per rock, N is in rocks per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the asteroid, will then have the units of grams/sec representing the rate at which the asteroid mass is growing. The full formula for the growth of the asteroid mass is then
dM/dt = 4 p R^2 m N V

Problem 3 – Answer: From Problem 1 we see that R(t) = $(3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have dM/dt = 4 π m N V $(3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}$ dM = 4 π m N V $(3/4\ \pi\ \rho)^{2/3}$ dt Then integrate both sides to get:
3 $M(t)^{1/3}$ = 4 π m N V $(3/4\ \pi\ \rho)^{2/3}$ t + c. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc? Answer M = 4/3 π $(50{,}000\text{ cm})^3$ x 3.0 gm/cc = **1.6 x 10^{15} grams.**

Problem 6 – The rock begins at t=0 with a mass of 1 rock, m = 5 grams. The cloud density N = 1.0 x 10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer? Answer: For t = 0, M(0) = m so the constant of integration is c = $m^{1/3}$ so c = 1.7.
Then M(t) = (4/3 (3.14) (5)(1.0 x 10^{-8})(100,000)$(3/(4(3.14)\ (3.0)))^{2/3}$ t +1.7 $)^3$
M(t) = $(0.0039\ t + 1.7)^3$

So to get M(t) = 1.6 x 10^{15} grams, solve for t to get t = 29,600,000 seconds or about **342 days!**