Black Holes and Tidal Forces

$$a = \frac{2GMd}{R^3}$$

A tidal force is a difference in the strength of gravity between two points. The gravitational field of the moon produces a tidal force across the diameter of Earth, which causes the Earth to deform. It also raises tides of several meters in the solid Earth, and larger tides in the liquid oceans.

If the tidal force is stronger than a body's cohesiveness, the body will be disrupted. The minimum distance that a satellite comes to a planet before it is shattered this way is called its Roche Distance. The artistic image to the left shows what tidal disruption could be like for an unlucky moon.

A human falling into a black hole will also experience tidal forces. In most cases these will be lethal! The difference in acceleration between the head and feet could be many thousands of Earth Gravities. A person would literally be pulled apart! Some physicists have termed this process spaghettification!

Problem 1 - The equation lets us calculate the tidal acceleration, **a**, across a body with a length of **d**. The acceleration of gravity on Earth's surface is 979 cm/sec^{2}. The tidal acceleration between your head and feet is given by the above formula. For M = the mass of Earth (5.9 x 10^{27} grams), R = the radius of Earth (6.4 x 10^{8} cm) and the constant of gravity whose value is G = 6.67 x 10^{-8} dynes cm^{2}/gm^{2} calculate the tidal acceleration, a, if d = 2 meters.

Problem 2 - What is the tidal acceleration across the full diameter of Earth?

Problem 3 - A stellar-mass black hole has the mass of the sun (1.9 x 10^{33} grams), and a radius of 2.9 kilometers. A) What would be the tidal acceleration across a human at a distance of 100 kilometers? B) Would a human be spaghettified?

Problem 4 - A supermassive black hole has 100 million times the mass of the sun (1.9 x 10^{33} grams), and a radius of 295 million kilometers. What would be the tidal acceleration across a human at a distance of 100 kilometers from the event horizon of the supermassive black hole?

Problem 5 - Which black hole could a human enter without being spagettified?

Answer Key:

Problem 1 - The equation lets us calculate the tidal acceleration, **a**, across a body with a length of **d**. The acceleration of gravity on Earth's surface is 979 cm/sec^{2}. The tidal acceleration between your head and feet is given by the above formula. For M = the mass of Earth (5.9 x 10^{27} grams), R = the radius of Earth (6.4 x 10^{8} cm) and the constant of gravity whose value is G = 6.67 x 10^{-8} dynes cm^2/gm^2 calculate the tidal acceleration, a, if d = 2 meters.

Answer: a = 2 x (6.67 x 10^{-8}) x **(**5.9 x 10^{27}**)** x 200 / (6.4 x 10^{8})^{3}
= 0.000003 x (200)
= 0.0006 cm/sec^{2}

Problem 2 - What is the tidal acceleration across the full diameter of Earth?
Answer: d = 1.28 x 10^{9} cm, so a = 0.000003 x 1.28 x 10^{9} = 3,840 cm/sec^{2}

Problem 3 - A stellar-mass black hole has the mass of the sun (1.9 x 10^{33} grams), and a radius of 2.9 kilometers. A) What would be the tidal acceleration across a human at a distance of 100 kilometers? B) Would a human be spaghettified?

Answer: a = 2 x (6.67 x 10^{-8}) x **(**1.9 x 10^{33}**)** x 200 / (1.0 x 10^{7})^{3}
= 50,700,000 cm/sec^{2}
Yes, this is equal to 50,700,000/979 = 51,700 times the acceleration of gravity, and a human would be pulled apart and 'spaghettified'

Problem 4 - A supermassive black hole has 100 million times the mass of the sun (1.9 x 10^{33} grams), and an event horizon radius of 295 million kilometers. What would be the tidal acceleration across a d=2 meter human at a distance of 100 kilometers from the event horizon of the supermassive black hole?

Answer: a = 2 x (6.67 x 10^{-8}) x **(**1.9 x 10^{41}**)** x 200 / (2.95 x 10^{13})^{3}
= 0.00020 cm/sec^{2}
Note that R + 2 meters is essentially R if R = 295 million kilometers.

Problem 5 - Which black hole could a human enter without being spaghettified?
Answer: The supermassive black hole, because the tidal force is far less than what a human normally experiences on the surface of Earth. That raises the question that as a space traveler, you could find yourself trapped by a supermassive black hole unless you knew exactly what its size was before hand. You would have no physical sensation of having crossed over the black hole's Event Horizon before it was too late.