Black Holes....II

$$T = t\sqrt{1 - \frac{2GM}{Rc^2}}$$

T = the time measured by someone located on a planet (seconds)

t = the time measured by someone located in space (seconds)

M = the mass of the planet (grams)

R = the distance to the far-away observer from the planet (cm)

And the natural constants are:
G = 6.67 x 10^{-8}
C = 3 x 10^{10}

The modern theory of gravity developed by Albert Einstein in 1915 leads to some very unusual predictions, which have all been verified by experiments.

One of the strangest ones is that two people will experience the passage of time very differently if one is standing on the surface of a planet, and the other one is in space. This is because the rate of time passing depends on the strength of the gravitational field that the observer is in.

For example, at the surface of a very dense neutron star, R = 20 km and M = 1.9 x 10^{33} grams, so

$$T = t\ (1- 0.15)^{1/2} = 0.92\ t$$

This means that for every hour that goes by on the surface of the neutron star (T = 3600 seconds), someone in space will see t = 3600 / 0.92 = 3913 seconds pass from a vantage point in space.

Problem 1 - The GPS satellites orbit Earth at a distance of R = 26,560 kilometers. If the mass of Earth is 5.9 x 10^{27} grams, use the formula to determine the time dilation factor. Be very careful with the small numbers in the 9th, 10th and 11th decimal places!

Problem 2 - What is the time dilation factor at Earth's surface?

Problem 3 - What is the ratio of the dilation in space to the dilation at earth's surface?

Problem 4 - At the speed of light (3 x 10^{10} cm/sec) how long does it take a radio signal from the GPS satellite to travel 26,560 kilometers to a hand-held GPS receiver?

Problem 5 - The excess time delay between a receiver at Earth's surface, and the GPS satellite is defined by the ratio computed in Problem 3, multiplied by the total travel time in Problem 4. What is the time delay for the GPS-Earth system?

Problem 6 - From your answer to Problem 5, how much extra time does the radio signal take compared to your answer to Problem 4?

Problem 7 - At the speed of light, how far will the radio signal travel during the extra amount of time?

Problem 8 - Is gravitational time delay an important phenomenon to include when using the GPS satellite system?

Answer Key:

Problem 1 - The GPS satellites orbit Earth at a distance of R = 26,560 kilometers. If the mass of Earth is 5.9 x 10^{27} grams, use the formula to determine the time dilation factor. Be very careful with the small numbers in the 9th, 10th and 11th decimal places!

Answer: (1 - 0.84/2.65x10^{9} cm)1/2 =
(1 - 3.1 x 10^{-10}) 1/2 =
(0.99999999969)1/2 = 0.99999999984

Problem 2 - What is the time dilation factor at Earth's surface?
(1 - 0.84/6.38x10^{8} cm)1/2 =
(1 - 1.3 x 10^{-9}) 1/2 =
(0.9999999987)1/2 = 0.99999999934

Problem 3 - What is the ratio of the dilation in space to the dilation at Earth's surface?
Answer - 0.99999999984 / 0.99999999934 = 1.0000000064

Problem 4 - How long does it take a radio signal from the GPS satellite to travel 26,560 kilometers to a hand-held GPS receiver?
Answer - Distance = 26,560 kilometers x (100,000 cm / kilometer) = 2.65 x 10^{9} centimeters.
Time = Distance / speed of light
= 2.65 x 10^{9} **cm** / 3 x 10^{10} cm/sec = 0.088 seconds.

Problem 5 - The excess time delay between a receiver at Earth's surface, and the GPS satellite is defined by the ratio computed in Problem 3, multiplied by the total travel time in Problem 4. What is the time delay for the GPS-Earth system?

Answer - 0.088 seconds * 1.0000000064 = 0.08800000056 seconds.

Problem 6 - From your answer to Problem 5, how much extra time does the radio signal take compared to your answer to Problem 4?
Answer - 0.08800000056 - 0.088 seconds = 0.00000000056 seconds.

Problem 7 - At the speed of light, how far will the radio signal travel during the extra amount of time?
Answer = 3 x 10^{10} cm/sec x 5.6 x 10^{-10} sec = 16.8 centimeters.

Problem 8 - Is gravitational time delay an important phenomenon to include when using the GPS satellite system?
Answer: Yes!