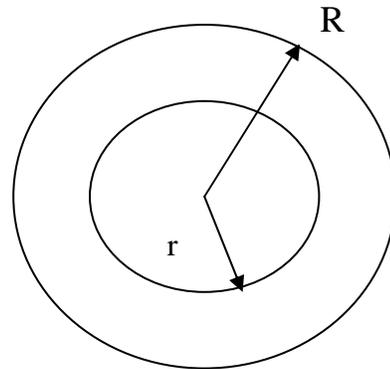


Astronomers can often make an estimate of what the interior of a planet looks like by carefully measuring the mass of the planet and its radius. They then create a model of the inside of the planet that matches the measured radius and mass. A simple model has a planet described as a perfect sphere, consisting of a low-density outer shell and a higher-density inner core. By using various kinds of matter (solids, gas, liquid) and different compounds (ice, iron, water etc) astronomers try various combinations of materials until the mass and radius of the model planet resembles the one actually studied. Here is a list of materials commonly found in planets:

Material	Density
Pure Iron	10.0 gm/cc
Basalt rock	5.0 gm/cc
Water	5.0 gm/cc
Silicate rock	3.0 gm/cc
Ice	1.0 gm/cc



Suppose a planet was discovered in another solar system that had a radius of $R = 6378$ kilometers and a mass of 5.97×10^{24} kilograms. The astronomer begins by considering a model with a core radius, r , of 3,500 kilometers. Using this model, answer the questions below.

Question 1 - What is the volume of the inner core in cubic centimeters?

Question 2 - What is the volume of the outer shell in cubic centimeters?

Question 3 - If the inner core were made of silicate rock, and the outer crust of ice what would be the total mass of the model planet?

Question 4 – How does the mass of the model planet compare to the actual mass of the planet?

Question 5 – Can you find a material in the above list for the core material that would make the model mass equal to the planets actual mass?

Question 6 – What else could you change in the model the astronomer started with to make a better estimate?

Answer - Extra Credit Problem

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Question 1 - What is the volume of the planet in cubic centimeters?

Answer :

$$\begin{aligned}V(\text{sphere}) &= 4/3 \pi R^3 = (1.33) \times (3.14) \times (6378)^3 \\ &= 1.08 \times 10^{12} \text{ cubic kilometers} \\ &= 1.08 \times 10^{27} \text{ cubic centimeters.}\end{aligned}$$

Question 2 - What is the volume of the inner core in cubic centimeters?

Answer:

$$\begin{aligned}V(\text{sphere}) &= 4/3 \pi R^3 = (1.33) \times (3.14) \times (3500)^3 \\ &= 1.79 \times 10^{11} \text{ cubic kilometers} \\ &= 1.79 \times 10^{26} \text{ cubic centimeters.}\end{aligned}$$

Question 3 - If the inner core were made of silicate rock (density = 3.0 grams/cm³), and the outer crust of ice (1.0 grams/cm³) what would be the total mass of the model planet?

Answer: The outer crust volume is the DIFFERENCE between the answers in Questions 1 and 2 ($1.08 \times 10^{27} - 1.79 \times 10^{26}$) = 9.01×10^{26} cubic cm.

$$\text{Model mass} = 1.0 \times (9.01 \times 10^{26}) + 3.0 \times (1.79 \times 10^{26}) = 1.44 \times 10^{24} \text{ kilograms}$$

Question 4 – How does the mass of the model planet compare to the actual mass of the planet?

Answer: The model mass is much smaller than the actual mass.

Question 5 – Can you find a combination of crust and core material in the table that would make the model mass almost equal to the planet's actual mass?

Answer: Students may use trial-and-error. Try iron in the core and silicate rock in the outer crust:

$$\text{Model mass} = 3.0 \times (9.01 \times 10^{26}) + 10.0 \times (1.79 \times 10^{26}) = 4.61 \times 10^{24} \text{ kilograms}$$

(This gives a model that is $(4.61 / 5.97) \times 100\% = 77\%$ less massive than the actual planet, which is a very good match for this activity – and the best match possible given the assumed core radius of 3,500 kilometers!)

Question 6 – What else could you change in the model the astronomer started with to make a better estimate?

Answer: You could change the radius of the core by making it bigger. You could change the composition of the outer crust by making it basalt instead of silicate rock. You could change the composition of the core by making it a mixture of iron and silicate (with a density between 3.0 and 10.0 gm/cm³). You could increase the number of zones in the model from two (crust and core) to three (crust, mantle, core) and see what happens.