Life-size model of the Webb Space Telescope.

In 2018, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, θ(R), for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on its surface is given by

$$T(R) = \frac{250}{\sqrt{R}} \text{ Kelvins}$$

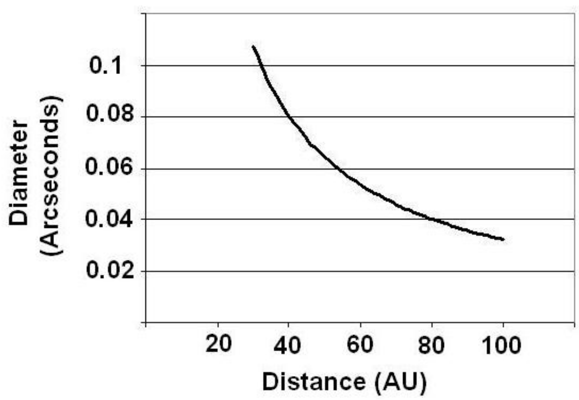
where R is the distance from the sun in AU. Create a graph that shows T(R) vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

Problem 3 - A body in other outer solar system with an angular size θ(R) emits most of its light energy in the infrared and has a temperature given by T(R) in Kelvins. Its brightness in units of Janskys, F, at a wavelength of 20 microns will be given by:

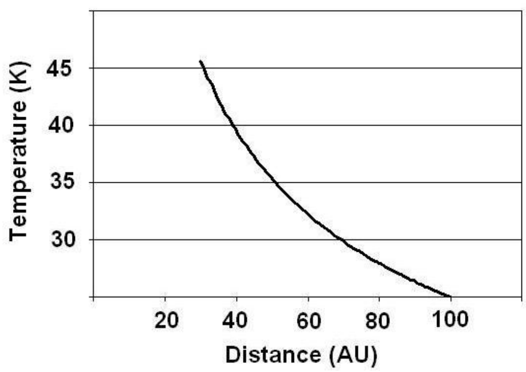
$$F(T) = \frac{120000}{(e^x - 1)}\theta(R)^2 \text{ Janskys} \qquad \text{where} \qquad x = \frac{720}{T(R)}$$

From the formula for θ(R) and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

Problem 1 - Answer: At 90 AU, the disk of a Pluto-sized body will be 0.035 arcseconds in diameter.



Problem 2 Answer: At 90 AU, the predicted temperature will be about 28 K.



Problem 3 - Answer: At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about 40 AU. More accurate estimates using more realistic emission properties for Pluto suggest 90 AUs as an actual limit.