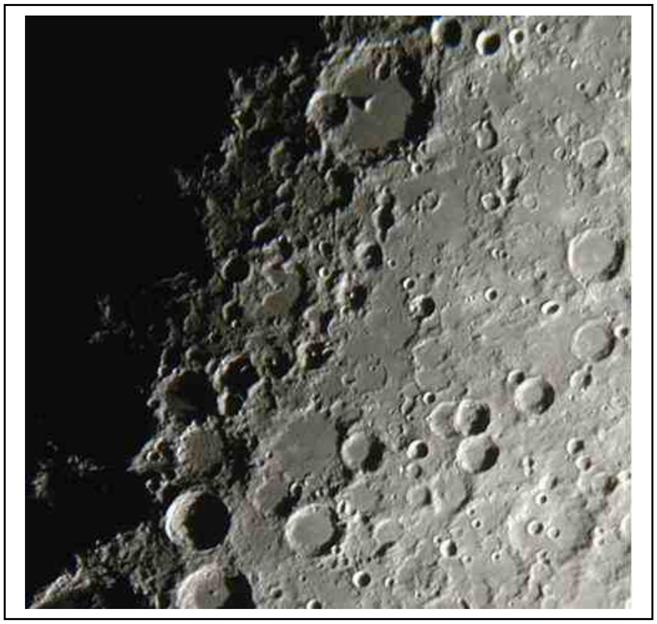


One of the first things that astronomers wish to learn about a planet or other body in the solar system is the number of craters on its surface. This information can reveal, not only the age of the surface, but also the history of impacts during the age of the body.

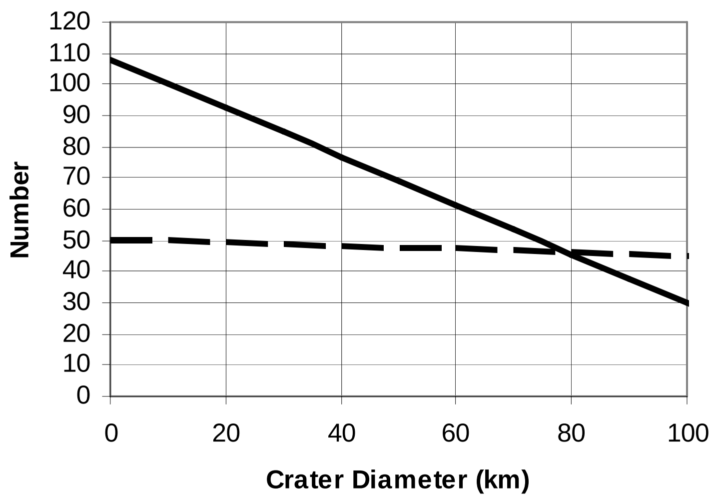
Bodies with no atmospheres preserve all impacts, regardless of size, while bodies with atmospheres or crustal activity, often have far fewer small craters compared to larger ones.

Studies of the number of craters on Venus and Mars have determined that for Venus, the number of craters with a diameter of D kilometers is approximated by $N = 108 - 0.78D$ while for Mars the crater counts can be represented by $N = 50 - 0.05 D$.

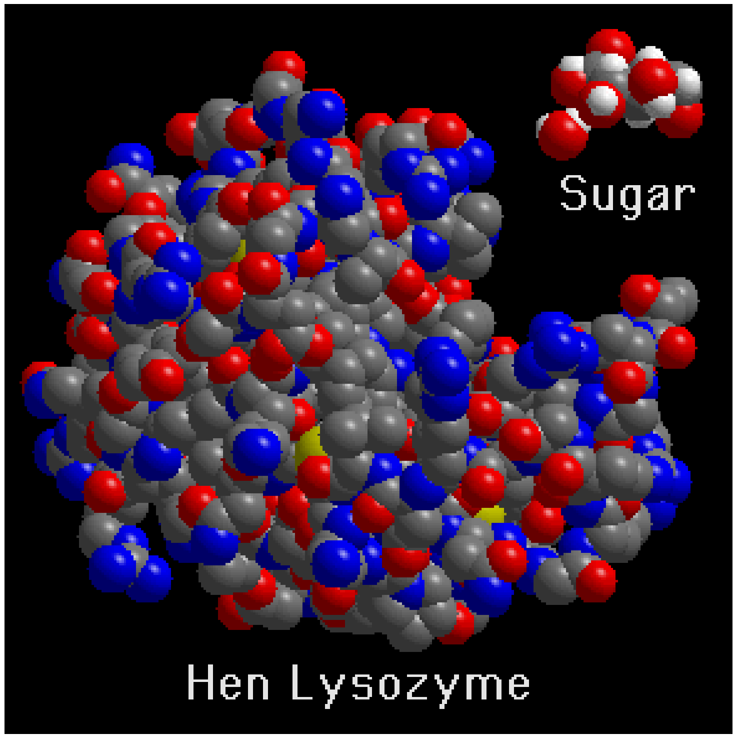
Problem 1 – Graphically solve these two equations to determine for what crater diameter the number of craters counted on the two planets is the same over the domain D:[0,100 km].

Problem 1 – Graphically solve these two equations to determine for what crater diameter the number of craters counted on the two planets is the same over the domain D:[0,100 km].

Answer:



Dashed line is the data for Mars, solid line is the data for Venus.
The intersection point is at D = 80 kilometers with N = 46 craters.



Problem 1 – The molecule sucrose has a mass of 342.30 AMU and consists of 12 carbon (C) , 11 oxygen (O) and 22 hydrogen (H) atoms. Another organic molecule called acetic acid has a mass of 60.03 AMUs and consists of 2 carbon, 4 hydrogen and 2 oxygen atoms. A third molecule, called benzoic acid, consists of seven carbon, 6 hydrogen and 2 oxygen atoms with molecular mass of 122.1 AMUs. What are the masses of the hydrogen, oxygen and carbon molecules individually?

Problem 1 – Answer: Solve:

$$\begin{aligned} 12C + 11O + 22H &= 342.3 \\ 2C + 2O + 4H &= 60.03 \\ 7C + 2O + 6H &= 122.1 \end{aligned}$$

Use substitution to reduce to a pair of 2 equations for 2 of the elements, then use substitution again to solve for one of the atomic masses.

$$\begin{aligned} 11/2 \times (7C + 2O + 6H &= 122.1) \\ -1\times(12C + 11O + 22H &= 342.3) \end{aligned}$$

$$\begin{aligned} 38.5C + 11O + 33H &= +671 \\ -12C - 11O - 22H &= -342. \end{aligned}$$

$$\mathbf{26.5C + 11H = 329}$$

$$\begin{aligned} & 7C + 2O + 6H = 122.1 \\ - & 2C + 2O + 4H = 60.03 \end{aligned}$$

$$\mathbf{5C + 2H = 62}$$

Then:

$$\begin{aligned} &\mathbf{11/2 \times (5C + 2H = 62)} \\ &\mathbf{-1 \times (26.5C + 11H = 329)} \end{aligned}$$

$$\begin{aligned} &\mathbf{27.5C + 11H = 341} \\ &\mathbf{-26.5C - 11H = -329} \end{aligned}$$

$$C = 12$$

So $27.5(12) + 11H = 341$ and $H = 1$

And $12(12) + 11O + 22(1) = 342$ so $O = 16$

C = 12 AMU H = 1 AMU O = 16 AMU

Graphing Systems of Linear Inequalities

The universe is a BIG place…but it also has some very small ingredients! Astronomers and physicists often find linear plotting scales very cumbersome to use because the quantities you would most like to graph differ by powers of 10 in size, temperature or mass. Log-Log graphs are commonly used to see the 'big picture'. Instead of a linear scale '1 kilometer, 2 kilometers 3 kilometers etc' a Logarithmic scale is used where '1' represents 10^1, '2' represents 10^2 …'20' represents 10^{20} etc. A calculator easily lets you determine the Log of any decimal number. Just enter the number, n, and hit the 'log' key to get m = log(n). Then just plot a point with 'm' as the coordinate number!

Below we will work with a Log(m) log(r) graph where m is the mass of an object in kilograms, and r is its size in meters.

Problem 1 - Plot some or all of the objects listed in the table below on a LogLog graph with the 'x' axis being Log(M) and 'y' being Log(R).

Problem 2 - Draw a line that represents all objects that have a density of A) 'N' nuclear matter (4×10^{17} kg/m^3), and B) 'W' water (1000 kg/m^3).

Problem 3 - Black holes are defined by the simple formula R = 3.0 M, where r is the radius in kilometers, and M is the mass in multiples of the sun's mass (1 M = 2.0×10^{30} kilograms). Shade-in the region of the LogLog plot that represents the condition that no object of a given mass may have a radius <u>smaller</u> than that of a black hole.

Problem 4 - The lowest density achievable in our universe is set by the density of the cosmic fireball radiation field of 4×10^{-31} kg/m^3. Draw a line that identifies the locus of objects with this density, and shade the region that excludes densities <u>lower</u> than this.

	Object	R (meters)	M (kg)
1	You	2.0	60
2	Mosquito	2×10^{-3}	2×10^{-6}
3	Proton	2×10^{-15}	2×10^{-27}
4	Electron	4×10^{-18}	1×10^{-30}
5	Z boson	1×10^{-18}	2×10^{-25}
6	Earth	6×10^{6}	6×10^{24}
7	Sun	1×10^{9}	2×10^{30}
8	Jupiter	4×10^{8}	2×10^{27}
9	Betelgeuse	8×10^{11}	6×10^{31}
10	Milky Way galaxy	1×10^{21}	5×10^{41}
11	Uranium atom	2×10^{-14}	4×10^{-25}
12	Solar system	1×10^{13}	2×10^{30}
13	Ameba	6×10^{-5}	1×10^{-12}
14	100-watt bulb	5×10^{-2}	5×10^{-2}
15	Sirius B white dwarf.	6×10^{6}	2×10^{30}
16	Orion nebula	3×10^{18}	2×10^{34}
17	Neutron star	4×10^{4}	4×10^{30}
18	Binary star system	1×10^{13}	4×10^{30}
19	Globular cluster M13	1×10^{18}	2×10^{35}
20	Cluster of galaxies	5×10^{23}	5×10^{44}
21	Entire visible universe	2×10^{26}	2×10^{54}

The figure below shows the various items plotted, and excluded regions cross-hatched. Students may color or shade-in the permitted region. This wedge represents all of the known objects and systems in our universe; a domain that spans a range of 85 orders of magnitude (10^{85}) in mass and 47 orders of magnitude (10^{47}) in size!

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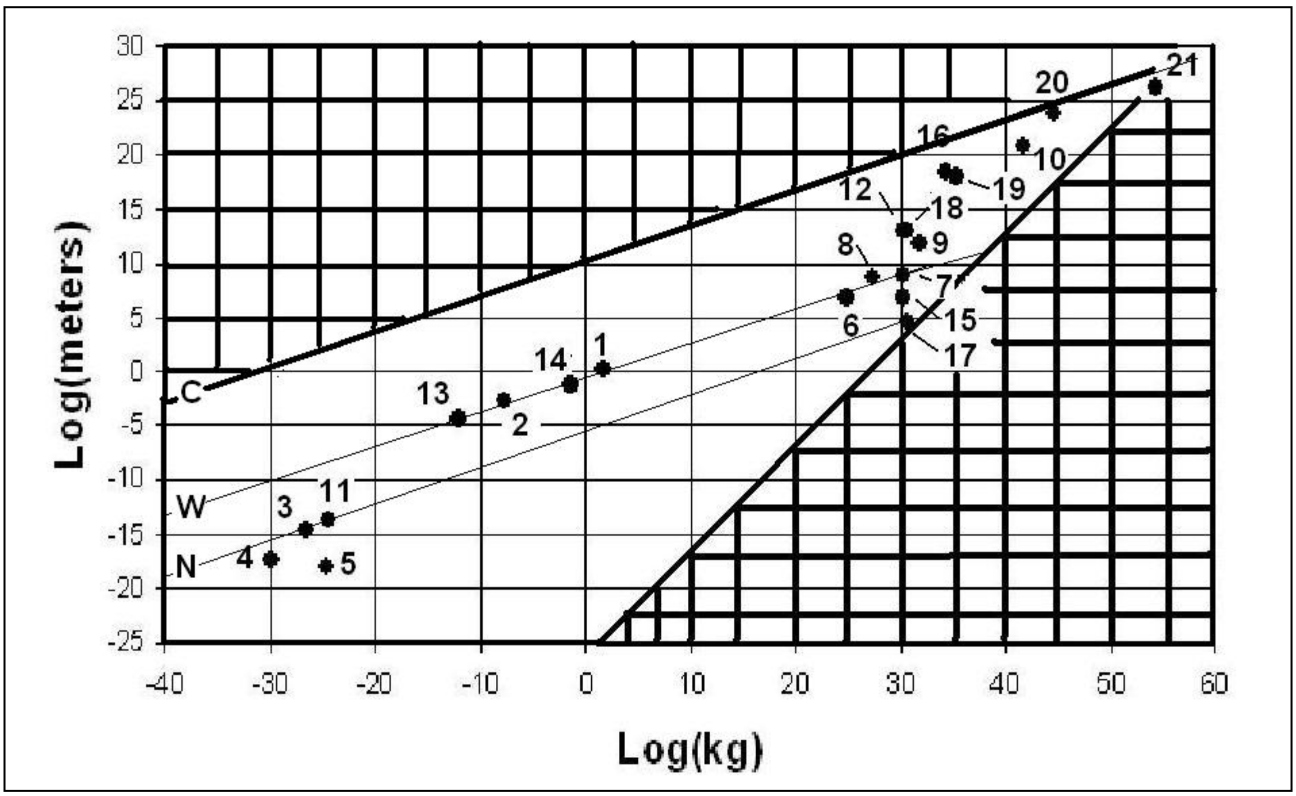
Inquiry: Can you or your students come up with more examples of objects or system that occupy some of the seemingly 'barren' regions of the permitted area?



Graphing Systems of Linear Inequalities

Astronomers and physicists often find linear plotting scales very cumbersome to use because the quantities you would most like to graph differ by powers of 10 in size, temperature or mass. Log-Log graphs are commonly used to see the 'big picture'. Instead of a linear scale '1 kilometer, 2 kilometers 3 kilometers etc' a Logarithmic scale is used where '1' represents 10^1, '2' represents 10^2 …'20' represents 10^{20} etc. Below we will work with a Log(T) log(D) graph where T is the temperature, in Kelvin degrees, of matter and D is its density in kg/m^3.

Problem 1 - Plot some or all of the objects listed in the table below on a Log-Log graph with the 'x' axis being Log(D) and 'y' being Log(T).

Problem 2 - A) Draw a line that includes the three black hole objects (#16, 17 and 18), and shade the region below and to the right that forbids objects denser or cooler than this limit. B) Draw a line, and shade the region that represents the quantum temperature limit where temperatures exceed T=10^{32} K.

Problem 3 - On this graph, plot the curve representing the temperature, T, and density D, of the Big Bang at a time, t, seconds after the Big Bang given by

$$T = 1.5\text{x}10^{10}\ t^{-\frac{1}{2}}\ \text{K} \qquad \text{and} \qquad D = 4x10^{8}\ t^{-2}\ \text{kg/m}^3$$

	Object or Event	D (kg/m^3)	T (K)
1	Human	1000	290
2	Bose-Einstein Condensate	$2\text{x}10^{-12}$	$2\text{x}10^{-7}$
3	Earth atmosphere @ sea level	1.0	270
4	Core of the sun	$1\text{x}10^{5}$	$1\text{x}10^{7}$
5	Core of Earth	$1\text{x}10^{4}$	$6\text{x}10^{3}$
6	Water at Earth's surface	$1\text{x}10^{3}$	270
7	Solar corona	$2\text{x}10^{-14}$	$2\text{x}10^{6}$
8	White dwarf core	$2\text{x}10^{9}$	$2\text{x}10^{4}$
9	Neutron star core	$2\text{x}10^{17}$	$4\text{x}10^{9}$
10	Quantum limit	$4\text{x}10^{94}$	$2\text{x}10^{32}$
11	Interstellar medium - cold	$2\text{x}10^{-27}$	$2\text{x}10^{4}$
12	Dark interstellar cloud	$2\text{x}10^{-23}$	40
13	Rocks at surface of the Earth	$3\text{x}10^{3}$	270
14	Liquid Helium	$1\text{x}10^{2}$	2
15	Cosmic background radiation	$5\text{x}10^{-31}$	3
16	Solar-mass Black Hole	$7\text{x}10^{19}$	$6\text{x}10^{-8}$
17	Supermassive black hole	100	$6\text{x}10^{-17}$
18	Quantum black hole	$3\text{x}10^{60}$	$1\text{x}10^{13}$
19	Controlled fusion Tokamak Reactor	$1\text{x}10^{-8}$	$2\text{x}10^{8}$
20	Intergalactic medium - hot	$2\text{x}10^{-27}$	$2\text{x}10^{8}$
21	Brown dwarf core	$2\text{x}10^{6}$	$1\text{x}10^{6}$
22	Cosmic gamma-rays (1 GeV)	$1\text{x}10^{-9}$	$1 \text{ x } 10^{13}$
23	Cosmic gamma-rays (10 billion GeV)	$1\text{x}10^{-17}$	$1 \text{ x } 10^{23}$
24	Starlight in the Milky Way	$2 \text{ x } 10^{-11}$	6,000

The figure below shows the various items plotted, and excluded regions cross-hatched. Students may color or shade-in the permitted region.

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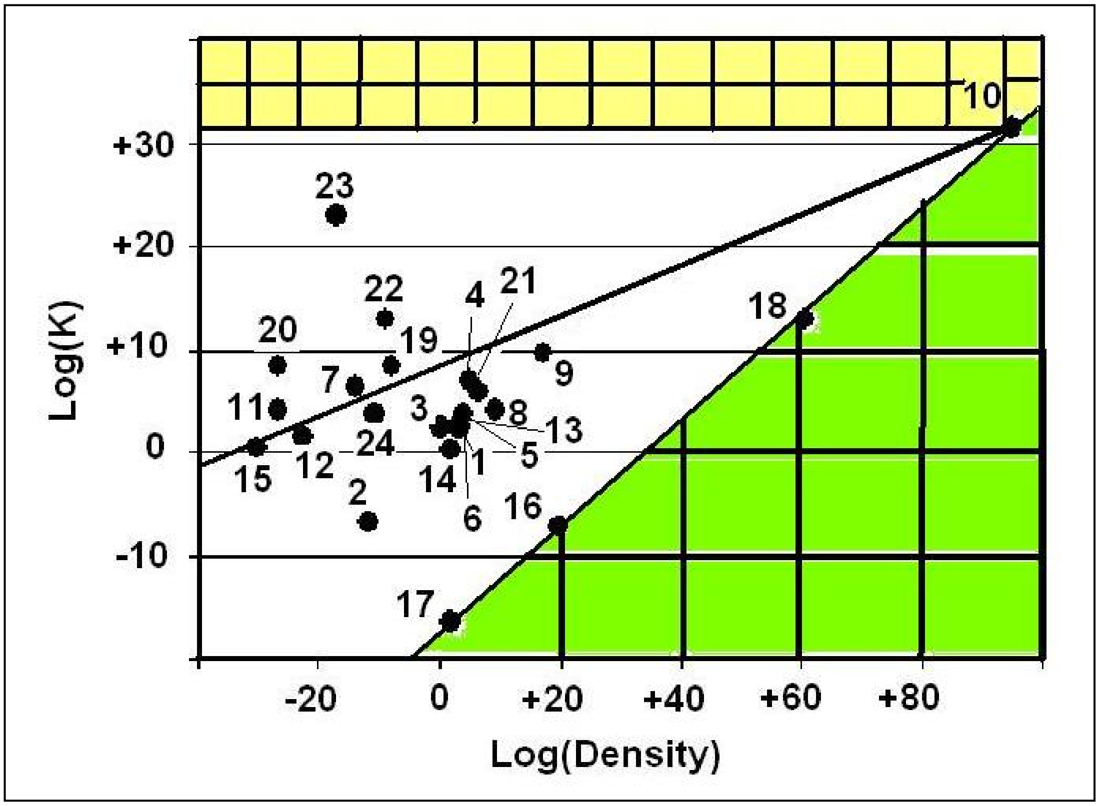
Inquiry: Can you or your students come up with more examples of objects or systems that occupy some of the seemingly 'barren' regions of the permitted area?



Graphing Systems of Linear Inequalities

Astronomers and physicists often find linear plotting scales very cumbersome to use because the quantities you would most like to graph differ by powers of 10 in size, temperature or mass. Log-Log graphs are commonly used to see the 'big picture'. Instead of a linear scale '1 kilometer, 2 kilometers 3 kilometers etc' a Logarithmic scale is used where '1' represents 10^1, '2' represents 10^2 …'20' represents 10^{20} etc. Below we will work with a Log(D) log(B) graph where D is the size of the system in meters, and B is its average magnetic field strength in Teslas.

Problem 1 - Plot some or all of the objects listed in the table below on a Log-Log graph with the 'x' axis being Log(L) and 'y' being Log(B).

Problem 2 – A) Draw a line that defines a region that excludes objects bigger than our entire visible universe (1×10^{26} meters), and shade this region; B) Exclude the region that satisfies the equation $y < 1/3x - 20$ which defines magnetic fields too weak to measure with existing technology.

Problem 3 - The strongest field allowed in stars or larger systems is given by $B = 10^{22} MR^{-2}$ where L is the size of the object in meters and M is the mass of the object in multiples of the sun's mass. Draw a line that passes through the points for Log(B) for M=1 and $L=10^9$ meters (a star) and $M=10^{12}$ and $L = 10^{22}$ meters (large galaxy). Which area would you shade to represent the excluded physical possibilities?

	Object or System	B (Teslas)	L (m)
1	Galactic Center region	1×10^{-7}	2×10^{18}
2	Solar wind	1×10^{-8}	2×10^{11}
3	Solar surface	1×10^{-2}	1×10^{9}
4	Sunspot	1×10^{0}	1×10^{7}
5	White dwarf	1×10^{2}	2×10^{7}
6	The entire Milky Way galaxy	1×10^{-9}	2×10^{21}
7	Neutron star	2×10^{8}	2×10^{4}
8	Magnetar star	2×10^{11}	2×10^{4}
9	Earth surface magnetic field	5×10^{-5}	1×10^{7}
10	Small toy magnet	1×10^{-2}	5×10^{-2}
11	Strong laboratory magnet	1×10^{1}	2×10^{1}
12	Gravity Probe-B measurements	2×10^{-18}	2×10^{1}
13	Human brain	1×10^{-12}	3×10^{-1}
14	Pulsed research magnet	1×10^{2}	2×10^{-2}
15	Explosive amplification of lab field	3×10^{3}	2×10^{1}
16	Magnetic A-type stars	1×10^{0}	1×10^{9}
17	Electron in hydrogen ground state	4×10^{-1}	1×10^{-10}
18	Supernova 1987A gas shell	1×10^{-6}	1×10^{16}
19	Solar system heliopause	2×10^{-11}	2×10^{13}
20	Intergalactic magnetic fields	1×10^{-11}	3×10^{23}
21	Single neuron synapse	1×10^{-10}	1×10^{-3}
22	Neodymium magnet	1×10^{0}	1×10^{-2}
23	Cygnus-A radio galaxy hot spot	2×10^{-8}	6×10^{19}

Problem 1 and 2 - The figure below shows the various items plotted, and excluded regions shaded. Students may color or shade-in the permitted region.
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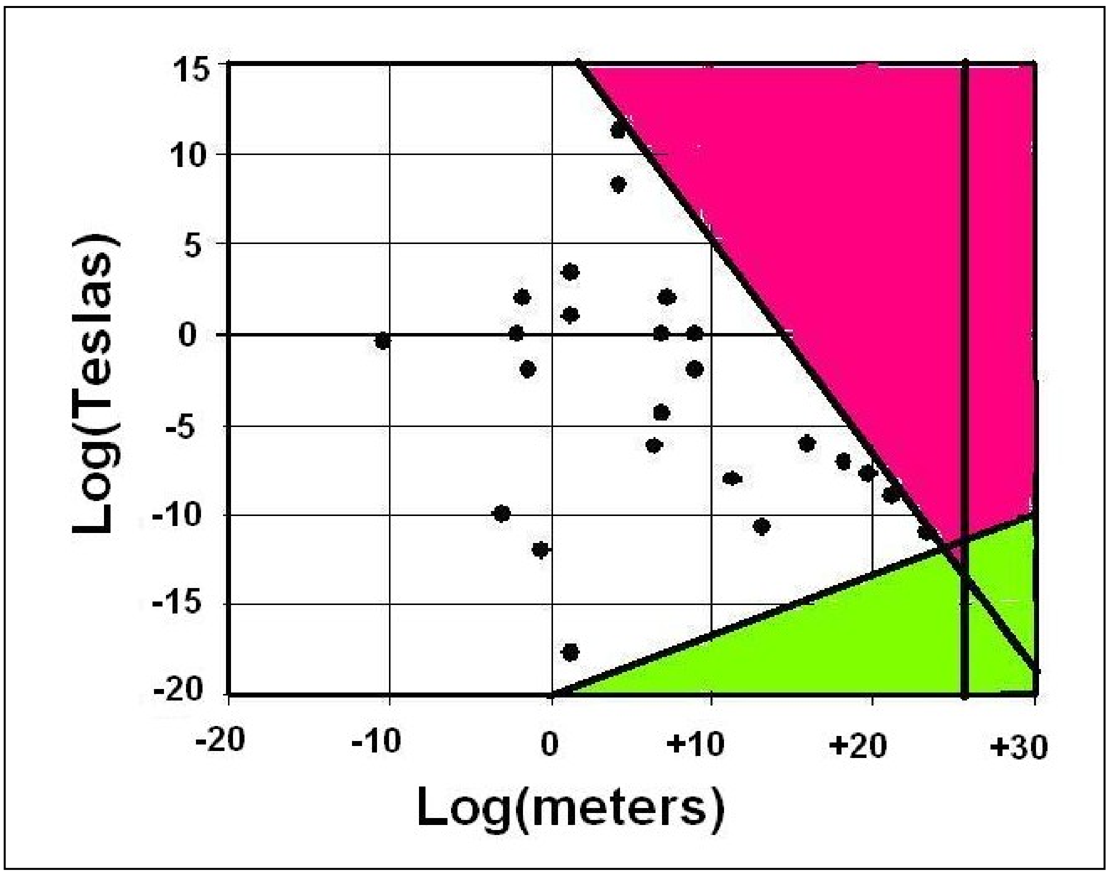
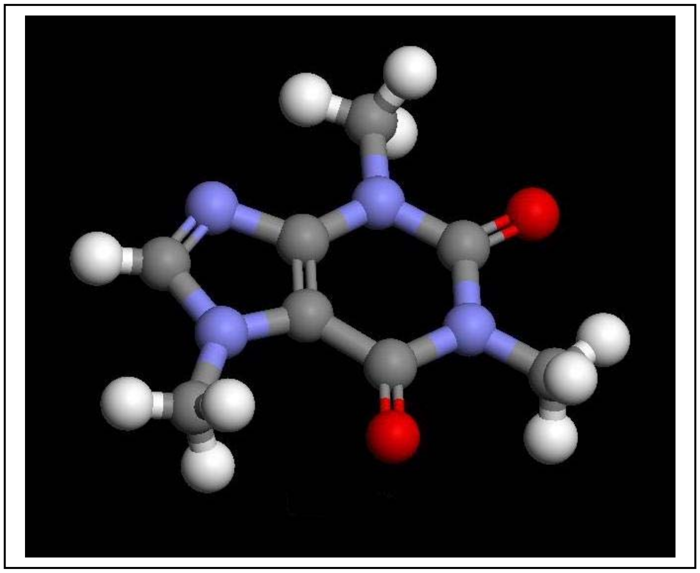
Problem 3 - Answer: The first point gives B = 10^{22} (1) $(10^{9})^{-2}$ = 10,000 Teslas for a star-like body and B = 10^{22} (10^{12}) $(10^{22})^{-2}$ = 10^{-10} Teslas for a galaxy-like object. For a LogLog graph, we have to take the Logs of both numbers to get (LogL,LogB) = (+9.0,+4.0) and (+22.0, -10.0). The line that passes through both points is given by the 2-point formula: (y-4) = (-10 – 4)/(22-9) (x-9) or after simplification y = 13.7 -1.1x. We want to exclude all possibilities below this line so y < 13.7 – 1.1x is the excluded region and, of course, when plotting use y = log(B) and x = log(D).

Inquiry: Can you or your students come up with more examples of objects or systems that occupy some of the seemingly empty regions of the permitted, unshaded, area?





A wide variety of molecules have been detected in space over the last 30 years.

Given the total masses of the molecules in AMUs, and the number of mystery atoms X, Y and Z that are involved, determine the identity of the atoms that comprise the molecules by setting up a set of 3 equations in 3 unknowns and solving them using algebraic methods (not by using matrices and their inverses).

Problem 1 - Acetic acid consists of 4 X atoms, 2 Y atoms and 2 Z atoms. Methyltriacetylene consists of 4 X atoms, 7 Y atoms, but doesn't have any Z atoms. Propanol consists of 6 X atoms, 3 Y atoms and 1 Z atom. The total atomic mass of the molecules are 60 AMU for acetic acid, 88 AMU for methyltriacetylene, and 58 AMU for propanol. What are the atomic masses of the atoms X, Y and Z? Use the table below to identify them.

Hydrogen	1	Sodium	23	Scandium	45
Helium	4	Magnesium	24	Titanium	48
Lithium	7	Aluminum	27	Vanadium	51
Beryllium	9	Silicon	28	Chromium	52
Boron	11	Phosphorus	31	Manganese	55
Carbon	12	Sulfur	32	Iron	56
Nitrogen	14	Chlorine	35	Cobalt	59
Oxygen	16	Argon	40	Nickel	59
Fluorine	19	Potassium	39	Copper	64
Neon	20	Calcium	40	Zinc	65

Problem 1 -

$$\begin{aligned} 4X + 2Y + 2Z &= 60 \\ 4X + 7Y + 0Z &= 88 \\ 6X + 3Y + 1Z &= 58 \end{aligned}$$

$$\begin{array}{r} -4x - 2y - 2z = -60 \\ 4x + 7y \qquad = 88 \\ \hline 5y - 2z = 28 \end{array} \qquad \begin{array}{r} 12x + 21y \qquad = 264 \\ -12x - 6y - 2z = -116 \\ \hline 15y - 2z = 148 \end{array}$$

Then solve:

$$\begin{array}{r} 5y - 2z = 28 \\ 15y - 2z = 148 \\ \hline \end{array}$$

Multiply first equation by -1 and add

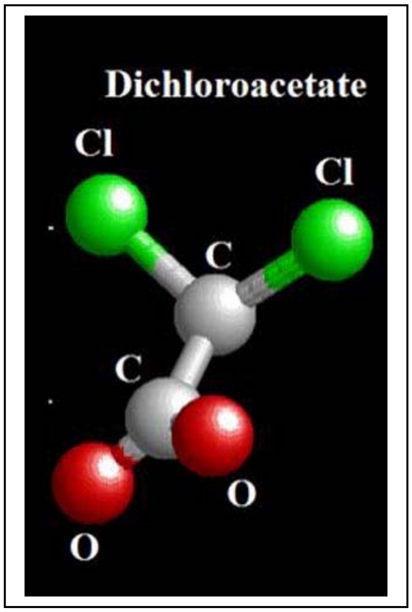
$$\begin{array}{r} -5y + 2z = -28 \\ 15y - 2z = 148 \\ \hline 10y = 120 \end{array} \qquad \text{so } \mathbf{y = 12}$$

Then from $15y - 2z = 148$ $\quad 15(12) - 2z = 148$ $\quad$ and so $\quad$ $\mathbf{z = 16}$

And from $4x + 7y = 88$ $\quad 4x + 7(12) = 88$ $\quad$ and so $\mathbf{x = 1}$

From the table **x = hydrogen; y = carbon and z = oxygen.**

$C_2H_4O_2$	Acetic Acid	$4(1) + 2(12) + 2(16) = 60$ AMU
C_7H_4	Methyltriacetylene	$4(1) + 7(12) + 0(16) = 88$ AMU
C_3H_6O	Propanol	$6(1) + 3(12) + 1(16) = 58$ AMU



A wide variety of molecules have been detected in space over the last 30 years.

Given the total masses of the molecule in AMUs, and the number of mystery atoms X, Y and Z that are involved, determine the identity of the atoms that comprise the molecule in the problem below by setting up a set of 3 equations in 3 unknowns and solving them using algebraic methods (not by using matrices and their inverses).

Problem 1 - Cyanotetra-acetylene consists of 1 X atoms, 9 Y atoms and 1 Z atoms. Aminoacetonitrile consists of 4 X atoms, 2Y atoms, and 2 Z atoms. Cyanodecapentayne consists of 1 X atoms, 11 Y atoms and 1 Z atom. The total atomic mass of the molecules are 123 AMU for cyanotetra-acetylene, 56 AMU for aminoacetonitrile, and 147 AMU for cyanodecapentayne. What are the atomic masses of the atoms X, Y and Z? Use the table below to identify them.

Hydrogen	1	Sodium	23	Scandium	45
Helium	4	Magnesium	24	Titanium	48
Lithium	7	Aluminum	27	Vanadium	51
Beryllium	9	Silicon	28	Chromium	52
Boron	11	Phosphorus	31	Manganese	55
Carbon	12	Sulfur	32	Iron	56
Nitrogen	14	Chlorine	35	Cobalt	59
Oxygen	16	Argon	40	Nickel	59
Fluorine	19	Potassium	39	Copper	64
Neon	20	Calcium	40	Zinc	65

Problem 1 -

$$\begin{aligned} 1X + 9Y + 1Z &= 123 \\ 4X + 2Y + 2Z &= 56 \\ 1X + 11Y + 1Z &= 147 \end{aligned}$$

$$\begin{array}{r} 1x + 9y + 1z = 123 \\ -1x - 11y - z = -147 \\ \hline -2y = -24 \end{array}$$

So **y = 12**

Eliminating y :

$$\begin{aligned} x + 9(12) + z &= 123 & \quad x + z &= 15 \\ 4x + 2(12) + 2z &= 56 & \quad 4x + 2z &= 32 \end{aligned}$$

Multiply by -4 :

$$\begin{array}{r} -4x - 4z = -60 \\ 4x + 2z = 32 \\ \hline -2z = -28 \end{array}$$

so **z = 14**

Then $x + z = 15$ yields $x + 14 = 15$ and so **x = 1**

From the table: **x = hydrogen, y = carbon and z = nitrogen**

HC_9N	Cyanotetra-acetylene	1 (1) + 9 (12) +1 (14) = 123 AMU
$H_4C_2N_2$	Aminoacetonitrile	4 (1) + 2 (12) + 2(14) = 56 AMU
$HC_{11}N$	Cyanodecapentayne	1 (1) + 11 (12) + 1 (14) = 147 AMU