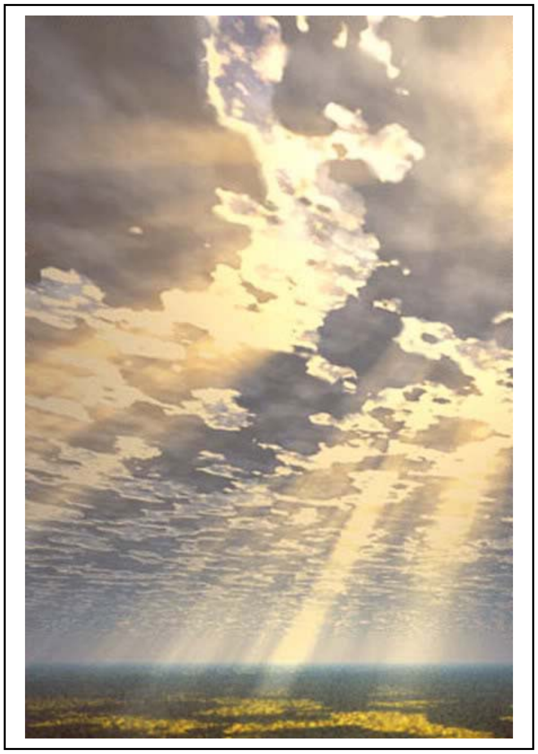


To work properly, a solar panel must be placed so that sunlight falls on its surface with nearly perpendicular rays. This allows the maximum amount of solar energy to fall on a given square-meter of the solar panel. Slanted rays are less efficient, and deliver less energy to the solar panel, so the amount of electricity will be lower.

The equation below accounts for the time of day and the latitude of the solar panel on Earth. The amount of sunlight that falls on a one-square-meter solar panel on June 21, at a latitude of L, and at a local time of T hours after midnight is given by the formula:

$$P(T) = 1370\cos(L - 23.5)\sin\left(\frac{2\pi T}{24} - \frac{\pi}{2}\right) \text{ watts}$$

for 6.00 < T < 18.00

Problem 1 – Graph this function for a 3-day time interval at a latitude of Washington DC, L = +39.0^{o}.

Problem 2 – What is the period of P(T)?

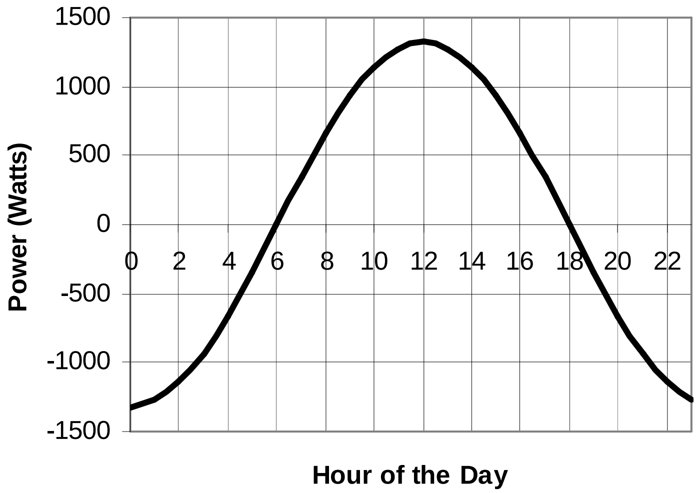
Problem 3 – What is the amplitude of P(T)?

Problem 4 - Explain why the shift of $\pi/2$ was included in P(T)?

Problem 5 - During how many hours of the day is the amount of power falling on the solar panel greater than 1,000 watts?

Problem 1 – Graph this function for a 3-day time interval at a latitude of Washington DC, L = +39.0^{o}.

Answer: The function at this latitude becomes $P(T)=1,320\ \sin(2\pi T/24 - \pi/2)$ watts which has the plot:

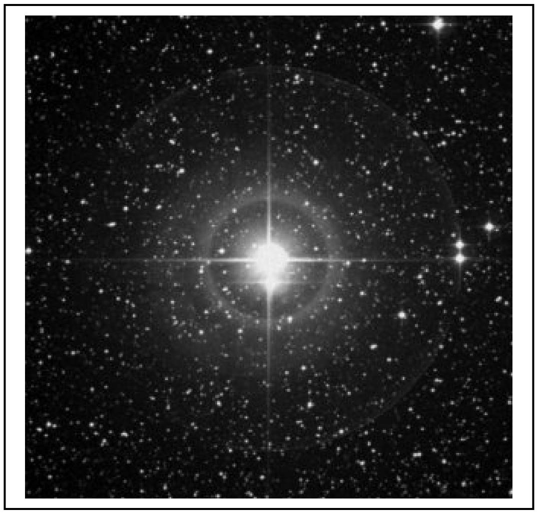


Problem 2 – What is the period of P(T)?
Answer: From the argument of the sin term we need $2\pi = 2\pi T/24$ so T = 24 hours is the period.

Problem 3 – What is the amplitude of P(T)? Answer: The amplitude is the coefficient in front of the sin term = 1,320 watts. **This can also be determined from the graph for which (Positive peak – negative peak)/2 = (+1320 –(-1320))/2 = 1320 watts.**

Problem 4 - Explain why the shift of $\pi/2$ was included in P(T)?
Answer: If no shift were included, the peak of the power would happen at T = 6.0 or 6:00 AM in the morning when the sun is still at the horizon! Adding a 6-hour shift = $2\pi/24 \times 6 = \pi/2$ which makes the peak of the power at Noon when the sun is highest above the horizon.

Problem 5 - During how many hours of the day is the amount of power falling on the solar panel greater than 1,000 watts?
Answer: From the graph, P(T) is above 1,000 watts between T= 9.0 and T = 15.0 or **6 hours.**



Cepheid variable stars are old, very luminous stars that change their radius periodically in time. Typical classical Cepheids pulsate with periods of a few days to months, and their radii change by several million kilometers (30%) in the process. They are large, hot stars, of spectral class F6 – K2, they are 5–20 times as massive as the Sun and up to 30000 times more luminous.

This image shows the variable star Delta Cephi.

The radius of the Cepheid variable star AH Velorum can be given by the following formula:

$R(t) = 71.5 + 2.4\sin(1.495t)$

where T is in days and R is in multiples of the radius of our sun (695,000 kilometers).

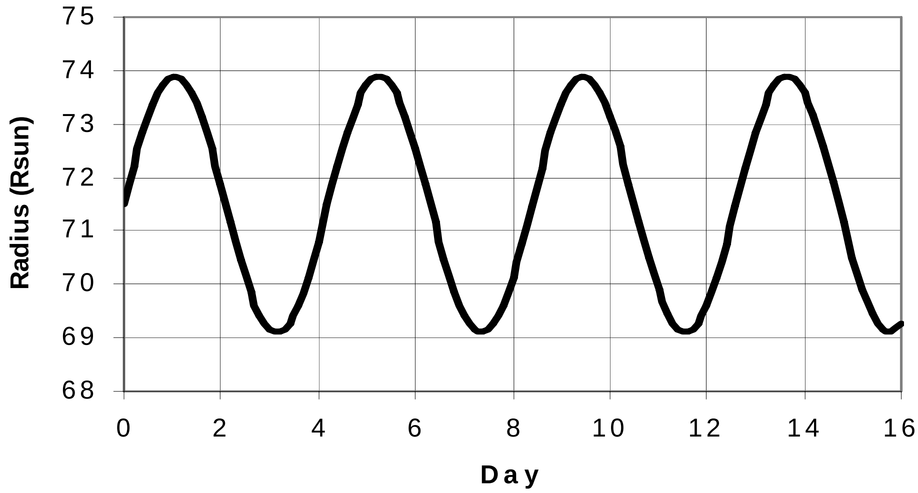
Problem 1 – Graph this function for 16 days. What is the period, in days, of the radius change of this star?

Problem 2 – What is the minimum and maximum radius of the star?

Problem 3 – What is the amplitude of the radius change?

Problem 4 – What is the radius of this star, in kilometers, after exactly one month (30 days) has elapsed?

Problem 1 – Graph this function for 16 days. What is the period, in days, of the radius change of this star?



Answer: Solve for $2\pi = 1.495t$ to get **t = 4.2** days as the period.

Problem 2 – What is the minimum and maximum radius of the star?
Answer: Rmax = 71.5 + 2.4
= **73.9 Rsun**
Rmin = 71.5 – 2.4
= **69.1 Rsun**.

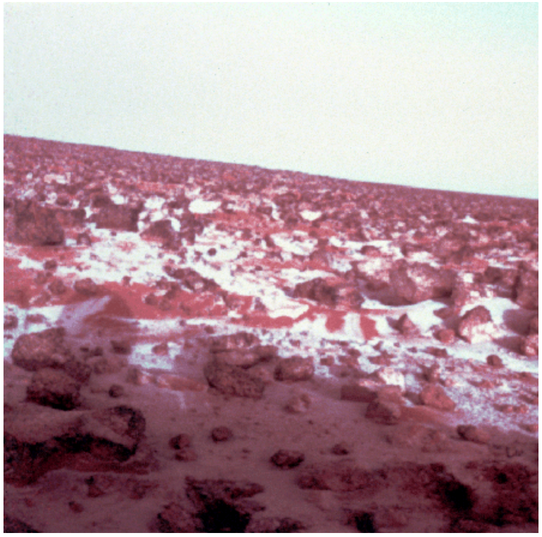
Problem 3 – What is the amplitude of the radius change?
Answer: Amplitude = (Maximum – minimum)/2
= (73.9 – 69.1)/2
= **2.4 Rsun**.

Problem 4 – What is the radius of this star after exactly one month (30 days) has elapsed?

Answer: 1 month = 30days so T = 30 and so:

R(30) = 71.5 + 2.4sin(1.495*30) = 71.5 + 2.4 (0.705) = 73.2 Rsun. Since 1 Rsun = 695,000 km, the radius of AH Velorum will be 73.2 x 695,000 = **50,900,000 kilometers**. (Note: the orbit of Mercury is 46 million kilometers).

Solving Trigonometric Equations



Although it has an Earth-like 24-hour day, and seasonal changes during the year, Mars remains a cold world with temperatures rarely reaching the normal human comfort zone.

This image, taken by the NASA'S Viking lander in 1976, shows frost forming as local winter approaches. This frost, unlike water, is carbon dioxide which freezes at a temperature of –109 F.

The formula that estimates the local surface temperature on Mars on July 9, 1997 from the location of the NASA Pathfinder rover is given by:

$$\mathbf{T(t) = -50 - 52 \sin(0.255\, t - 5.2)} \quad \text{Fahrenheit}$$

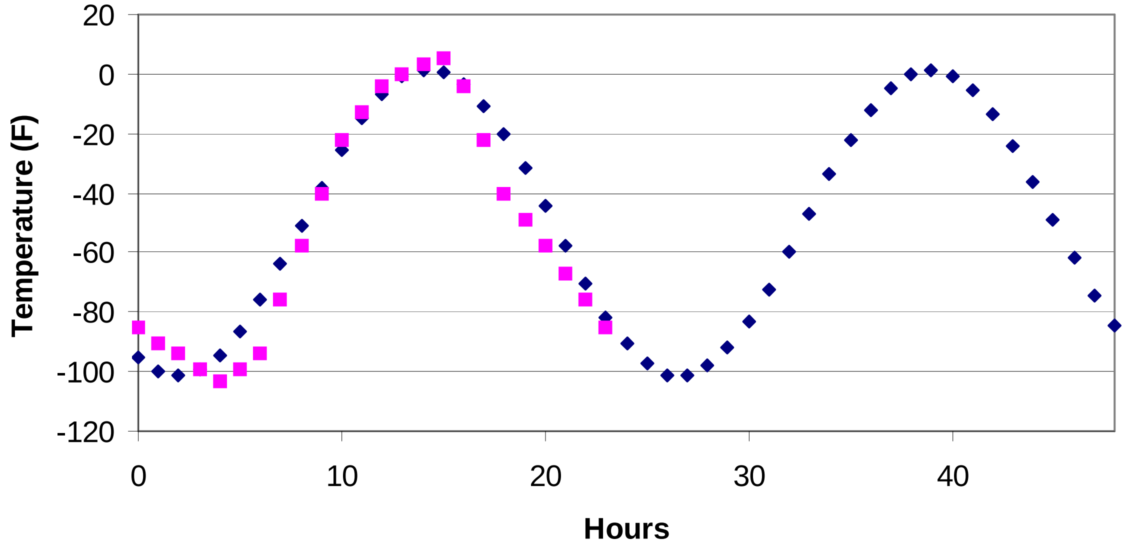
where t is the number of hours since local midnight.

Problem 1 – Graph the function for a 48-hour time interval.

Problem 2 – What is the period of the function?

Problem 3 – The martian day is 24.5 hours long. During what time of the day, to the nearest hour, is the temperature above -20 Fahrenheit, if t=0 hours corresponds to a local time of 03:00 AM?

Problem 1 – Graph the function for a 48-hour time interval.



The squares represent the actual temperature data, and the diamonds represent the function T(F).

Problem 2 – What is the period of the function?

Answer: $2\pi = 0.255$ P
$6.242 = 0.255P$
Period = 24.5 hours.

Problem 3 – The martian day is 24.5 hours long. During what time of the day, to the nearest hour, is the temperature above -20 Fahrenheit if t=0 hours corresponds to a local time of 03:00 AM?

Answer: $-20 = -50 - 52 \sin(0.255\, t - 5.2)$
$30 = -52 \sin(0.255\, t - 5.2)$
$-0.577 = \sin(0.255\, t - 5.2)$
$-0.577 = \sin(x)$

This happens for two values of the angle, x = -0.615 radians in Quadrant 4 and x = -2.186 radians in Quadrant 3.

Then $-0.615 = 0.255\, t - 5.2$ **t = 18 hours** so the time is 03:00 + 18 = **21:00**

And $-2.186 = 0.255\, t - 5.2$ so **t = 12 hours** so the time is 03:00 + 12 = **15:00**

Satellites are designed to make accurate measurements of many kinds of physical quantities including temperatures, the intensity of gravity and electromagnetic fields and other quantities.

At the same time, satellites spin at a rate of 2-5 rpm to keep them stable and avoid tumbling randomly as they travel along their orbit.

The result is that every measurement is 'modulated' by the periodic rotation of the spacecraft.

When one coordinate system is rotated with respect to another the original coordinates are transformed according to

$X' = x \sin(\theta) - y \cos(\theta)$

$Y' = x \cos(\theta) + y \sin(\theta)$

where θ is the rotation angle, measured counter-clockwise from the x-axis

Problem 1 – The original magnetic field has the components x = 35 nanoTeslas and y = 94 nanoTeslas, and the satellite measures the field to be x' = 100 nanoTeslas and y' = 0 nanoTeslas. What is the viewing angle, θ, of the magnetometer in degrees?

Problem 1 – The original magnetic field has the components x = 35 nanoTeslas and y = 94 nanoTeslas, and the satellite measures the field to be x' = 100 nanoTeslas and y' = 0 nanoTeslas. What is the viewing angle of the magnetometer in degrees?

Answer:

$X' = x \sin(\theta) - y \cos(\theta)$ becomes $100 = 35 \sin(\theta) - 94 \cos(\theta)$

$Y' = x \cos(\theta) + y \sin(\theta)$ becomes $0 = 94 \sin(\theta) + 35 \cos(\theta)$

Solve by elimination:

$100/35 = \sin(\theta) - 94/35 \cos(\theta)$

$0 = \sin(\theta) + 35/94 \cos(\theta)$

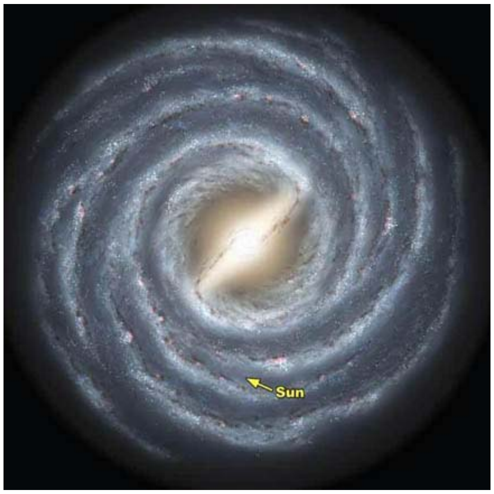
subtracting to get:

$100/35 = - 94/35 \cos(\theta) - 35/94 \cos(\theta)$

so $\cos(\theta) = -0.935$

and so $\boldsymbol{\theta = 159^{o}}$

Note: Students may check the result, but need to make consistent use of significant figures. The magnetic field measurements are to two significant figures. The angle measurements should be to the nearest integer…no decimals



Artistic rendition of Milky Way: (Courtesy NASA/JPL-Caltech/R Hurt, SSC/Caltech)

In a rotating galaxy, the speed of rotation at a given distance from the nucleus can be determined by knowing the distribution of mass in the galaxy.

Astronomers use this fact, together with the measured distance to a star or nebula and its angular distance from the center of the galaxy, to determine its distance from the center of the galaxy.

This method has been used in the Milky Way to map out the locations of many stars, star clusters and nebula in the Milky Way as seen from Earth.

A formula that estimates the distance, r, from an object to the center of the Milky Way, given its distance from the Sun, L, the observed angle, θ, between the object and the center of the Milky Way, and the distance from the Sun to the center of the Milky Way, R, is given by the Law of Cosines as:

$$r^2 = L^2 + R^2 - 2LR\cos(\theta)$$

Problem 1 – Astronomers measure the distance to the star cluster Berkeley-29 as 72,000 light years. This cluster is located at a 'longitude' angle of $\theta = 198^o$. If the distance from the Sun to the galactic center is 27,000 light years, how far is Berkeley-29 from the center of the Milky Way?

Problem 2 - Astronomers want to find young star-forming regions in the Perseus Spiral Arm of the Milky Way. They can only obtain optical images from objects within 10,000 light years of the Sun. If the Perseus Spiral Arm is located 35,000 light years from the center of the Milky Way, what are the two galactic longitude angles, θ, that they can search to find these objects?

Problem 1 – **Answer:** R = 27,000 Light years , L = 72,000 Light years, and θ = 198 so

$$r^2 = (72,000)^2 + (27,000^2 - 2(72000)(27,000)\cos(198)$$

so $$r^2 = 5.91\times10^{9} + 3.69 \times 10^{9}$$ and so **r = 98,000 light years**.

See Digital Sky Survey image of the cluster below. The cluster is located on the right-hand edge as a faint group of stars.

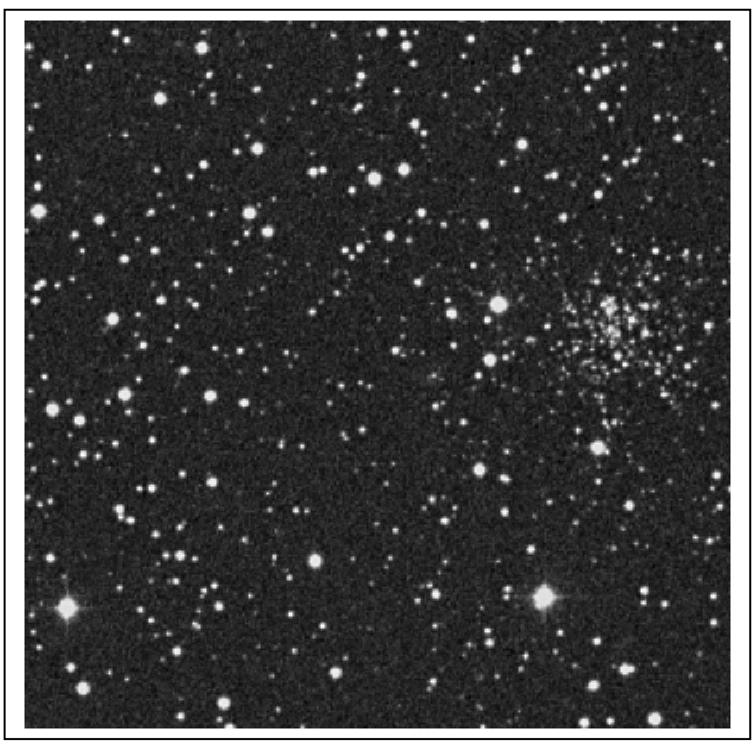
Problem 2 - Astronomers want to find young star-forming regions in the Perseus Spiral Arm of the Milky Way. They can only obtain optical images from objects within 10,000 light years of the Sun. If the Perseus Spiral Arm is located 35,000 light years from the center of the Milky Way, what are the two galactic longitude angles, θ, that they can search to find these objects?

Answer:

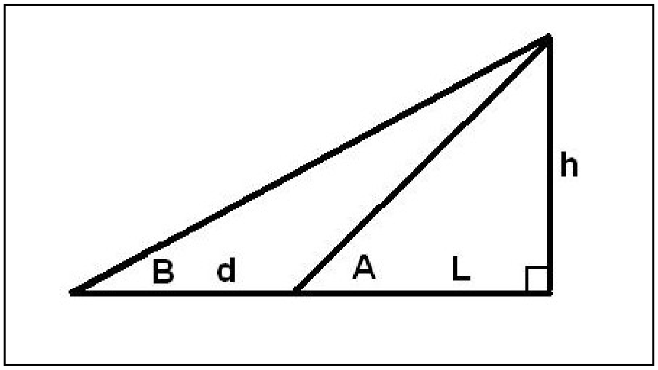
$$(35,000)^2 = (10,000)^2 + (27,000^2 - 2(10,000)(27,000)\cos(\theta)$$

so θ = **137^{o}**.

The solution is symmetric about the axis connecting the Sun with the Galactic center. That will mean that a second angle, 360-137 = **223^{o}** is also a possible solution.



Solving Trigonometric Equations



A very practical problem in applied geometry is to determine the height of an object as shown in the figure to the left. The challenge is to do this when you cannot physically determine the distance, L, because it may be partially obstructed by the object itself!

A related problem is to determine the width of a river when the distance, L, includes the river, which cannot be crossed.

Suppose that you are able to determine the two angles, A and B, and the length, D.

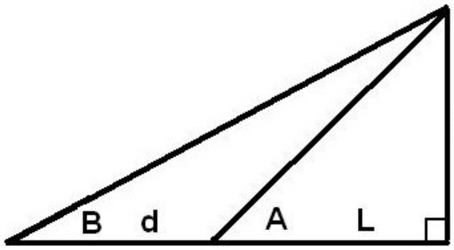
Problem 1 – Explain in words a situation in which, h, represents the height of a mountain and the method by which you were able to determine, A, B and d.

Problem 2 – Using trigonometric functions, define all of the angles in the problem in terms of h, L and d.

Problem 3 – What is the equation that defines h in terms of the two measured angles and the distance, d?

Problem 4 - How would you re-interpret this geometry if the distance, L, represented the width of a river?

Problem 5 – At the base of Mount Everest, a surveyor measures the angle to the summit and gets $A = 2.53^o$. He moves d = 50 kilometers and measures the angle $B= 2.03^o$. What would he estimate as the height of Mount Everest, in meters, and why could he not measure L directly and use a simpler formula?



Problem 1 –Answer: **Standing at a location L from the mountain, you measure the angle, A, with a theodolite. Then you walk d meters directly away from the mountain and measure a second angle, B.**

Problem 2 – Answer:
Tan A = h/L
Tan B = h/(L + d)

Problem 3 – Answer:

Tan(B) = h/(L+d) so L = hcot(B) - d
Tan(A) = h/(hcot(B)-d)

H (cotB – cotA) = d so
H = d/(cotB – cotA) or
H = d TanA TanB/(Tan A – TanB)

Problem 4 - Answer: **The distance, h, represents a known distance between two points on the opposite side of the river located parallel to the river on the other shore. At a point on your side of the river on your shore, you measure the angle A. Then you move perpendicular to the river a distance d on your side and measure the angle B.**

Problem 5 - At the base of Mount Everest, a surveyor measures the angle to the summit and gets A = 2.53^{o}. He moves d = 50 kilometers and measures the angle B= 2.03^{o}. What would he estimate as the height of Mount Everest, and why could he not measure L directly and use a simpler formula? Answer:
H = 50 km Tan(2.53)Tan(2.03) / (Tan(2.53) – Tan(2.03)) = 50 km (0.00157/0.00874) = 8.98 kilometers or **8,980 meters**.
The simpler formula requires a measurement of L which is obstructed and inaccessible because it is partially inside the mountain.

Electricity consumption varies with the time of year in a roughly sinusoidal manner. Electrical energy is measured in units of kilowatt-hours. One kWh = 1000 joules of energy used.

Because most of this energy comes from burning fossil fuels, we can convert electricity consumption in kWh into an equivalent number of tons of carbon dioxide released as coal or oil are burned to heat water in a steam turbine which then generates the electricity.

Problem 1 – The electrical energy used each month from a single-family home in suburban Maryland can be found from the monthly electric bills which yield the following data:

t	1	2	3	4	5	6	7	8	9	10	11	12
E	1100	980	850	835	900	1150	1400	1638	1755	1650	1450	1200

Write a trigonometric model based on the cosine function that approximates the electric power usage with a function E(t), where t is the month number (For example: January = 1) and E is the energy in kilowatt-hours. Do not use technology to 'fit' a curve, but determine the period, amplitude and appropriate phase and time-shifts from the data table.

Problem 2 – Graph the model for 24 months. If 700 kg of carbon dioxide are produced in order to generate 1000 kWh from fossil fuels, about how many tons of carbon dioxide were generated by the electrical energy consumption of this during its minimum and maximum months for E(t)?

Problem 1 –

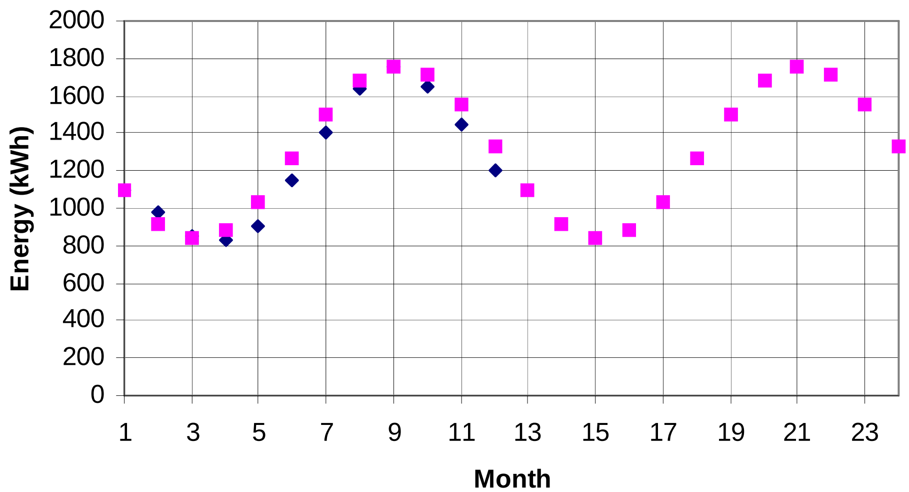
t	1	2	3	4	5	6	7	8	9	10	11	12
E	1100	980	850	835	900	1150	1400	1638	1755	1650	1450	1200

Answer:
Amplitude = (maximum – minimum)/2 = (1755 – 835)/2 = **460 kWh**
Shift = (maximum + minimum)/2 = (1755 + 835)/2 = **1295 kWh**
Period: **12 months**
Phase: For a cosine function, cos(x = '0') gives E = 1755-1295 = +460, but this value occurs for t = 9, so we can either retard x by 360/12x9 = 270o so x = 360t/12 - 270, or we can advance x by 90 degrees so x = 360t/12 + 90
The phase is either **p = 270 or -90**.

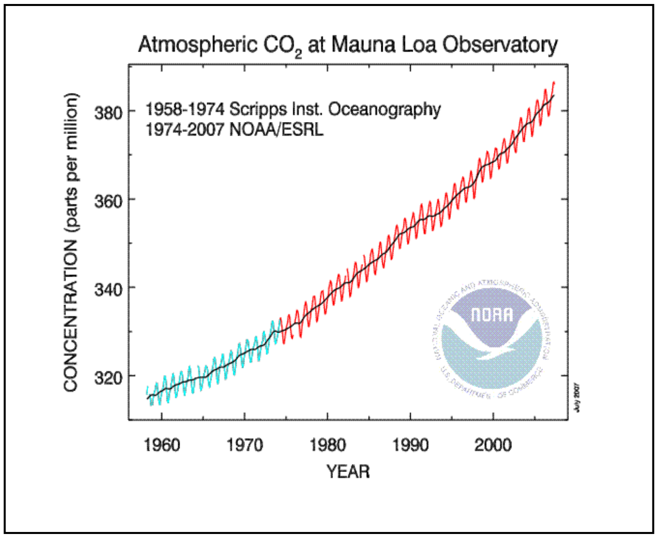
The function is then, for example, **E(t) = 1295 + 460 cos(360t/12 – 90)**

Problem 2 – Graph the model for 24 months. If 700 kg of carbon dioxide are produced in order to generate 1000 kWh from fossil fuels, about how many tons of carbon dioxide were generated by the electrical energy consumption of this during its minimum and maximum months for E(t)?

Answer: See graph below. **The maximum month was September for which 1755 kWh were used, equal to 1.2 tons of carbon dioxide. The minimum month was April for which 835 kWh were used, equal to 0.6 tons of carbon dioxide were generated**.



Note: Adding up the monthly energy usages we get 14,900 kWh per year, which equals about 10.4 tons of carbon dioxide. This is equal to the area under the curve for E(t) converted into tons of carbon dioxide….an extension problem in calculus.



The amount of carbon dioxide in the atmosphere continues to rise each year, contributing to global climate change and specifically a steady increase in global temperatures.

In addition to the steady increase, seasonal sinusoidal variations are also evident as shown in the 'Keeling Curve' graph to the left.

Problem 1 – A section of the data between 2006 and 2008 is shown in the table below.

t	2006.6	2006.7	2006.9	2007.0	2007.2	2007.3
C	377.7	376.6	377.1	380.7	383.6	383.9
t	2007.5	2007.6	2007.8	2007.9	2008.0	2008.2
C	381.6	377.9	376.3	377.0	380.5	383.5

Write a trigonometric model based on the sine function that approximates the carbon dioxide change in parts per million (ppm) with a function C(t), where t is the year. Do not use technology to 'fit' a curve. Assume a period of 1.0 years, and determine the amplitude and appropriate phase and time-shifts from the data table.

Problem 2 – Graph the model between 2007 to 2009.

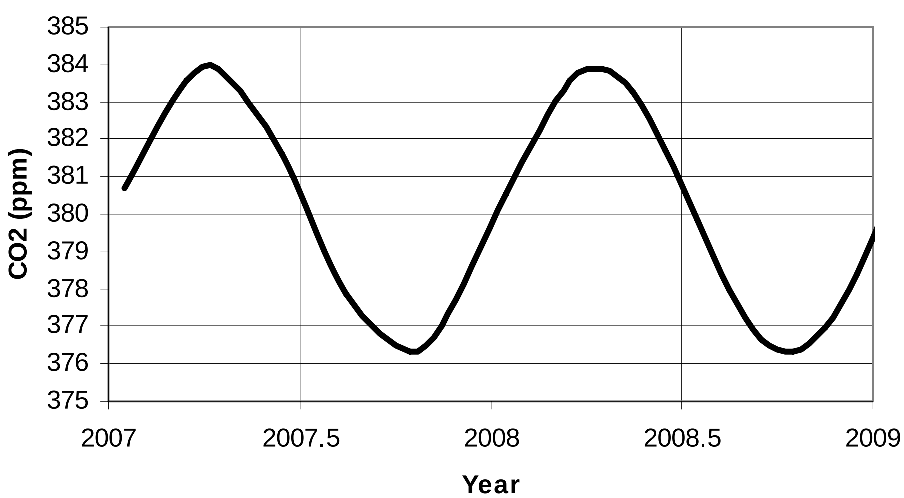
Problem 3 - During what time of the year is the additional carbon dioxide at it's A) minimum? B) maximum?

Problem 1 – A section of the data between 2006 and 2008 are shown in the table below.

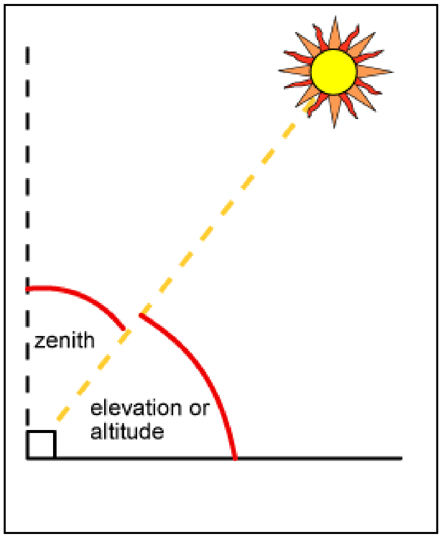
t	2006.6	2006.7	2006.9	2007.0	2007.2	2007.3
C	377.7	376.6	377.1	380.7	383.6	383.9
t	2007.5	2007.6	2007.8	2007.9	2008.0	2008.2
C	381.6	377.9	376.3	377.0	380.5	383.5

Answer:
Amplitude = (maximum – minimum)/2 = (383.9 – 376.6)/2 = **3.7 ppm**
Shift = (maximum + minimum)/2 = (383.9 + 376.6)/2 = **380.3 ppm**
Period: **12 months**
Phase: For a sine function, sin(x = '1') gives C = 383.9-389.3 = +3.6, but this value occurs for t = 2007.3.
We can either retard x by 360/1.0x(2007.3 – 2007) = 108^o so x = 360t + 108,
(Note: each year = 360^o so subtract the 2007 to get the residual degrees in the shift)
Or we can advance x by (360 – 108) = 252^o so x = 360t - 252
The phase is either **p = 108^o or -252^o**.
The function is then, for example, **C(t) = 380.3 + 3.7 sin(360t – 108)**

Problem 2 – Graph the model between 2007 to 2009.



Problem 3 - During what time of the year is the additional carbon dioxide at it's A) minimum? B) maximum? Answer**: It is at its minimum (376.4 ppm) in September, and at is maximum (384.0 ppm) in March. These are near the start of Fall and Spring near the tie of the Equinoxes.**



A new kind of telescope has been designed that uses a vat of rotating liquid mercury instead of a glass mirror. Liquid mercury mirrors can be made extremely large. However, they cannot be tilted away from a horizontal position to view stars or objects at other locations in the sky.

Viewing stars or other objects that are only directly overhead forces astronomers to wait for an object to be carried by Earth's rotation so that it passes directly through the telescope's field of view. At that time, the elevation angle of the object is exactly 90 degrees from the southern horizon.

A trigonometric equation lets astronomers predict the elevation angle of an object:

$$\sin(e) = \sin(d)\sin(L) + \cos(d)\cos(L)\cos(T)$$

where **e** is the elevation angle, **d** is the declination coordinate of the object, **L** is the latitude of the telescope and **T** is the hour angle of the object where T=0 is due south on the north-south meridian, -180 is 180^{o} east of the meridian and +180 is 180^{o} west of the meridian.

Astronomers want to make sure that, in addition to other research, that several prime targets will be visible to the telescope. These objects are: 1) M-13: The Globular Cluster in Hercules at a declination of $+36.5^{o}$ and 2) M-31: The Andromeda Galaxy at a declination of $+41.0^{o}$.

Problem 1 - What are the two equations for sin(e) for M-13 and M-31?

Problem 2 – Using technology (calculator or spreadsheet) at what latitude, L, will A) M-13 pass exactly through the Zenith ($e=90^{o}$) on the meridian (T=0)? B) M-31 pass exactly through the Zenith ($e=90^{o}$) on the meridian (T=0)?

Problem 3 – Graph e(L) for both sources over the domain T:[-5^{o}, $+5^{o}$] and range e:[$+80^{o}$, $+90^{o}$]. Suppose the field of view of the telescope is a circle about 10^{o} in diameter. At what latitude will the two objects pass within 3^{o} of the zenith?

Answer Key

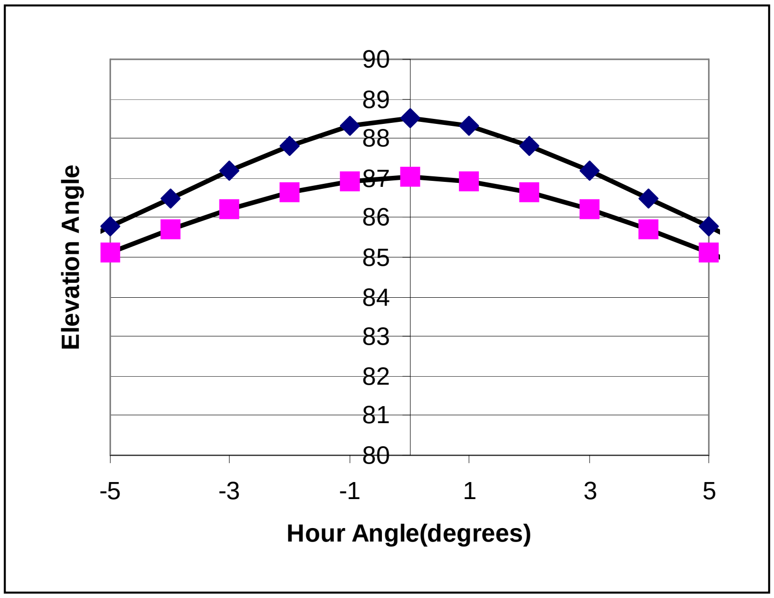
Problem 1 - What are the two equations for sin(e) for M-13 and M-31?
Answer:
M-13: **sin(e) = 0.595 sin(L) + 0.804 cos(L) cos(T)**
M-31: **sin(e) = 0.656 sin(L) + 0.755 cos(L) cos(T)**

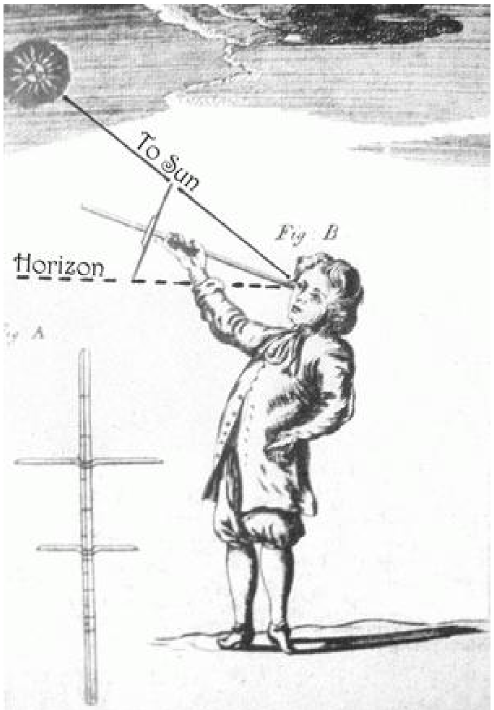
Problem 2 – At what latitude, L, will M-13 pass exactly through the Zenith ($e=90^o$) on the meridian (T=0)?
Answer: A) sin(90) = 0.595 sin(L) + 0.804 cos(L) cos(0)
X(L) = 0.595 sin(L) + 0.804 cos(L)
A calculator can be programmed with x(L) and plotted. The intercept at x=1.0 gives the latitude, L. and so **L = +36.5^{o}**.

B) sin(90) = 0.656 sin(L) + 0.755 cos(L) cos(0)
X(L) = 0.656 sin(L) + 0.755 cos(L)
and so **L = +41.0^{o}**
Students will note that for a latitude equal to the declination of the source, the source will pass exactly through Zenith at e=90.

Problem 3 – Graph e(L) for both sources over the domain T:[-5^o, $+5^o$] and range e:[$+80^o$, $+90^o$]. Suppose the field of view of the telescope is a circle about 10^o in diameter. At what latitude will the two objects pass within 3^o of the zenith?

Answer: The graph below shows that, if the latitude is (41.0 + 36.5)/2 = **+38.8^{o}**, both objects will pass within 90^o-87^o = 3^o of the Zenith.





Credit: Illustration from Jonas Moore ,"A New System of Mathematicks", 1681

Ancient astronomers used instruments called cross staffs to measure directly the angular distances between points in the sky, which could be pairs of stars, or planets and the sun. Today, this process is much simpler because we know the coordinates of objects in the sky from numerous catalogs that include as many as 500 million objects.

The positions of stars in the sky are measured in terms of their declination angle, δ, from -90^{o} to $+90^{o}$, and their Right Ascension in units of time from $0^{h}:00^{m}$ to $24^{h}:00^{m}$. We can convert the RA time into an angular value α = RA*360/24.

Sometimes, astronomers need to know the angular distance D, between two objects in the sky ($\alpha1$,$\delta1$) and ($\alpha2$,$\delta2$).

$$\cos D = \sin \delta2 \sin \delta1 + \cos \delta2 \cos \delta1 \cos (\alpha2 - \alpha1)$$

Problem 1 – Suppose that two objects have the same Right Ascension. What is the simplified formula for the sky angle D?

Problem 2 – The two brightest stars in the sky are Sirius (6^{h} 41^{m}, -16^{o} 35’) and Canopus (6^{h} 22^{m}, -52^{o} 38’). What is the angular separation of these stars in the sky?

Problem 1 – Suppose that two objects have the same Right Ascension. What is the simplified formula for the sky angle D?

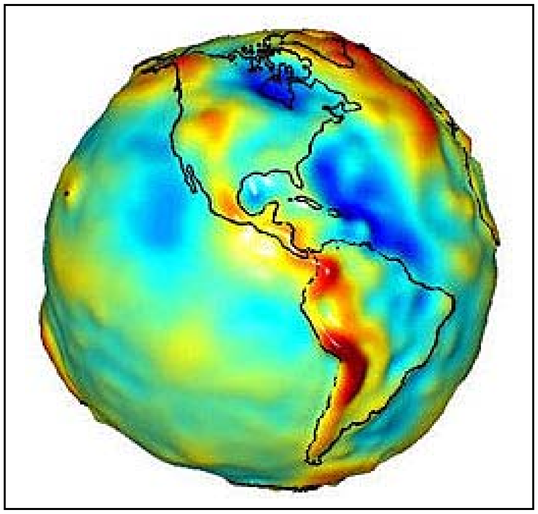
Answer: $\alpha 2 = \alpha 1$ so cos(0) = 1 and so
$\cos D = \sin \delta 2 \sin \delta 1 + \cos \delta 2 \cos \delta 1$
Then using cos(A-B) we have
$\cos D = \cos (\delta 2 - \delta 1)$ so
$D = \delta 2 - \delta 1$

Problem 2 – The two brightest stars in the sky are Sirius ($6^h\ 41^m$, $-16^o\ 35'$) and Canopus ($6^h\ 22^m$, $-52^o\ 38'$). What is the angular separation of these stars in the sky?

Answer: Convert the RA and Declination coordinates in decimal form to angles in degrees:
$\alpha 1 = 6^h\ 41^m = 6.68 \times 360/24.0 = 100^o$ and
$\delta 1 = -16 - 35/60 = -16.58^o$
$\alpha 2 = 6^h\ 22^m = 6.26 \times 360/24.0 = 94°$
$\delta 2 = -52 - 38/60 = -52.63^o$

Then $\cos D = \sin (-52.63)\sin(-16.58) + \cos(-52.63)\cos(-16.58) \cos (94 - 100)$
$\cos D = (-0.795)(-0.285) + (0.607)(0.958)(0.995)$
$\cos D = (0.227) + (0.579)$
$\cos D = 0.806$
Then **D = 36.3 degrees.**

Using Double-Angle Formulas



The acceleration of gravity on Earth changes depending on the distance from the center of Earth, and the density of the rock or water. It also changes because the rotation of Earth decreases the acceleration at the equator compared to the acceleration at the geographic North Pole. Although satellites such as NASA's GRACE can measure this acceleration to high precision, as shown in the figure to the left, it is still convenient for some applications to use a simple formula to estimate the acceleration.

A trigonometric equation models the dominant component of the acceleration of gravity on the spherical Earth globe from the equator to the pole, given by:

$$\mathbf{A(\theta) = 978.0309 + 5.1855 \sin^2(\theta) - 0.0057 \sin^2 (2\theta) \quad cm/sec^2}$$

Problem 1 – Re-write the formula only in terms of the sine and cosine of the latitude angle, θ.

Problem 2 – A geologist wants to search for underground deposits in a location at a latitude of 37^{o}. What will be the acceleration of gravity to which he must calibrate his accelerometer in order to look for differences that could indicate buried materials?

Problem 3 – Graph this function over the domain θ:$[0^{o},90^{o}]$. A geologist wants to survey a region with an instrument that looks for differences in gravity from a pre-set value of 980 cm/sec^2. At what latitude can he perform his surveys with this instrument?

$A(\theta) = 978.0309 + 5.1855 \sin^2(\theta) - 0.0057 \sin^2(2\theta)$ cm/sec^2

Problem 1 – **Answer:** Since $\sin 2\theta = 2 \sin\theta \cos\theta$

We have $A(\theta) = 978.0309 + 5.1855 \sin^2(\theta) - 0.0057 (2 \sin(\theta)\cos(\theta))^2$ cm/sec^2

$A(\theta) = 978.0309 + 5.1855 \sin^2(\theta) - 0.0228 \sin^2(\theta)\cos^2(\theta)$ cm/sec^2

Problem 2 – Answer: For q = 37^o,

A(37) = 978.0309 + 5.1855 $\sin^2(37) - 0.0057 \sin^2(74)$ cm/sec^2

A(37) = 978.0309 + 1.8781 – 0.0053

A(37) = 979.9037 cm/sec^2

Problem 3 – Answer: See graph below. The value of '980' occurs for a latitude of **θ = +38.0^o.**