



$\frac{\partial f}{\partial y}$ $\operatorname{tg} x \cdot \operatorname{cotg} x = 1$ $2x^2yy' + y^2 = 2$

$B = \begin{pmatrix} 2 & 1 & -1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix}$

$a^2 = b^2 + c^2 - 2bc \cos$

$2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$ $\operatorname{tg} x = \frac{\sin x}{\cos x}$

$\lambda x - y + z = 1$
 $x + \lambda y + z = \lambda$
 $x + y + \lambda z = \lambda^2$



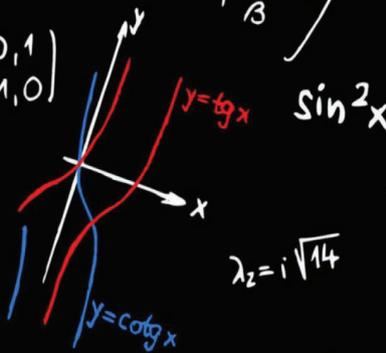
$m \frac{\sqrt{h^3+1+n}}{\sqrt[3]{3h^2+2n-1}}$

$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$

$y = \sqrt[3]{x+1}$; $x = \operatorname{tg} t$

$x_1 = \begin{pmatrix} \alpha + \beta + \gamma \\ \alpha \\ \beta \end{pmatrix}$

$c = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix}$



$\alpha, \beta, \gamma \in \mathbb{C}$
 $\gamma = 2^{-x} + 1, \epsilon = 0.005$

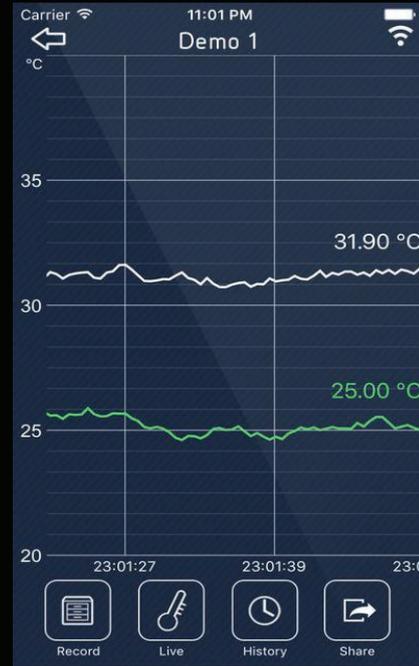
$z^2 - xy z = e ; A [0; e; 1]$
 $= \frac{2}{5}$
 $k\alpha + \beta \neq 0 ; \mu \neq 0$

$\frac{2x}{x^2+2y^2} = 2$ $z = \frac{1}{x} \operatorname{ar} t$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix} ; x=0, y=1, z=2$ $y' - \frac{\sqrt{y}}{x+2} = 0 ; y|_0$

$A = [1; 0; 3]$ $\cos \varphi = \frac{(1; 0)}{\sqrt{1^2+0^2}}$



A Guide to Smartphone Sensors

Experimeter's Guide To Smartphone Sensors

Dr. Sten Odenwald

Version 6.0: May 2019



Introduction

Smartphones have now become ubiquitous instruments for communications and information retrieval, but as part of their functionality they also contain a variety of sensors to determine their orientation, location and meteorological conditions. Over the years, hundreds of ‘apps’ have been designed to access this hidden information, turning smartphones into powerful measurement platforms, and usually at no cost. Meanwhile, mathematics plays a key role in all scientific investigations because research and observation usually involve taking some form of data, and often this data is in numeric form. The task then becomes one of organizing the data, looking for patterns in it, and finding an explanation for these patterns. These explanations, called theories, make predictions for future experiments and observations, which then have to be tested to see if what was predicted is in line with what was observed. All of these steps involve working with numbers.

The first part of this Guide examines a wide variety of apps that measure interesting physical parameters such as temperature, pressure, motion, light intensity and many more quantities. The objective is to assess how well these apps and sensors actually measure the indicated parameters so that formal and informal educators and citizen scientists can gauge their reliability. Common apps and smartphone platforms are compared to professional-grade measurements to determine reliability and accuracy. A collection of 45 experiments are also provided for using these apps to make a variety of interesting real-world measurements.

In the second part of this Guide, you will use data created from a variety of smartphone apps and examine this data using a variety of mathematical operations. Smartphone apps can be downloaded that measure a variety of interesting physical properties such as magnetic fields, pressure, temperature and acceleration. This guide will show you how to access this data, which you can record in an Excel spreadsheet and then manipulate to search for patterns or to establish the accuracy and precision of the smartphone sensors.

Beginning with the concept of ‘significant figures’ you will investigate the statistical processes of using mean and median, and standard deviation to explore how to analyze data. These skills will then be extended to discuss how to explore the accuracy and precision of a measurement, calibrating measured data, and testing a prediction against measurements to determine their statistical significance.

Dr. Sten Odenwald

NASA Space Science Education Consortium

May, 2019 (Version 1.0)

Table of Contents

Alignment with Standards	8
ISTE Technology in Education	8
Math and Science Education NGSS.....	8

I. A Guide to Smartphone Sensors and Experiments

Magnetism	10
Experiment 1: Comparing Smartphone and Geomagnetic Coordinates	12
Experiment 2: Finding the Smartphone Magnetometer!	15
Experiment 3: Can magnetism be shielded?	16
Experiment 4: Measuring the parameters of a magnet	17
Experiment 5: Detecting Geomagnetic Storms.....	19
Clinometer	21
Experiment 1: Measurement of sun angle	22
Experiment 2: Finding your Latitude using the Sun.....	23
Experiment 3: Finding the altitude of a rocket or a cloud	25
Radiation	27
Experiment 1: Measuring your Local Background Radiation.....	30
Experiment 2: Measuring Radiation Dosages In a Jet Plane.....	31
Experiment 3 – Other Radiation Sources.....	31
Gravity	32
Experiment 1: Rotation of Earth	34
Pressure	37
Experiment 1: Air Pressure at Different Altitudes	39
Experiment 2: Monitoring Weather Changes	40
Experiment 3 – A Simple Pressure Thermometer.....	43
Position-Distance	45
Experiment 1: How accurate are the distance meters?.....	46
Compass Bearing	47
Experiment 1: Comparison with Analog Compass.....	48

Experiment 2: Comparison with Magnetometer Direction	49
Sound Intensity	52
Experiment 1: Creating a Sound Level Library	54
Experiment 2: Finding the Zero-Level.....	55
Experiment 3: Smallest Detectable Difference	56
Other Experiment Suggestions	56
Acceleration	57
Experiment 1: Comparison of Static Acceleration	59
Experiment 2: Measuring Controlled Acceleration with a Turntable	60
Experiment 3: The location of the acceleration sensor.....	64
The Gyroscope	66
Experiment 1: Simple controlled rotations using a turntable.....	67
Light Intensity	71
Experiment 1: Exploring Lumens and Lux.	73
Experiment 2: Light Flux and Electrical Power.....	75
Experiment 3: A Catalog of Every-Day Light Levels	76
Experiment 4: The World of Dim Lighting	77
Experiment 5: Reflectivity and Albedo	78
Experiment 6: Atmospheric Aerosols and Haze.....	79
Experiment 7: Detecting the seasonal insolation change.....	82
Color Temperature	86
Experiment 1: App-based Spectrophometers: How good are they?.....	90
Experiment 2: Creating a catalog of common spectra	91
Time	93
Experiment 1: Measuring your smartphone drift rate.	94
Experiment 2: Absolute Time Comparison	97
Experiment 3: Synodic Rotation Period of Earth by Gnomon Shadow.....	98
Experiment 4: Sidereal Rotation Period of Earth by the Stars.....	100
Experiment 5: Sidereal Rotation Period of Earth by Pendulum.....	101
Astrophotography	103
Experiment 1: Finding the Resolution of Your Camera.....	108

Experiment 2: Star Photography.....	111
Experiment 3: Sun Photography	120
Experiment 4: Lunar photography.....	122
Experiment 5: Stellar Photometry with JPEG images	126
Experiment 6: Stellar Photometry with RAW images	131

II. A Guide to Smartphone Mathematics

Working with real-world measurements.....	139
Significant Figures	139
Problem 1 – Significant Figures.....	140
Problem 2 – Significant Figures in Formula Calculations	140
Accuracy versus Precision	141
How Smartphones Create Data.....	143
Analog and digital data	145
Problem 3 – Minimum ADC bits.....	147
Problem 4- Data Word Size	147
Problem 5 – Memory Fill Rate	147
A/D Converters	147
Problem 6 – Number of ADC Comparitors.....	149
Problem 7 – Reference Voltage Intervals	149
Quantization (Digitization) Noise.....	149
Problem 8 – Calculating Quantization Noise	150
How Smartphones Corrupt Data.....	151
Electrical Glitches.....	152
Problem 9 – Glitches and Drifts Oh My!	154
Problem 10 – Investigating glitch causes.....	154
Other Kinds of Variations	154
Problem 11 – When is data not valid?	155
Problem 12 – Peak to Peak Variations.....	157
How the Environment Corrupts Data	158
Local magnetic changes	158

Problem 13 – When not to ‘go long’	159
Barometric changes due to weather	159
Problem 14 - Pressure and Weather	161
The Zen of Data Analysis	162
Scaling and Rounding	162
Problem 15 – Scaling from one set to another	163
Problem 16 – Scaling and Rounding.....	163
Statistical Analysis.....	164
Computing Mean, Median, Mode.....	164
Problem 17 – Calculating Means with the Associative Law	166
Data Histogramming and Median Filtering.....	167
Problem 18 – Median Filtering	168
Problem 19 – Binning data to remove outliers.....	168
The effect of digitizing the data	169
Problem 20: Exploring Digital Data	170
Standard Deviations and Measurement Uncertainty	171
Problem 21 Degrees of Freedom	172
Problem 22 Sigma and Significance.	173
Problem 23 – Calculating M and σ	174
Problem 24 Pressure data analysis	176
Data Modeling and Significance Testing.....	180
Method of least-squares.....	181
The Fischer F-Test	183
The R ² coefficient of determination statistic	184
Noise Analysis.....	186
Problem 25 – Exploring copy variance: σ_c	186
A Variety of Smartphone Data Types	188
Problem 26 – Data Rates and File Size.....	188
Linear Accelerometer.....	188
G-force meter on stationary tabletop	190
Barometer – indoors.....	191

Magnetometer – on tabletop	192
Problem 27 – Magnetic Bearing Changes.....	194
Inclinometer – on tabletop	196
Light meter.....	197
Sound meter	198
Calibrating Sensors	200
Sound Calibration	200
Relative Performance Tests	201
iPhone and Samsung comparison.....	202
Absolute Performance Tests	202
Comparison with multiple Samsung phones	206
Figure Credits:.....	207

Alignment with Standards

ISTE Technology in Education

Knowledge Constructor: Students critically curate a variety of resources using digital tools to construct knowledge, produce creative artifacts and make meaningful learning experiences for themselves and others.

- **3a** - Students plan and employ effective research strategies to locate information and other resources for their intellectual or creative pursuits.
- **3b** - Students evaluate the accuracy, perspective, credibility and relevance of information, media, data or other resources.
- **3c** - Students curate information from digital resources using a variety of tools and methods to create collections of artifacts that demonstrate meaningful connections or conclusions.
- **3d** - Students build knowledge by actively exploring real-world issues and problems, developing ideas and theories and pursuing answers and solutions.

Math and Science Education NGSS

From: Appendix F: <https://www.nextgenscience.org/get-to-know>

A Science Framework for K-12 Science Education provides the blueprint for developing the Next Generation Science Standards (NGSS). The Framework expresses a vision in science education that requires students to operate at the nexus of three dimensions of learning: Science and Engineering Practices, Crosscutting Concepts, and Disciplinary Core Ideas. The Framework identified a small number of disciplinary core ideas that all students should learn with increasing depth and sophistication, from Kindergarten through grade twelve. Key to the vision expressed in the Framework is for students to learn these disciplinary core ideas in the context of science and engineering practices.

Practice 4 Analyzing and Interpreting Data

Once collected, data must be presented in a form that can reveal any patterns and relationships and that allows results to be communicated to others. Because raw data as such have little meaning, a major practice of scientists is to organize and interpret data through tabulating, graphing, or statistical analysis. Such analysis can bring out the meaning of data—and their relevance—so that they may be used as evidence. Engineers, too, make decisions based on evidence that a given design will work; they rarely rely on trial and error. Engineers often analyze a design by creating a model or prototype and collecting extensive data on how it performs, including under extreme conditions. Analysis of this kind of data not only informs design decisions and enables the prediction or assessment of performance but also helps define or clarify problems, determine economic feasibility, evaluate alternatives, and investigate failures. (NRC Framework, 2012, p. 61-62) As students mature, they are expected to expand their capabilities to use a range of tools for tabulation, graphical representation, visualization, and statistical analysis. Students are also expected to improve their abilities to interpret data by identifying significant features and patterns, use mathematics to represent relationships between variables, and take into account sources of error. When possible and feasible, students should use digital tools to analyze and interpret data. Whether analyzing data for the purpose of science or engineering, it is important students present data as evidence to support their conclusions.

Magnetism

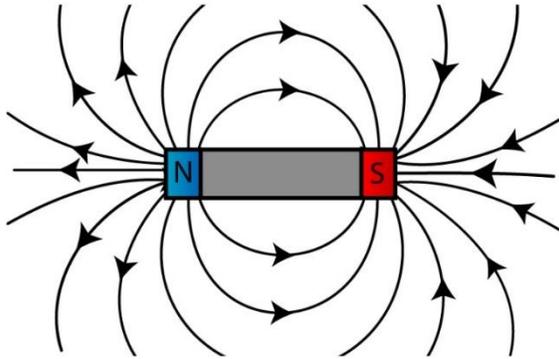


Figure 1: Bar magnet field.

Magnetism is a force that is found across the universe in a variety of objects from stars and planets to galaxies. Unlike gravity, which is a force that operates in one direction along the line connecting the centers of two bodies, magnetic forces are 'vector' forces that require three numbers, one for each vector component in 3-dimensional space, to define their intensity.

Because magnetism is a physical quantity defined by BOTH a magnitude and a direction, it requires a bit more work to describe fully. Because the direction can be anywhere in 3-dimensional space, we have to define magnetism by its direction along three axes perpendicular to each other. Just as 'velocity' measures both the speed and direction of a body in motion, the magnetic field is a similar 'vector' quantity that has to be defined both by its strength and its direction. We need three plots to determine how magnetism changes in time, each plot representing a direction.

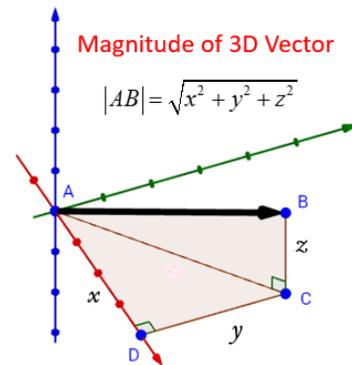


Figure 2: Magnitude of a 3 dimensional vector.

Smartphone apps record the magnetic strength along the three smartphone coordinate axis and usually present these numbers on a display or show them graphically. In these experiments, we will learn about magnetism using your smartphone and the appropriate apps, which you can obtain from the Apple (iOS) or GOOGLE (Android) online stores.

Warning: You should not expose your smartphone to strong magnetic fields for extended periods of time. It will recalibrate the magnetometer and will stop it from working. It will take days before the system gets recalibrated to earth's magnetic field, and if you accidentally magnetize some of the internal metal parts your magnetometer apps will never work properly.

Which app to use.

- **Teslameter 11th (Android; iOS)** – Takes advantage of the built in magnetometer in your iPhone/iPad, allows you to monitor the strength of magnetic field. Displays the raw 3 axes x, y and z magnetometer values; Record and export the data to email for further analysis.
- **Tesla Recorder (Android; iOS)** - Uses the magnetometer sensor of your mobile device to transform your phone or tablet into a fun and easy to use metal detector / electromagnetic scanner. Automated recording for long-time measurements; Diagram showing real-time measurement of electromagnetic field strength in all three dimensions (x, y, z). Record and export the data to email for further analysis.
- **Tesla Field Meter (Android; iOS)** – This application only measures the magnitude of the magnetic field. The Tesla Meter will respond to a static magnetic field as well as the magnetic field radiated through the application of alternating current. In AC mode, Earth's static field vanishes in measurement. If sensor is moved, Earth's static field will appear as changing field. (Cost \$0.99) Record and export the data to email for further analysis.
- **Sensor Kinetics (Android; iOS)** –Sensor Kinetics is a complete Physics class about all of the motion sensors on your iPhone or iPad. The advanced viewers show real-time measurements from the magnetometer. Tap the sensor title line with the chart icon to activate the chart viewer. Each chart viewer provides detailed scrolling graphs for the three relevant axes of the associated sensor. Record and export the data to email for further analysis.

Additional Resources:

Calibrating magnetometer sensors:

Odenwald, 2018, <https://ieeexplore.ieee.org/document/8425973>

CrowdMag citizen science project:

NGDC, <https://www.ngdc.noaa.gov/geomag/crowdmag.shtml>

More experiments with magnetism:

Rick Hoadley, <http://www.coolmagnetman.com/magindex.htm>

Experiment 1: Comparing Smartphone and Geomagnetic Coordinates

The figure below shows the relationship between the direction that a magnetic field is pointing and its magnitude in a Cartesian coordinate system. The origin of the coordinate system is the physical point in space where the field is being measured. The thick line connected to the Origin represents the magnetic field vector. Its length represents the magnitude of the magnetic field.

To describe its direction, professional magnetometers are lined up so that they measure

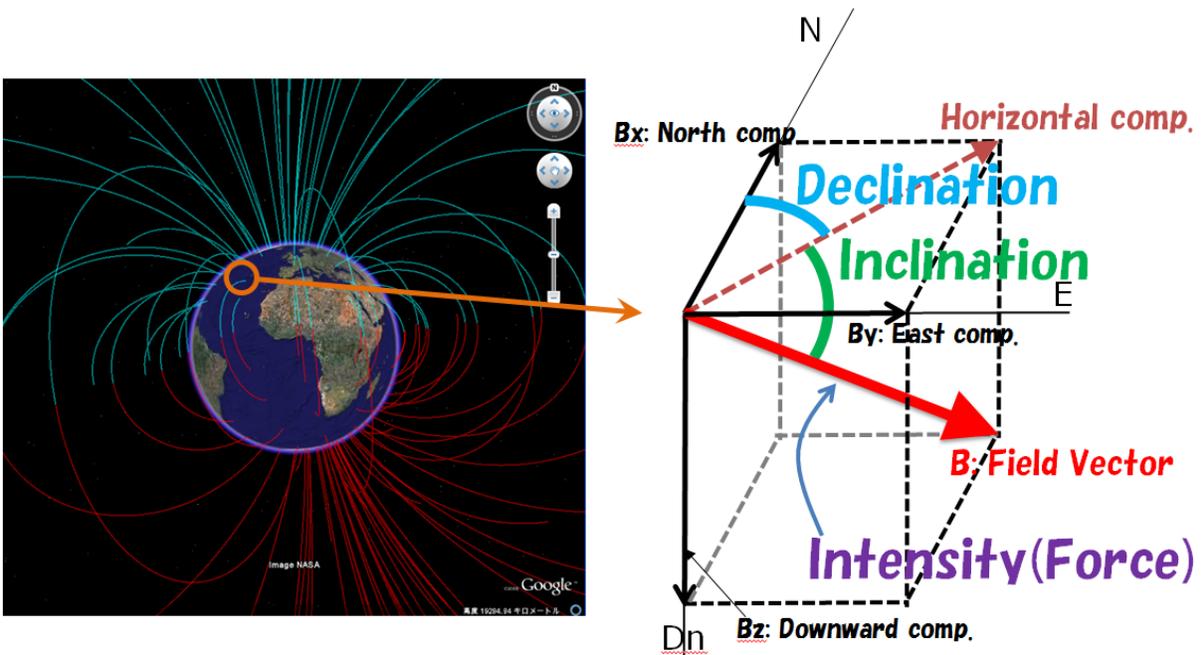
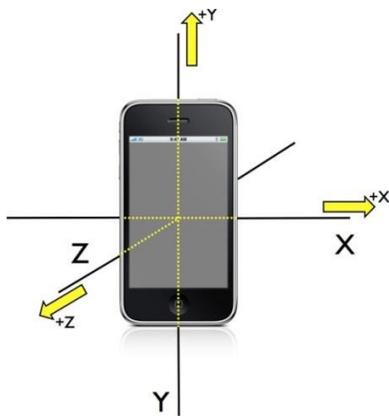


Figure 3. Geometry of Earth's magnetic field.

the magnetic coordinates in the geophysical coordinate system (X,Y,Z) , while smartphones measure in terms of their body coordinate system (x,y,z) .



Step 1: First, place your smartphone on a leveled table-top. That insures that your smartphone's Z axis is parallel with the geophysical Z axis, and that your smartphone's X and Y axis are in the same horizontal plane as the geophysical X and Y axis.

Step 2: Orient your phone to magnetic north. To do this, rotate your smartphone on the table-top until the Bx value reaches a minimum, which should be near zero. Your smartphone is now aligned with the geomagnetic coordinate system.

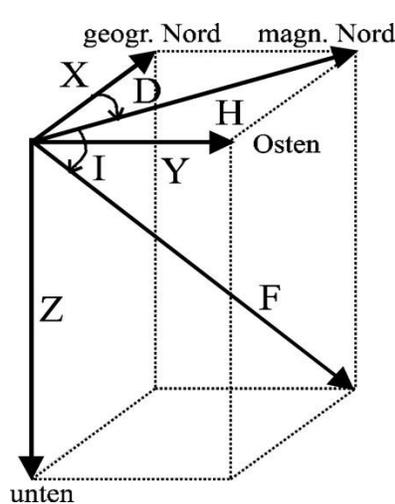
Geomagnetic X: represents the magnetic field strength in the direction of the north magnetic pole. A positive X-value means that part of the magnetic field is pointing north. A negative X-value means that part of the magnetic field is pointing south. For your smartphone, this direction will be the phone's long-axis (By) with positive values increasing to the north and negative values directed south.

Geomagnetic Y: represents the magnetic field strength 90 degrees from the x-direction in the "magnetic east" direction. A positive y-value means that part of the magnetic field is pointing towards magnetic east. A negative y-value means that part of the magnetic field is pointing towards magnetic west. For your smartphone, this direction is along the short axis (Bx) with positive values directed eastwards and negative values westwards.

Geomagnetic Z: represents the magnetic field strength in the local nadir direction (vertically down) with negative values towards zenith and positive values towards nadir. For your smartphone, the z-axis values (Bz) are reversed and positive towards zenith and negative towards nadir.

To summarize: Geomagnetic X = smartphone y ; Geomagnetic Y = smartphone x ; Geomagnetic Z = smartphone - z

This means that the magnetic field components (bx,by,bz) measured by the smartphone magnetometer are related to the standard geomagnetic coordinate components (Bx,By,Bz) according to: $B_x = b_y$; $B_y = b_x$; $B_z = -b_z$. For example, if you measure with an app that (0.1 μT , 18.3 μT , -46.6 μT) for (bx,by,bz) the standard geomagnetic components are $B_x = 18.3 \mu\text{T}$; $B_y = 0.1 \mu\text{T}$; $B_z = 46.6 \mu\text{T}$.



according to: $B_x = b_y$; $B_y = b_x$; $B_z = -b_z$. For example, if you measure with an app that (0.1 μT , 18.3 μT , -46.6 μT) for (bx,by,bz) the standard geomagnetic components are $B_x = 18.3 \mu\text{T}$; $B_y = 0.1 \mu\text{T}$; $B_z = 46.6 \mu\text{T}$.

In the horizontal geomagnetic X-Y plane, the angle between geomagnetic X-axis and the Y-axis is the compass bearing to magnetic North and we can find this from $D = \text{arcTan}(B_x/B_y)$ which for this example gives $D = 89.7^\circ$, which is nearly due-East.

In the vertical geomagnetic plane, a free, movable compass needle points in a direction that is not actually the true north. Instead it follows the orientation of Earth's magnetic field and also slants upward or downward in the vertical plane. While the angle between the compass direction and the magnetic north direction in a horizontal plane is called **D** - the magnetic declination (positive value for eastward), the angle between the compass direction and horizontal plane is called **I** - the inclination (positive value for downward) or 'dip angle', where.

$I = \text{arcTan}(B_z/H)$ where $H^2 = B_x^2 + B_y^2$ In the example above for which $D = 89.7^\circ$, we have $H = 18.3 \mu\text{T}$ and $B_z = 46.6 \mu\text{T}$ so $I = 68.6^\circ$.

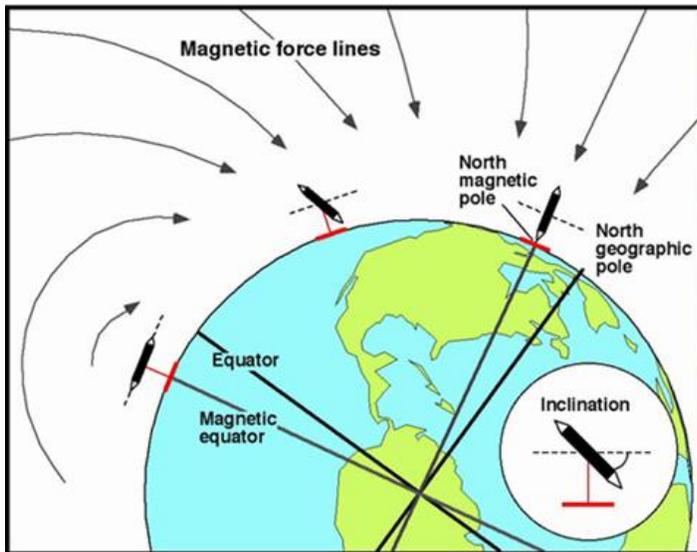
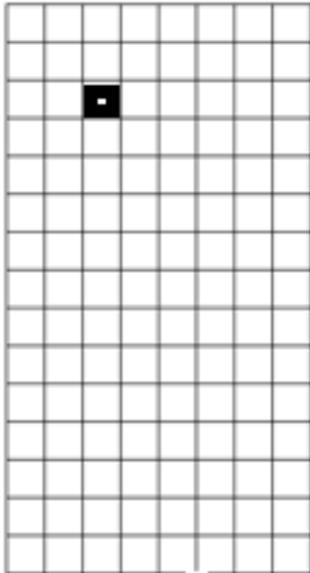


Figure 4. Magnetic force lines and compass needle orientation.

In other words, if you trace a single line of Earth's magnetic field at this geographic location, it will make an angle of 68.6° with the ground and will be tilted downwards in the direction of magnetic North in the Northern Hemisphere. You can measure this angle yourself by creating a compass needle that is free to swing in the vertical direction rather than the horizontal direction. Alternatively, just rotate your compass vertically and measure it!

Experiment 2: Finding the Smartphone Magnetometer!

The biggest technical challenge to mapping magnetic fields with a smartphone is that your smartphone is physically much larger than the magnet whose field you are trying to map. For the case where the smartphone is much smaller than the grid spacing you are sampling, there is no need for a detailed knowledge of the sensor location inside the smartphone case. This applies to mapping the geomagnetic field over meter-scales. This may also apply to the recent surge in interest in determining a customer's location in a store by creating an indoor map of the magnetic environment and then measuring the customer's magnetic coordinates. But when we want to map the field of a small object like a bar magnet, to make an accurate map, you need to know exactly where the smartphone's magnetometer is located inside its case so that you can define what spot in space is being measured relative to your magnet. It would be nice if the magnetometer was located directly under the exact center of the case, but typically they may be located anywhere within the body of the phone from one model to the next!



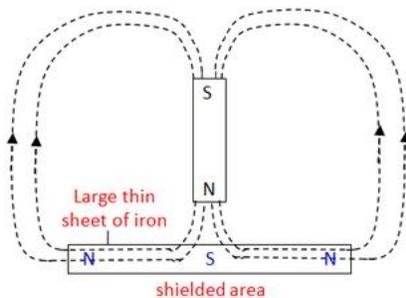
A typical magnetometer 'chip' on a smartphone circuit board is only a few millimeters square. Smartphone apps that detect pipes in the wall or electrical wires use the magnetometer as a sensor. If we use a very thin metallic object (iron nail, jeweler's screwdriver) we can hover it over the surface of the smartphone and locate the chip to millimeter-accuracy by watching the B_z values suddenly increase to a maximum.

Figure 5 is a map of the front of an iPhone 6s in standard orientation with +y up and +x to the right. The on-off bottom is on the bottom edge and the front camera is at the top. The outside dimensions are $y=150$ mm and $x=75$ mm. Using *Teslameter 11th*, the maximum value for B_z was found at 27 mm from the left edge of case and 24 mm from the top edge of case.

Figure 5. Location of magnetometer relative to smartphone case.

Experiment 3: Can magnetism be shielded?

Magnetic fields can be shielded by using magnetic or ferromagnetic materials such as iron, which divert external magnetic field lines into the material and away from the detector. The degree of shielding that a material provides is quantified by its Relative Permeability. Substances such as air, wood and aluminum have relative permeabilities of 1.0, while pure iron has a value of 5000, and *mu-metals* have values near 300,000. What this means is that pure iron is 5000 times more effective in 'trapping' and deviating magnetic field lines compared to an equal thickness of air or aluminum.



To see how various kinds of materials shield the smartphone magnetometer from external magnetic fields, we can place the smartphone inside various containers, or above various materials, specifically made from large-permeability materials (typically metals). Thickness is an important factor so foils and thin plates will not be effective. We will use the app: *Teslameter 11th*.

Figure 6. Magnetic field being shielded by iron plate.

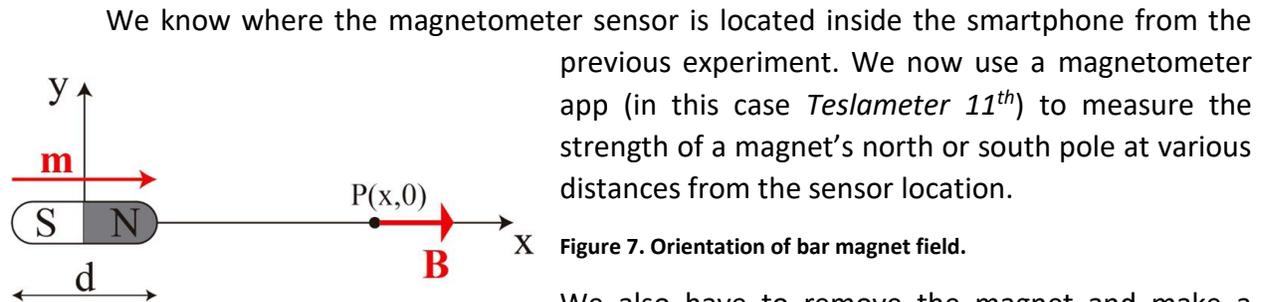
Cast-Iron Skillet - Use an ordinary 10-inch cast-iron kitchen skillet. In this experiment with the skillet not present, the local magnetic field components were $B_x=+1.7$, $B_y=+20.3$ and $B_z=-52.1$ and $|B| = 55.8 \mu\text{T}$. With the app running inside the skillet at 1-cm from the bottom, the readings were $B_x=-20.9$, $B_y=8.6$ and $B_z=0.1$ and $|B|=23.4 \mu\text{T}$. Clearly the z-component had been completely shielded and the horizontal plane components had been altered substantially with this open container arrangement. Moreover, as the smartphone was moved closer to the bottom of the skillet, the component field strengths increased from their minimum at 1-cm. This behavior agrees with the expectation that the geomagnetic field was being trapped and amplified in the thin cast-iron medium. With the smartphone directly in contact with the skillet bottom, the maximum detected value was $|B|=287 \mu\text{T}$, with field components of $B_x=-226$, $B_y=-158$, $B_z=78$.

Experiment 4: Measuring the parameters of a magnet

The magnetic field of a dipole along the polar direction is given by the formula

$$B = \frac{\mu_0 mX}{2\pi \left(X^2 - \left(\frac{d}{2} \right)^2 \right)^2}$$

where d is the length of the bar magnet, m is the value of the dipole moment, and μ_0 is the magnetic permeability of free space. This equation shows that the magnetic intensity \mathbf{B} decreases about as the inverse-cube of the distance between the magnetic pole and the sensor for d being small compared to the distances x . We can verify (or alternatively discover) this behavior using a small magnet and a smartphone.



We know where the magnetometer sensor is located inside the smartphone from the previous experiment. We now use a magnetometer app (in this case *Teslameter 11th*) to measure the strength of a magnet's north or south pole at various distances from the sensor location.

Figure 7. Orientation of bar magnet field.

We also have to remove the magnet and make a measurement of Earth's background magnetic field and subtract this value from our magnet measurements. These corrected data can then be plotted using *Excel*, and a best-fit power law 'trendline' defined for each magnet.

The set-up is shown below, we use for convenience the smartphone Y-axis (not the x-axis) and move the magnet along this axis for a series of measurements. We also make sure that the distance for each measurement includes the actual location of the sensor along the Y-axis, which is 24 mm from the top edge of the case.

Table 1 on the next page shows magnetic measurements for two types of common 'bar' magnets. The graphs below show this information together with an *Excel* power-law trend line. In both cases the points fall along smooth power-laws but with exponents of -2.6 for the small magnet and -2.1 for the large magnet.

Table 1 Magnetometer data for test magnets.

Distance (cm)	Small Alnico (μT)	Small Alnico (μT)	Steel/Iron (μT)
	D=3 cm	D=0.7 cm	D=15 cm
23	-7.8		-24.7
21	-8.5		-28.3
19	-9.8	1.2	-33.7
17	-11.4	1.7	-41.2
15	-14.1	3.0	-51.6
13	-18.3	4.3	-67.5
11	-26.1	7.0	-93.3
9	-41.3	12.4	-138.9
7	-74.7	26.4	-223.8
6		41.1	-302.3
5	-172.3	71.5	-420.0
4	-303.0	132	-629.0
3	-570.7	284	
Background	-5.0	-5.5	-9.5

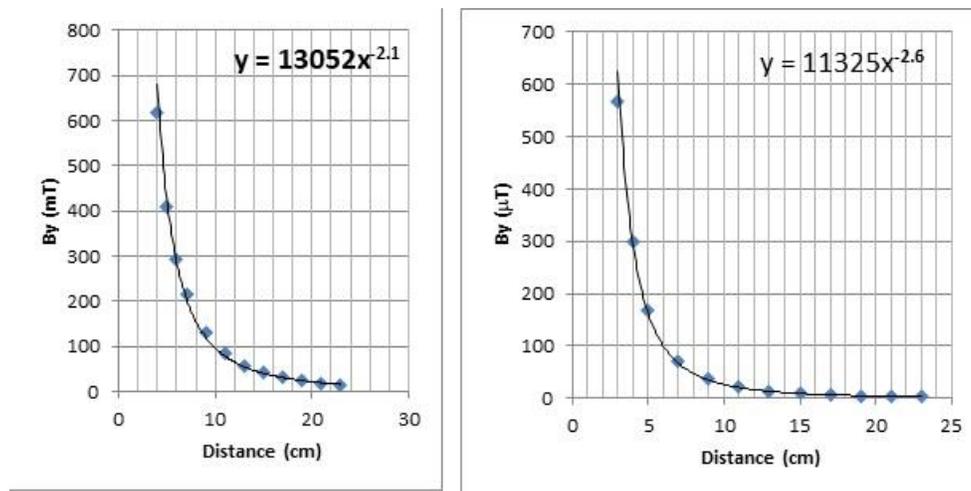


Figure 8. Plots of decrease in magnetic field strength fitted to a simple power-law.

This is not the usual 'inverse-cube' law that is predicted because the magnets are large in size compared to the distances defined by x . The large bar magnet is $d=15\text{cm}$ long and the small

magnet is only $d=3$ cm long, so the smaller magnet should follow a law that is closer to inverse-cube, which it does with the exponent of -2.6 being closer to -3.0 than for the larger magnet.

For more information on how to do this experiment with another set up see:

Enrique Arribas, Isabel Escobar, Carmen P Suarez, Alberto Najera and Augusto Beléndez. Article · *Eur. J. Phys.* 36 (2015) 065002 (11pp) October 2015 DOI: 10.1088/0143-0807/36/6/065002 https://www.researchgate.net/publication/281427984_Measurement_of_the_magnetic_field_of_small_magnets_with_a_smartphone_A_very_economical_laboratory_practice_for_introduutory_physics_courses

Experiment 5: Detecting Geomagnetic Storms

When a coronal mass ejection colides with earth's magnetic field, it causes changes at ground level that can easily be detected by professional-grade magnetometers, and with smartphone magnetometers for some of the strongest storms. The typical magnetic signature will be a change by about 1 mT over the course of 3-6 hours. To make this measurement is a challenge, but along the way you learn a lot about local sources of magnetic noise and the sensitivity of smarttrphone systems.

- Step 1) Select a smartphone app that allows for recrding of the data and exporting as a .csv spreadsheet.
- Step 2) Place the smartphone on a tabletop or other surface that has been leveled.
- Step 3) Start the app and allow it to record data for 3-4 hours. Make sure the smartphone is fully-charged.
- Step 4) Stop the recording process and email the data as a .csv file to yourself.
- Step 5) Download the .csv file to your computer and open it using MS Excel. You will need to usd the 'text to columns' function to expand the date with one value per column.
- Step 6) Combine the X and Y values into a horizontal field value, $BH = (X^2+Y^2)^{1/2}$
- Step 7) Plot the Bh and Z magnetic values on the same plot by highlighting the two columns and using the standard plotting function. An example of the result should look like the graph below, taken during calm non-storm conditions.

The graph in Figure 9 displays BH as the lower blue line and $|Z|$ as the top red line. The y-axis is the magnetic values in units of μT and the horizontal axis is the sample number, with one sample made every second. The total duration was 316 seconds or about 5 minutes.

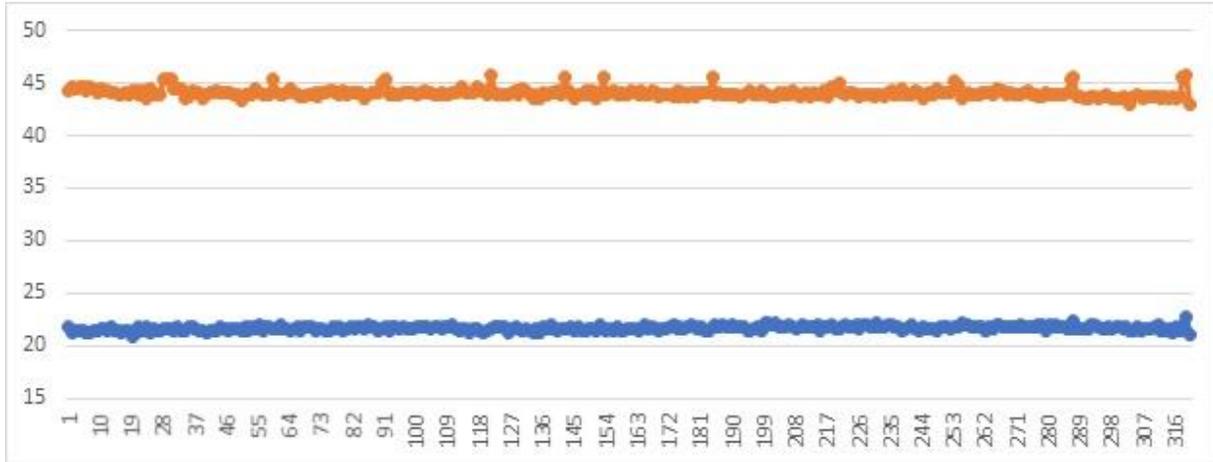


Figure 9. Plots of horizontal and vertical magnetic field strengths.

For longer measurements you will notice many different kinds of artifacts. You are looking for smooth changes in the baseline values that appear and disappear over the course of several hours and have a maximum change of about 1 μT . Use a service such as <https://www.spaceweatherlive.com/en/auroral-activity/kp-index> to see if a storm is occurring.

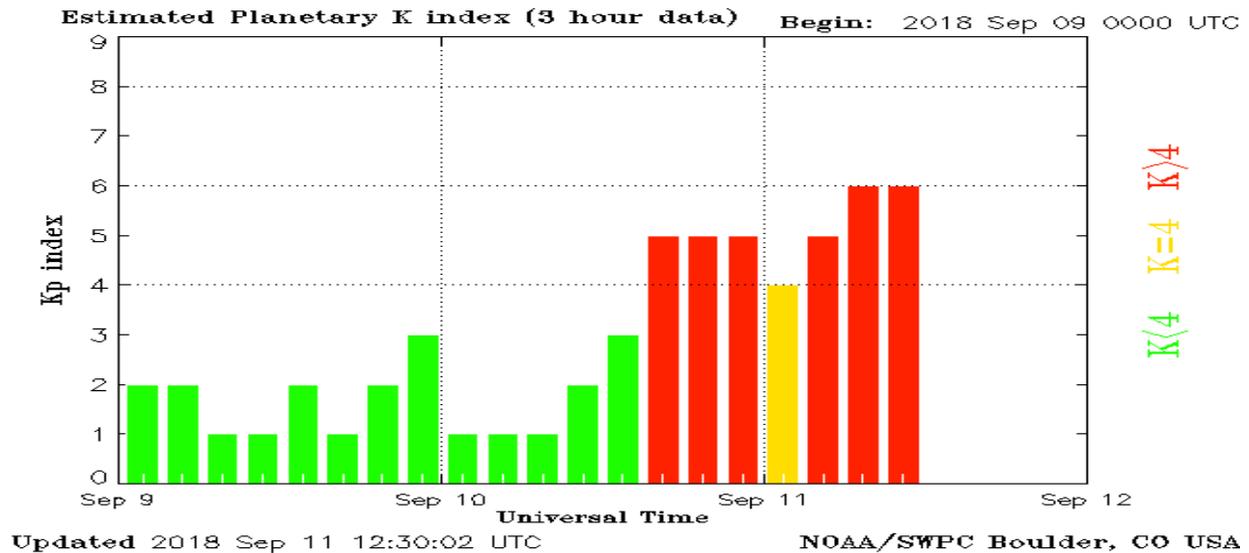


Figure 10. Plot of Kp index during a magnetic storm on September 11, 2018.

For more details on detecting magnetic storms with smartphones, see the paper by Odenwald, S.F., 2018, "The Feasibility of Detecting Magnetic Storms With Smartphone Technology" <https://ieeexplore.ieee.org/document/8425973>

Clinometer

A clinometer is a device, rather ancient in origins, that measures the angle between the horizontal plane and some point located perpendicular to this plane, such as the crown of a tree and the top of a roof from street-level, or the height of the sun above the horizon.

- **SeeLevel (\$0.99 - Android; iOS)** - Measure angles and align objects viewed through the camera. SeeLevel also includes a distance/height estimation tool and traceable level. With SeeLevel you can for example: - Find the angle relative to the horizon (like a sextant) - Get the vertical line to the ground (like a plumb) - Estimate the tilt of objects in the distance - Calculate the height of, or distance to, an object (Inclinometer). - The unit of the input and output distances are always the same so you can use any unit you wish (meter, feet, yards etc) - Use it as a conventional level, independent of the camera - Zoom in the measured object to improve accuracy - If your device has a front camera you can use the clinometer to measure the level from the ground.
- **Dioptra (Android)** - a camera position and angle measurement tool for navigation, surveying, positioning, and measurement. Instructions: Press the on-screen camera button to save an image. View saved images in your device's gallery app. Dioptra™ provides the following information like a theodolite optical instrument - yaw indicator (compass) - pitch indicator (tilt degrees) - roll indicator (tilt degrees) - along with - GPS position - azimuth and bearing to photo subject
- **Theodolite Droid (Android)** – This is a free app for the Android which gives you a collection of useful surveying tools such as a GPS, maps, inclinometer, camera, magnetometer, etc. plus you will also get the longitude, latitude, altitude, azimuth, address (if available), and the date and time can be recorded when you take a photo with the app. The app is very useful for pros but it can also be handy for hikers, cyclists, and travelers alike. The information that you have gathered can then be easily shared to your friends through Facebook or e-mail. If in the event that you do not have internet access, the app will just store the info first until you have a connection to the net and then work on it.

Additional Resources:

Measuring Height: <https://www.instructables.com/id/Using-a-clinometer-to-measure-height/>

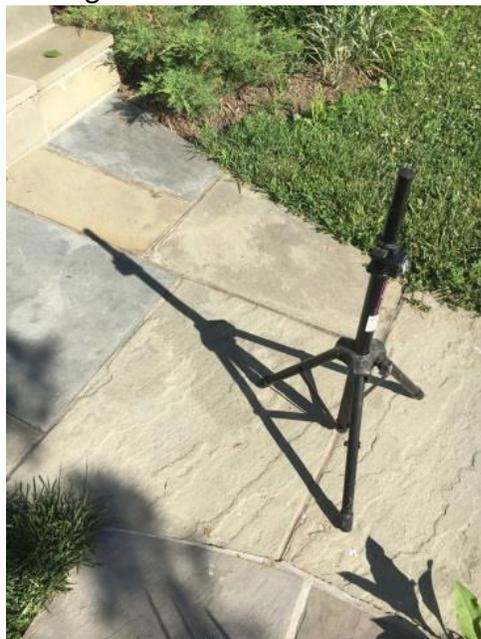
A Simple Astrolabe: <https://paksc.org/pk/science-experiments/physics-experiments/how-to-make-simple-astrolabe-clinometer/>

Measuring road grades:

https://www.nrcs.usda.gov/wps/portal/nrcs/detail/pr/newsroom/factsheets/?cid=nrcs141p2_037322

Experiment 1: Measurement of sun angle

For millennia, humans have measured the angle of the sun above the horizon to tell time or to track the seasons. They have done this using nothing more than a stick driven vertically into the ground called a *gnomon*. To measure this angle, students can draw a scale model of the height of the gnomon and the shadow it casts, and from this triangle and with the help of a protractor,



directly measure the shadow angle. If the height of the gnomon is AB and the length of the shadow is BC , the gnomon angle is just $\angle ACB$. We can also use basic trigonometry to determine $\angle ACB = \theta = \tan(AB/BC)$.

Figure 11. Gnomon shadow.

For a gnomon, you can use the bottom of a music stand and take off the holder. Set its height at 24 inches and use this as a standard and portable gnomon. You then measure the shadow length every hour from sunrise to sunset. You can also use the *SeeLevel* app and its 'clinometer' feature to measure the sun's elevation angle, which for June 30 (DOY181) looks like the graph in

Figure 12.

The solid blue line indicates the mathematical model for DOY 181. The red squares are the elevation measurements from *SeeLevel*, and the blue diamonds are measurements from the gnomon shadow length. The most difficult measurements were close to sunrise and sunset when the shadow measurements were imprecise and the solar disk was blurry and faint in the camera field-of-view. Nevertheless, very good agreement between the predicted and measured elevation angles were obtained in the time interval of 4 hours before Noon (-4) and 4 hours after Noon (+4).

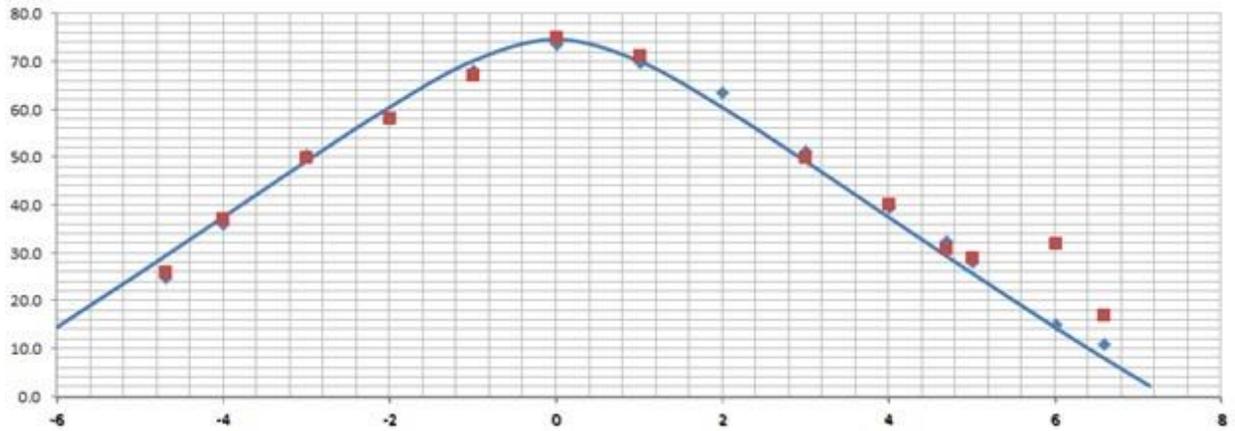
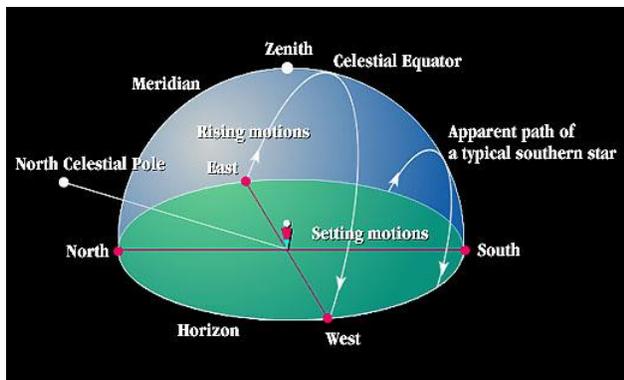


Figure 12. Plot of gnomon shadow length in inches during daytime hours relative to noon at 0.0.

Experiment 2: Finding your Latitude using the Sun

Objects in the sky are located on the ‘celestial sphere’ which is a coordinate system that



resembles our Earth’s latitude and longitude system where Earth is at the center of the celestial sphere. The sky coordinates are declination (latitude) and Right Ascension (longitude). Earth rotates, which means that the celestial sphere (and sky objects) rise in the east and set in the west. The north-south longitude line connecting your location with the north and south poles in longitude is a fixed coordinate line on Earth’s surface called the

Meridian, but at any given moment, the reflection of this line on the celestial sphere is a unique Right Ascension, whose coordinates are measured in hours, minutes and seconds.

You can see that the angle above the horizon changes continuously during a day. This is caused by two effects. First the RA and Dec coordinates of the sky are tilted relative to your local coordinates expressed as surveyor’s azimuth and elevation. This is because Earth’s rotation axis is tilted, and you are viewing the sky from a particular latitude on Earth, where the tilt is just the angle of the North Star above your horizon. If you were standing on the North Pole of Earth, the North Celestial Pole about which the celestial sphere rotates, would be directly over-head. All sky objects with a declination of exactly $+90^\circ$ North would be directly overhead. In fact, Polaris has a



declination of 89.25° North, so although it looks like the point about which stars rotate, it actually follows a tiny 'orbit' around the NCP about the size of the full moon! Here is an example of this effect captured by Ted Quakenbush (Wikipedia).

Figure 14. Trailing star images as Earth rotates

If you are at Earth's equator, the celestial equator will be directly overhead (Declination 0°). So, as you move towards increasing northern latitudes, the celestial equator will rotate to your south on the meridian line and the NCP will get higher on the meridian towards due-North. This is why measuring the angle of Polaris above your northern horizon will almost-directly give you your latitude, which every Boy Scout knows!

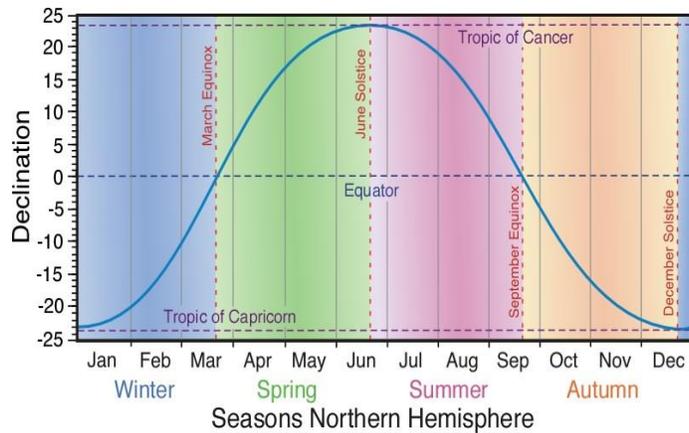
How do we figure out how high above the southern horizon a celestial object will be? You just subtract your latitude from the object's declination! To see why this is so, it helps to draw a picture. If the object were at a declination of 0° on the equator, and if your latitude were 25° North, the equator would have shifted 25° south of zenith and the object would be at an elevation angle of $90^\circ - 25^\circ = 65^\circ$. If the object were at a declination of $+15^\circ$, you first correct for the celestial equator shift of -25° south of zenith, then you add the declination of the object to get $65^\circ + 15^\circ = 80^\circ$. Similarly, if its declination is -15° south, its elevation angle would be $65^\circ - 15^\circ = 50^\circ$. In general for observers in the Northern Hemisphere, the elevation angle is given by $90^\circ - \text{latitude} + \text{declination}$.

Now the problem is that a smartphone camera is not sensitive enough to detect stars, but we have two possible solutions:

1 – We can point the smartphone towards the North Star and use an app to measure the smartphone's tilt angle.

At night, find Polaris using any popular method. Start the *SeeLevel* app and select the clinometer feature. At arm's length, tilt your smartphone until the center of the screen is at the same angle as the direction towards Polaris. Read the displayed elevation angle, θ . Your latitude is $90^\circ - \theta$. For example, from Kensington Maryland, the clinometer angle to Polaris appeared to be 51° so the latitude is 39° .

2 – We can measure the elevation angle of the sun at Noon on a specific day for which we know its actual declination. This is a trickier measurement to make because you need a table showing the Sun's declination for each day of the year. Graphically this is shown in Figure 15.



For example, on March 21 the declination is 0°. If we use the clinometer feature and measure a noon-time sun angle in Kensington Maryland of 51°. The latitude would be latitude = 90° – elevation + declination or 39°.

Figure 15. Declination of the sun during the year.

Experiment 3: Finding the altitude of a rocket or a cloud

The basic principle involves the properties of a right triangle as shown in the figure below.

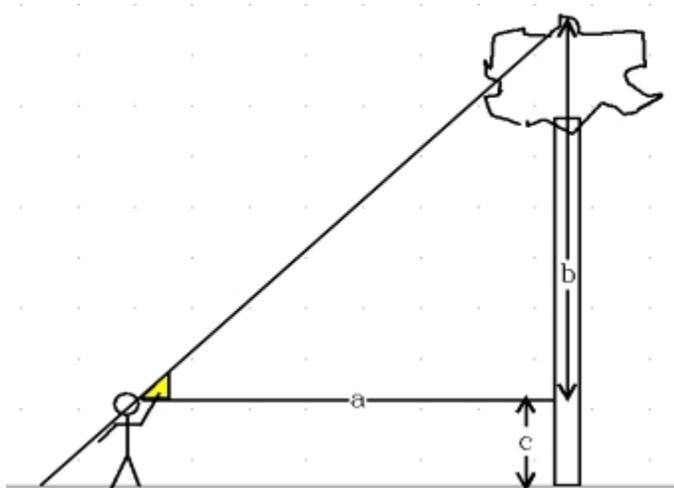


Figure 16. Diagram showing geometry of tree height sighting.

The clinometer would be used to measure the angle shown in yellow. You will also need to know the distance to the base of the tree, a, which can be paced-out. Then

$$\tan(\theta) = b/a \text{ so}$$

$$b = a \tan(\theta).$$

A similar method can be used to determine the length, c, so that the final height = b+c.

This procedure works for toy or hobby rocket launches when the rocket travels more-or-less vertically from the launch pad, but usually this is not the case, so there are two additional formulae

we can use to take into consider more complex launch situations when two people are combining their sighting data. This also applies to determining the altitude of a cloud.

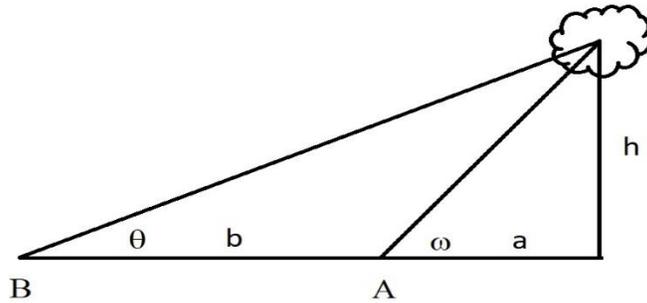


Figure 17. Diagram of cloud height sighting for two observers.

If you and your partner line up with the launch pad so that your partner is exactly between you and the launch pad...or base of the cloud, you can use this formula where $L = a+b$, and you each are measuring with the clinometer two angles θ and ω :

$$h = L \left(\frac{\text{Tan}\theta \text{Tan}\omega}{\text{Tan}\omega - \text{Tan}\theta} \right)$$

If you and your partner are not lined up with the point where the cloud or rocket is at its highest altitude, you need a more complicated formula

$$h = L \left(\frac{\text{Tan}\alpha \text{Tan}\beta}{\text{Tan}\beta \text{Tan}\gamma + \text{Tan}\alpha \text{Tan}\delta} \right)$$

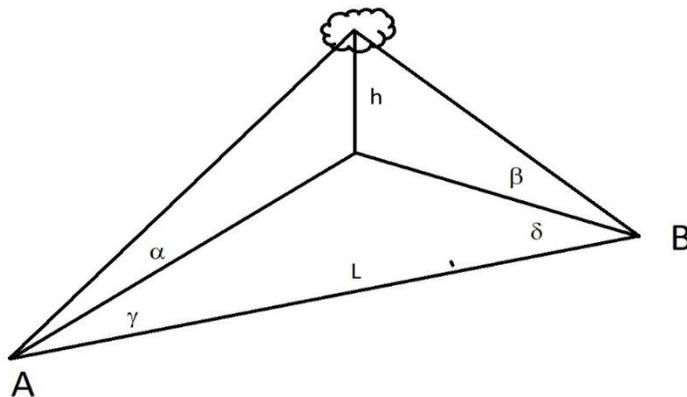


Figure 18. Diagram of cloud height sighting when two observers not on same line.

The two observers A and B not only have to use the clinometer to measure the angles α and β , but need to know how far apart they are, L , and the angles that each of them make with respect to the point just below the height to be measured, γ , δ laid out on the ground. You can measure these two angles by using a compass and taking the differences in the bearings between the cloud/rocket and your other partner.

Radiation

Radiation comes in two forms: particles and electromagnetic. Particle radiation includes protons, neutrons, electrons and the nuclei of many kinds of atoms such as helium and iron. Electromagnetic radiation includes all forms of light, which travel at 300,000 km/sec and include gamma rays, x-rays, ultraviolet, infrared and radio-forms. Radiation in all its forms can be measured accurately in terms of units called Grays and Sieverts, and to understand the basics of radiation dosimetry we need to define some terms and concepts.

Dose is a measure of the amount of total energy that is absorbed by matter over a period of time. This matter can be human tissue, or sensitive computer circuitry. The unit for dose is the Gray (1 Gy = 1 Joule of energy deposited in 1 kilogram of matter). The term 'Dose Equivalent' compares the amount of absorbed energy in Grays to the amount of tissue damage it produces and is measured in Sieverts (Sv). Each type of radiation, for the same exposure level in Grays, produces a different amount of damage. Mathematically, this is represented by the equation:

$$\text{Dose Equivalent (in Sieverts)} = \text{Dose (in Grays)} \times Q.$$

X-rays and gamma-rays, produce 'one unit' of tissue damage, so for this kind of radiation $Q = 1$, and this is also the case for beta radiation. For alpha particles, $Q = 15-20$, and for neutrons, $Q = 10$. Typically, our total, annual radiation dose is about 3.7 milliSv, or alternatively in terms of dose rate this is about 0.4 $\mu\text{Sv/hr}$. During the 2011 Fukushima reactor meltdown in Japan, residents in Tokyo some 240 km away temporarily experienced levels of 0.8 $\mu\text{Sv/hr}$. By comparison, if you are traveling in a commercial jet at an altitude of 33,000 feet, you can expect a dose rate of about 2 $\mu\text{Sv/hr}$ for equatorial and mid-latitudes and about 7.0 $\mu\text{Sv/hr}$ for polar latitudes for a few hours. For more information about background radiation, sources and units take a look at the SpaceMath@NASA 'Radiation Math' book at

https://www.nasa.gov/audience/foreducators/topnav/materials/listbytype/Radiation_Math.html

During a single year, you receive about 380 milliRems of accumulated radiation dose (3,800 $\mu\text{Sieverts}$), which corresponds to a dose rate of 0.4 $\mu\text{Sv/hr}$. Most apps provide dosage measurements in the SI units of $\mu\text{Sv/hr}$. During the Fukushima reactor meltdown in Japan, residents in Tokyo located 239 km away, temporarily experienced levels of 0.8 mSv/hr. If you are traveling in a commercial jet at an altitude of 33,000 feet, you can expect a dose rate of about 2 $\mu\text{Sv/hr}$ for equatorial latitudes and about 7.0 $\mu\text{Sv/hr}$ for polar latitudes.

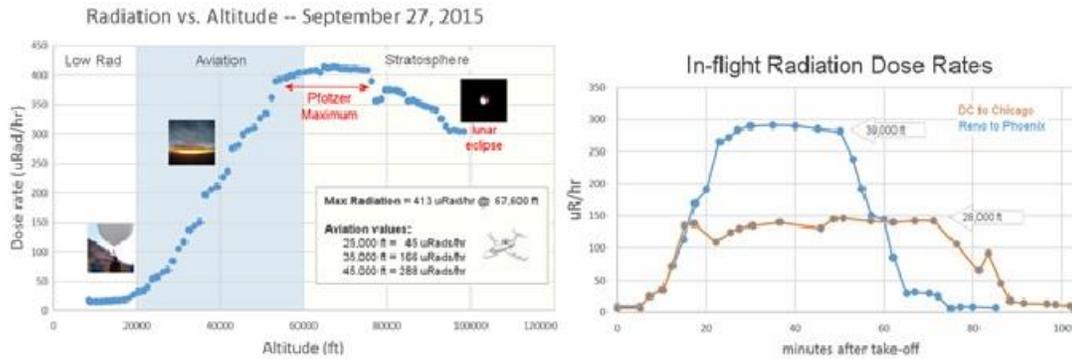
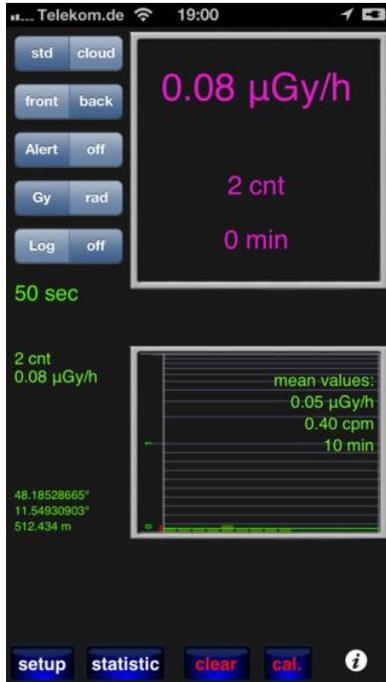


Figure 19. Radiation dosages versus altitude.

The detection of radiation in order to measure dose rates depends on the type of material in the detector, the energy of the particles, and the type of particle involved, so there is no single detection system that works for all possibilities. High energy gamma-rays and neutrons can penetrate matter relatively easily, but the heavier, charged alpha particles can be easily shielded before they reach the detector. There are two ways to measure particle radiation and gamma rays using smartphones. One can obtain a plug-in module that converts your smartphone into a Geiger counter, or one can use the smartphone camera as a track detector. Each method has its advantages and disadvantages.

Camera methods involve closing off the front and back camera apertures so that the camera chip is in fully-dark mode. High-energy particles such as gamma rays and neutrons will collide with one or more pixels in the camera array chip and cause them to 'light up' with excess charge. Once the data has been corrected for the unavoidable 'dark noise' from the pixels themselves, the result is a count of the number of hits per sampling interval usually in counts per minute (CPM). This can be related to the level of radiation measured in $\mu\text{Sv}/\text{hour}$ in your environment after calibration. External sensors usually plug into the audio jack of your smartphone. They are small-volume, solid-state devices that react to energetic particles by producing a voltage or current spike that is picked up through the smartphone headphone jack and counted.

- **RadioactivityCounter (iOS – Android) - \$4.99** This application is a real working radioactivity counter without any extra hardware needed. It will turn your phone camera into a Geiger Müller Counter to measure radioactivity after proper calibration. The camera sensor can measure gamma radiation (Cesium-137 for example) and some beta radiation (depends on the energy if the beta rays can pass through the plastic parts - for example Helium-3 or potassium-14 are weak sources). It cannot measure alpha rays (Polonium-210) which will be blocked by the smartphone case.



Radioactivity Counter setup – Cover camera lenses with black electrical tape.

Step 1 – select the ‘front’ camera, select the Alert ‘off’. Select ‘Gy’

Step 2 – Cover the camera lenses completely with black tape so that the top right camera screen is completely dark. You may also want to perform this calibration work in a darkened room to avoid stray light. Do not move the smartphone during the calibration process. Also, do not use an unventilated basement room because radon gas is common and will skew the baseline measurements to a higher background level in the calibration process. Also, place the smartphone on a wood surface or table top, but not on a granite counter since granite contains radioactive elements.

Figure 20. Screen for Radioactivity Counter app.

Step 3 – Calibration: Press the ‘clear’ button and the ‘cal’ button on the lower right, the app will go through a pre-programmed process and count-down for 90 seconds with ‘wait...calibration’ on the screen. At the end, it will then show a screen where you choose a 4-minute or 10-minute calibration run. Select ‘10-minutes’. It will then cycle through 10, 1-minute sampling periods. In each 1-minute period it will tell you the number of gamma-ray flashes it counted.

Step-4 – At the end of calibration, a message appears describing how the information has been saved. Click ‘OK’. On the main screen, click ‘clear’. The app will pause, then begin taking a series of 1-minute samples until you stop the app. After each minute it will update the values in the top-right screen with the measured background values. Let the app run for 5 or 10 minutes in order to build up reliable statistics since you will typically only be detecting a few counts every minute.

Additional Resources:

Calibrating a smartphone dosimeter:

Odenwald, S., 2019. Smartphone Sensors for Citizen Science Applications: Radioactivity and Magnetism. *Citizen Science: Theory and Practice*, 4(1), p.18. DOI: <http://doi.org/10.5334/cstp.158>

Radiation Basics:

Radiation Dosimetry:

<http://www-naweb.iaea.org/nahu/DMRP/documents/Chapter3.pdf>

Medical dosimetry:

<https://www.radiologyinfo.org/en/info.cfm?pg=safety-xray>

Experiment 1: Measuring your Local Background Radiation

This is the most basic, and invaluable, measurement that a dosimeter can make because if you do not know how much radiation exposure you should normally expect to receive, it is hard to gauge when a reading is outside a safe range. Here is the count series for a sample series made in Williamstown, MA spanning 30 minute CPM averages each lasting 1 minute.

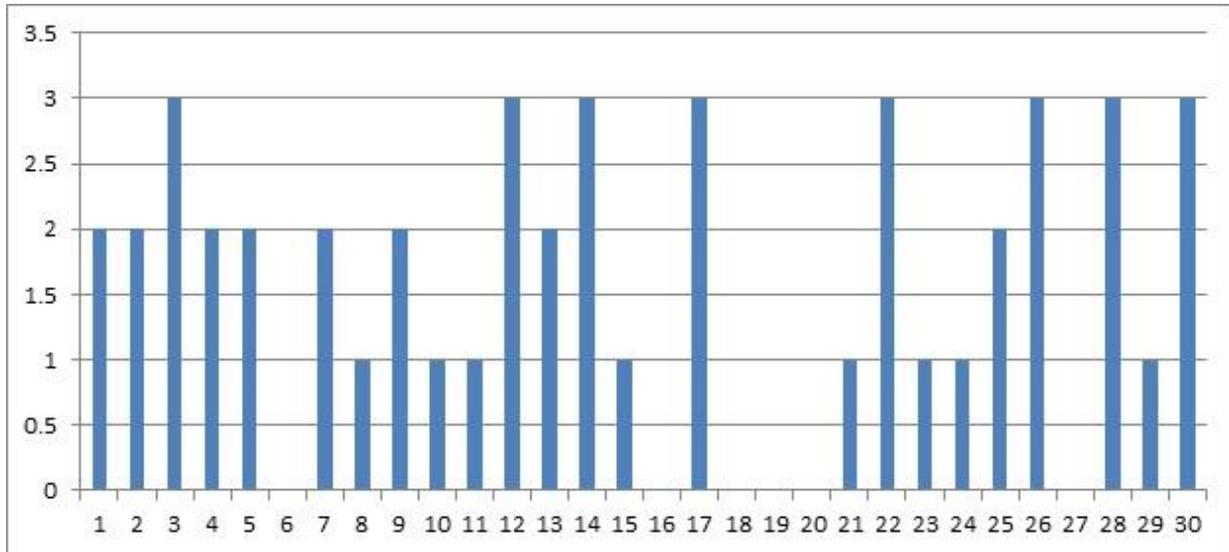


Figure 21. Radioactivity Counter background data.

The average background count rate near sea level is 2 CPM. The calibration performed for setting up *Radioactivity Counter* reports that this equals $0.07 \mu\text{Gy/hr}$. A series of measurements from the summit of Mt Greylock: Counts = 0,4,1,2,2,2,2,2,4,1 for an average of $20/10 = 2$ CPM so the reported dose is $0.08 \mu\text{Gy/hr}$. The 1000-meter elevation change does not affect the CPM compared to similar levels recorded in Williamstown, MA at 190-meter elevation.

Experiment 2: Measuring Radiation Dosages In a Jet Plane

In Table 2, Observer 1 flew from LA to Houston at 39,000 feet and measured 55.3 CPM, which *Radioactivity Counter* reported as 74.3 $\mu\text{Sv/hr}$. Observer 2 flew from Baltimore, Maryland to Charlotte, North Carolina at 36,000 feet and measured 11 cpm (10 $\mu\text{Gy/h}$), 8 cpm (6.3 $\mu\text{G/hr}$) and 4.5 cpm (1.95 $\mu\text{G/hr}$) for an average of 8 cpm and 6.0 $\mu\text{G/h}$. This is significantly higher than at ground-level (2cpm and 0.07 $\mu\text{G/h}$).

Table 2. Dose rates versus altitude

Location	CPM	$\mu\text{Gy/hr}$
Home on Table	3.0	0.090
Observer 1 (13,000 m)	55.3	74.3
New York (52 m)	1.9	0.07
Williamstown (190 m)	1.7	0.06
Mt Greylock (1048 m)	2.3	0.08
Observer 2 (8800 m)	8.0	6.0

Experiment 3 – Other Radiation Sources

In the *YouTube* program, the app was compared to several known radiation sources. More details about sensitivity and calibration can be found at the developer's website including the CPM to $\mu\text{Sv/hr}$ conversion curves for various smartphones. http://www.hotray-info.de/html/radioa_ios.html

Background in Denmark: 0.16 $\mu\text{Gy/hr}$ and 11 cpm (17, 5, 10, 12 in 1-minute samples) was registered on the app and a similar reading registered on professional dosimeter.

Cesium-137 was used = 3.17 $\mu\text{Sv/hr}$ and 3.25 $\mu\text{Sv/hr}$ on two professional dosimeters. After 5 minutes, the app registered CPMs = 27, 38, 109, 47, 42 with average = 51 cpm and a dose of 4.95 $\mu\text{Gy/hr}$.

Mineralized uranium ore: 283 $\mu\text{Sv/hr}$ and 265 $\mu\text{Sv/hr}$ on the two professional dosimeters. After 5 minutes the app says 1342 cpm and 324.8 $\mu\text{Sv/hr}$.

Gravity

The gravity sensor measures the acceleration effect of Earth's gravity on the device enclosing the sensor. It is typically derived from the accelerometer, where other sensors (e.g. the magnetometer and the gyroscope) help to remove linear acceleration from the data. The Gravity unit are in m/s^2 like the accelerometer, and they are measured along the X,Y, and Z axes. In modern mobile devices, the physical sensor that measure acceleration is the accelerometer. The accelerometer, however, measures all the accelerations that affect the device, which are the sum of the gravity acceleration and the actual linear acceleration that are associated with the movement of the device. A crude estimate of the gravity on hand-held device can be made on the accelerometer reading using a low-pass filter that minimizes the linear acceleration. Modern mobile devices refined the gravity measurement by creating a virtual sensor that is implemented as a sensor-fusion of several basic physical sensors, the accelerometer, the gyroscope, and the magnetic sensor.

- **Physics Toolbox Sensor Suite** (Android and iOS; Vieyra Software) is a massive collection of 20 different sensor functions all in one app. All sensor data can be recorded by pressing a button. Data can be saved in a .csv file, and transferred via e-mail or shared in Google Drive or Dropbox. Clicking on the (i) icon within each mode provides information about the sensor name, vendor, and current data collection rate, in addition to a description of what kind of data is collected by the sensor, its physical operating principle, and links to additional resources, including webpages and videos that illustrate how it works. The *G-Force meter* function gives a real-time display of the g-acceleration components to 0.001 gs ($0.01 m/sec^2$). The Android model allows the display cadence to be adjustable from 420 Hz to 5 Hz. The iOS model operates in a fixed mode with a cadence of about 200 Hz.
- **Gravity Meter** (Android: Atcle) offers a simple display that shows measurements to 0.001 m/s^2 , and also provides for two values in which one is 'calculated from the G-sensor' and the second from the 'Acc sensor'. It can also correct these values by adjusting them to the local standard gravitational acceleration determined from your current GPS latitude, and provides a 'Gravity Chart' that presents the value of g for various international cities.
- **GravityMeter** (Android: Teknik Elektronika SMK) is a static display of your local acceleration calculated from a model given your current GPS coordinates, and is not suitable for actual dynamic observations.
- **MRE Acceleration Meter** (Android: MRE Programming) after calibration by gently shaking it, its Gravity Calculator mode calculates your local gravitational acceleration to 6 decimal places in m/sec^2 using your GPS or manually-entered coordinates. It uses the World Geodetic System 1984 which models Earth as a smooth ellipsoid and from your coordinates calculates a model-based local acceleration. The app also has an Acceleration

Meter mode which after shaking gently to calibrate provides a digital and graphical display of your local acceleration vector components including a measure of the local gravitational acceleration using the smartphone's 'gravity sensor'. The digital display shows the local vertical component along the smartphone's current Z axis to four-digits in m/sec^2 .

- **Accelerometer: g-force meter** (Android: Miodne Projects) provides a digital display to 0.01 'g-units' of the average, maximum and minimum readings along with a graphical display showing the elapsed observation time and the acceleration to 0.01 g's.
- **Accelerometer Meter** (Android: Keuwlsoft) provides a realtime display to 0.01 m/sec^2 and the sampling cadence can be changed with a slider from slow to fast from 0.5 to 0.03 seconds. The exported data logging to a .csv file. The app says that the file is saved in the 'root directory'. Use an app like File Manager (Android: ZenUI ASUS Computer Inc.) to access this data file. In the upper left corner of the top tool bar, tap the /// icon, which will open the File Manager menu. Tap 'Internal Storage', and scroll down to the 'Accelerometer' .txt file. Tap the file icon to open. It will show a 7-column data file, one line for each time interval, and accelerations give to 0.000001 m/sec^2 .
- **iAccelMeter** (iOS: BITCAIRN) offers a dynamic display of your smartphones general acceleration components but is not useful for these studies.
- **Accelerometer** (iOS: DreamArc) realtime display with data exporting. The measurement cadence can be changed with a slider from 1 to 30 Hz. The units are in measured and displayed to 0.001 gs. The exported .csv file, however, gives readings to 0.000001 gs .
- **Accelerometer** (iOS: Physics Toolbox) gives a realtime display of the g-force components to 0.00001 gs . The display cadence is 10 milliseconds so the 0.001 digit is unreadable in general. The display can be paused, however by merely taping the screen.
- **GravityMeter** (iOS: Perfectly Simple Apps, LLC), was specifically available for gravity measurements, and once calibrated using the in-app procedure, allows you to measure the local gravitational acceleration to 0.01 meters/s^2 It requires a short 60-second calibration period before using. It is no longer available in the AppStor by 2018 because the developer needs to update it for iOS 11.
- **Mobile Science** (iOS: Indiana University) offers an acceleration app that gives readings to 0.01 in 'gs' with adjustable samples-per-second to 10 and mailable output as a .csv file. It is not a downloadable app but is used online.
- **SparkVue** (iOS: PASCO) allows for data logging and .csv emailing. It is used by physics teachers (<https://quantumprogress.wordpress.com/2012/05/24/using-smartphone-apps-to-take-physics-day-to-the-next-level/>) for student projects. The display offers measurements to 0.1 m/sec^2 .
- **Gauges** (iOS: Emidio Cunha) provides a suite of sensor measurements but its acceleration metering only shows 0.1 m/sec^2 accuracy. However to adjust the gauges and record data you need to purchase the app for \$2.99.

Additional Resources:

Calibrating a smartphone sensor:

Odenwald, S., 2019. Smartphone Sensors for Citizen Science Applications: Radioactivity and Magnetism. *Citizen Science: Theory and Practice*, 4(1), p.18. DOI: <http://doi.org/10.5334/cstp.158>

Citizen Science Applications:

Earth Rotation Detector: <http://www.anecdata.org>

Terrestrial gravimetry:

GFZ Potsdam: <https://www.gfz-potsdam.de/en/section/global-geomonitring-and-gravity-field/topics/terrestrial-and-airborne-gravimetry/>

Gerlach, et al., <http://mediatum.ub.tum.de/doc/1367754/772561.pdf>

Absolute Gravity Database: <https://www.slideserve.com/aliza/bureau-gravimetrique-international-bgi-r-ole-and-services-to-terrestrial-gravimetry>

US Water Storage by Grace measurements

<https://calmit.unl.edu/grace-terrestrial-water-storage>

Experiment 1: Rotation of Earth

A combination, the equatorial bulge and the effects of the Earth's rotation mean that sea-level gravitational acceleration increases from about 9.77 m/s^2 at the Equator to about 9.83 m/s^2 at the poles, so an object will weigh about 0.5% more at the poles than at the Equator. At a level of 0.01 m/s^2 the most significant effect comes from Earth rotation and equatorial bulge. At the North and South poles along the rotation axis, the value for the acceleration at the surface will be fully due to the gravitational bulk mass of Earth, however at the equator, centrifugal force will reduce the surface acceleration in a predictable way. The theoretical difference is 0.06 m/s^2 , which should be detectable with smartphone accelerometers.

Sophisticated ‘geoid’ models have been developed that are incorporated into Earth rotation models such that the local acceleration at arbitrary latitude, longitude and altitude points can be predicted. Table 4 shows the results of using the *SensorOne* model to determine surface acceleration for a range of latitudes compared to the simple spherical-Earth rotation model. The formulas used by this calculator are based on the International Gravity Formula 1980 (IGF) which determines the gravity from the position of latitude, and the Free Air Correction (FAC) which corrects for height above sea level:

$$\mathbf{g} = \mathbf{9.780327 (1 + 0.0053024\sin^2\Phi - 0.0000058\sin^22\Phi) - 3.086 \times 10^{-6} \times h}$$

The IGF model is based on the 1980 Geodetic Reference System (GRS 80) whose Earth model has a 6,378,137m semi-major axis and a 1/298.257222101 flattening at the poles. This

system was adopted at the XVII General Assembly of the International Union of Geodesy and Geophysics (IUGG) in Canberra, Australia, 1979. This model is rotationally-symmetric so the surface acceleration is longitude-independent. Absolute gravity measurements from ground-based observatories are available from the Bureau Gravimétrique Internationale (BGI, 2018) and are shown in the table below for measurements near a longitude of 90 West. <http://bgi.omp.obs-mip.fr/data-products/Gravity-Databases/Absolute-Gravity-data>

Table 3. Comparison of gravity data and models

Latitude	BGI	IGF1980
80	-9.830	-9.831
70	-9.824	-9.826
60	-9.820	-9.819
50	-9.813	-9.811
40	-9.800	-9.802
30	-9.793	-9.793
20	-9.787	-9.786
10	-9.773	-9.782
0	-9.778	-9.780

With the help of crowdsourcing, we arranged for students in three classrooms at different latitudes to measure the local surface acceleration in Maine (Ms. Sue Lamden’s class, latitude +43.9°), Nebraska (Mr. George Griffith’s class, latitude +40°) and Hawaii (Ms. Melissa Montoya’s class, latitude +19.9°). The students also noted the smartphone type. The large number of students in each class made it possible to match equal numbers of the major iPhone types between the classes. Because of the non-recording nature of the *GravityMeter* display, which was in use at that time, students were asked to note the smallest value, the largest value, and the most frequently-displayed values for the surface acceleration when the smartphone was placed on a horizontal tabletop. Using the smallest and largest values as a peak-to-peak (P2P) estimator, this will be related to the standard deviation by $\sigma = P2P/\sqrt{2}$. The most common value is assumed to represent the average measurement. The resulting observations are shown in Figure 22, where the classroom medians are $\langle g \rangle = 9.80 \text{ m/s}^2$ for Maine and 9.78 m/s^2 for Hawaii. In addition to the classroom data, we also obtained measurements by individuals.

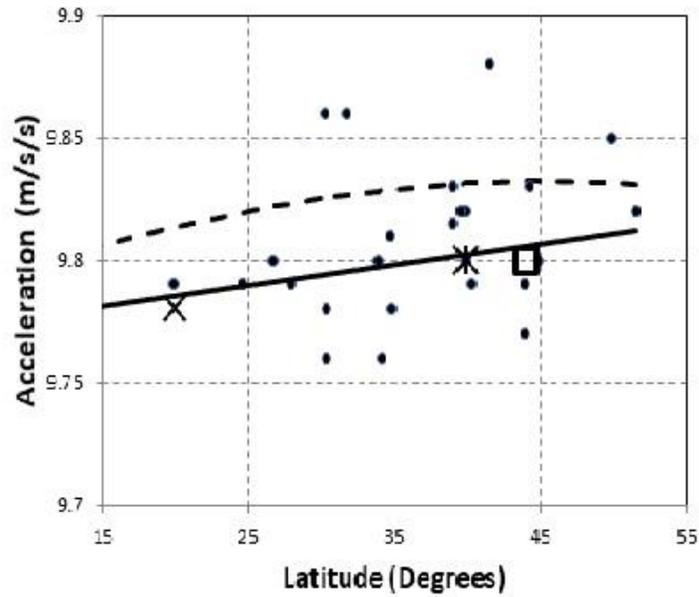


Figure 22 Data from 2017: Individual measurements (dots), Hawaii (cross), Maine (square) and Nebraska (triangle). The nominal (irreducible) measurement uncertainties are $\pm 0.01 \text{ m/s}^2$. The median value for each school measurement is shown. The expected surface gravity at each latitude based on the BGI model (dashed line) and the IGF1980 model (solid line) are also shown.

All that can be concluded with this small sample is that higher latitudes detect a higher value for the surface acceleration, and that the rate-of-change with latitude is consistent in sign and magnitude with theoretical expectations.

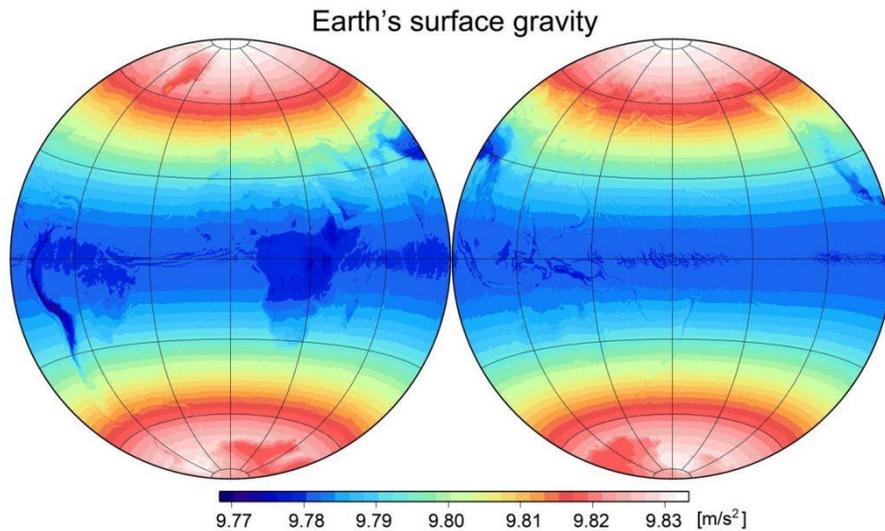
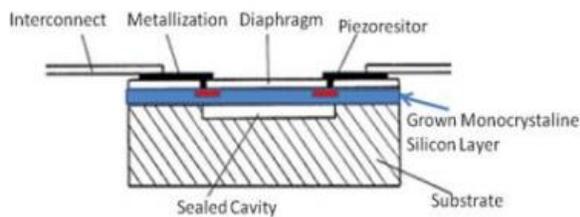


Figure 23. Plot of earth's surface acceleration strength showing variation from pole to equator.

Pressure

Pressure is actually a tensor quantity not a scalar. Pressure is a force applied to a surface area, but that surface area can be oriented in 3-D space. A true pressure sensor would measure the force perpendicular to the face of the smartphone, and this pressure would change as you changed the orientation of the smartphone's face. Force is a vector quantity so it has the components (F_x , F_y , F_z), but each of these three force components can be applied to one of three orientation planes (x - y , x - z , y - z) so there are a total of 9 components to the pressure tensor. The normal to the x - y plane is along the z axis, so this results in measures of the pressure tensor components P_{xz} , P_{yz} , P_{zz} . The smartphone pressure sensor does not actually measure physical pressure but measures an isotropic scalar quantity derived from barometric sensors. This is approximately the magnitude of the pressure tensor, $P = \sum(P_{ij} P_{ij})$, which is a scalar quantity.



Like the accelerometer, the pressure sensor is a microminiaturized, electromechanical (MEMS) transducer designed to function on the micron-scale within the confines of a chip measuring no more than a few millimeters on a side. Like all pressure transducers, it relies on a deformable diaphragm spanning a sealed cavity, and a

Figure 24. Diagram of a pressure sensor.

metrology system. The deformation of the diaphragm is sensed by a piezoelectric system whose resistance varies with the amount of deformation. Alternative metrology systems involve capacitance changes between the diaphragm and the substrate that also lead to a detectable output signal.

- **Air Pressure (Android)** - This application shows the current local air pressure and altitude, measured by the internal pressure sensor (if available) or a nearby weather station for which it uses your current location: the pressure is automatically updated; set a reference altitude: current altitude, sea level or custom altitude; pressure units: hPa/psi, Torr/in, atm, altitude above sea level and percent of average pressure at a reference altitude; compare with reference pressure at a reference altitude; the location is automatically updated when you move; metric and imperial units.
- **BarometerPlus (Android; iOS)** – Barometer Plus app measures air pressure and altitude using the built-in barometric pressure sensor on your device. It can be used for virtually all of your outdoor and indoor activities, such as climbing, trekking, hiking and even stair climbing. Because air pressure changes normally indicate change in the weather, so you can use this app to monitor and predict short term weather changes. Contains: barometer,

altimeter. Unit support for pressure (mb, inHg, kPa, atm, Torr, psi, hPa, mmHg), altitude (meter, feet). Personalization graphic in analog classic style. Calibrate values output by sensors. Calibrate the app by GPS altitude, GPS location and nearest airport information/METAR, manually enter QNH. Pressure tracking and history graph. Notification about pressure change. Useful when you are in weather sensitive zone like desert, mountain, sea. Export history to CSV format and customize notification change value (Pro feature). You should take the phone far from computer, cables and other magnet source in order to get accurate result. Altimeter always show approximately result. Because it count from air pressure (barometer) with some equation. Even Altimeter on any airplane, so in the plane they use many method to get altitude like GPS, radar, know altitude from the Maps, etc.

- **Barometer and Altitude (iOS)** – (Friends of the Web) The Barometer and Altimeter is a free app that helps in detecting estimating elevation and also for weather prediction when calculating air pressure. In addition to this it can also be used with iOS 8 user and widget integration for it.
- **Barometer-14th (iOS)** - gives local atmospheric pressure, obtained from the barometric sensor. Real time display atmospheric pressure information in both Analog and Digital modes. Shows current location based on GPS information (or from the nearest Wifi station). Supports multiple units: hPa, mBar, etc..
- **Barometer - Altimeter (Android; iOS)** – (Baueresis) Shows you the atmospheric pressure and altitude using the barometer sensor of your device.
- **Barometer (iOS)** –This is the first easy to use barometer App for your iPhone 6, iPhone 6 Plus and iPad Air 2 Check your height based on the current pressure and nearest weather station or by a manual entered reference. Automatically update reference pressure from nearest weather station (using openweathermap.org) - Today widget for quick look of current pressure, height and temperature and more to come soon.

Additional Resources:

Calibrating smartphone pressure sensors:

Odenwald, 2018, Smartphone Sensors for Citizen Science Applications:II”, Journal of Citizen Science, (In Press).

Other resources:

Air pressure and density: <http://mathscinotes.com/2016/01/a-mathcad-utility-function-for-air-pressure-and-density/>

Air pressure and height: https://www.engineeringtoolbox.com/air-altitude-pressure-d_462.html

Experiment 1: Air Pressure at Different Altitudes

Run each app sequentially and take a measure of local air pressure at ambient elevation (ca 300 feet) over the course of 5 minutes. An example is shown in Table 4. The Breckenridge, Colorado measurement was made at an altitude of 3000 meters.

Table 4. Air pressure data using different apps.

App Name	Cost	Measurement (2/14 @ 15:30)	GSFC (2/22 @ 13:00)	Top of 11-meter cooling tower	Summit Mt Greylock (1054 meters)	Colorado (3000 meters)
<i>My Altitude</i>					898.45	697.57
<i>Baro</i>	Free	999.8 hPa	1007.6	1006.2	898.5	
<i>Air Pressure</i>	Free	999.8 hPa	1007.6	1006.3	898.4	
<i>Barometer Plus</i>	Free	999.81 hPa	1007.63	1006.25	898.45	697.49
<i>Barometer 14th</i>	Free	999.73 hPa	1007.6	1006.18	898.42	
<i>Barometer & Altitude</i>	Free	999.70 hPa	1007.64	1006.25	898.4	
<i>Barometer-Altimeter</i>	Free	999.63 hPa	1007.62	1006.24	898.5	
Ave		999.75 hPa			898.45	
Rms (1-sigma)		+/- 0.07 hPa			+/-0.05 hPa	

The measurements with the different apps are remarkably similar, and sensitive enough to detect an 11-meter vertical shift in altitude. On Mt Greylock summit the official elevation is 3,491 feet or 1,064 meters. The various apps give the following answers:

- Radiation Counter.....1048 meters;
- Barometer Plus 1054.4 meters;
- Altimeter..... 1057 meters;
- Barometer.....1054 meters;
- Barometer..... 981.7 meters;
- My Altitude..... 1055 meters;
- Air Pressure..... 1003 meters.

Discounting Air Pressure and Barometer, the apps average altitude is 1055 meters with an rms of 2 meters. Given the actual elevation of 1064, these apps are systematically 9 meters lower than the geographic altitude.

Experiment 2: Monitoring Weather Changes

How quickly does atmospheric pressure change as your local weather changes? First of all, we have to check that the smartphones are capable of measuring small changes in pressure. The following graph shows three different smartphones measuring 12 minutes of pressure using the *Physics Toolbox* app. The top curve is for the iPhone 6s, the middle points are for the Samsung 8s and the bottom graph is for the Samsung Note 5. The digitization noise is 1 Pa but the smartphone pressure sensors show different levels of jitter ‘noise’ with the Samsung Note 5 at ± 3 Pa, Samsung 8s with ± 2 Pa and the iPhone 6s with ± 5 . Clearly the phone you use will influence how well you can detect small, fast changes in atmospheric pressure.

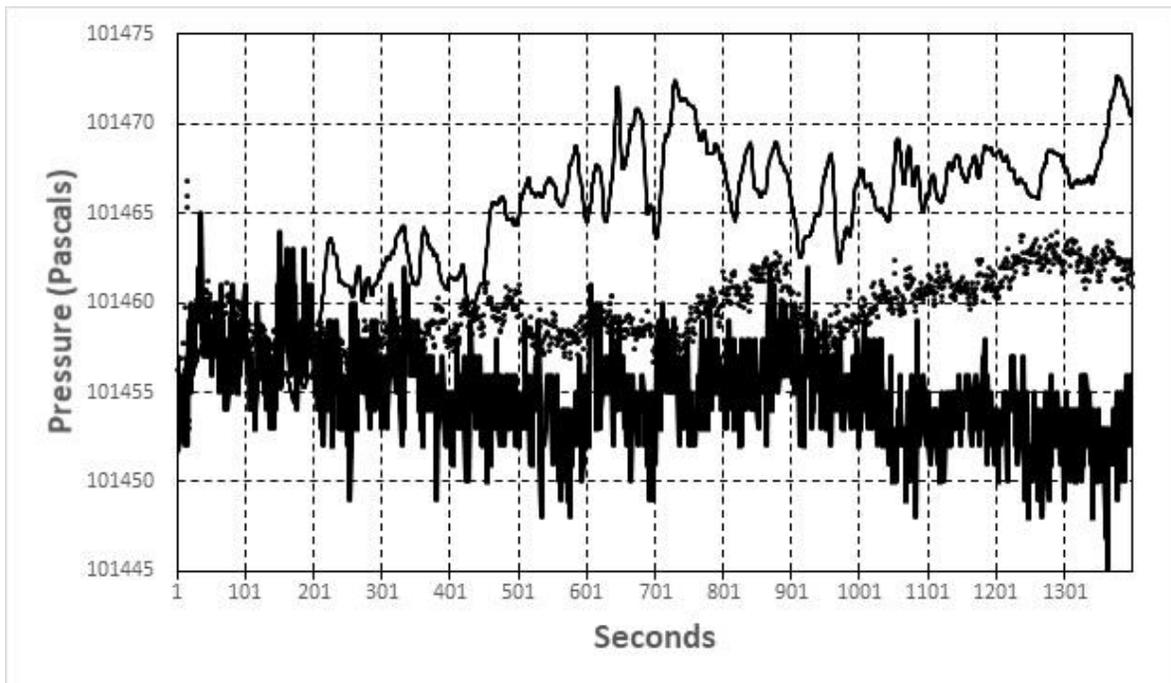


Figure 25. Data for pressure measurements

The quietist smartphone in our study, the Samsung 8s was operated with the *Physics Toolbox* ‘pressure’ app for one hour during morning clear sky and afternoon increasing cloudiness weather conditions.

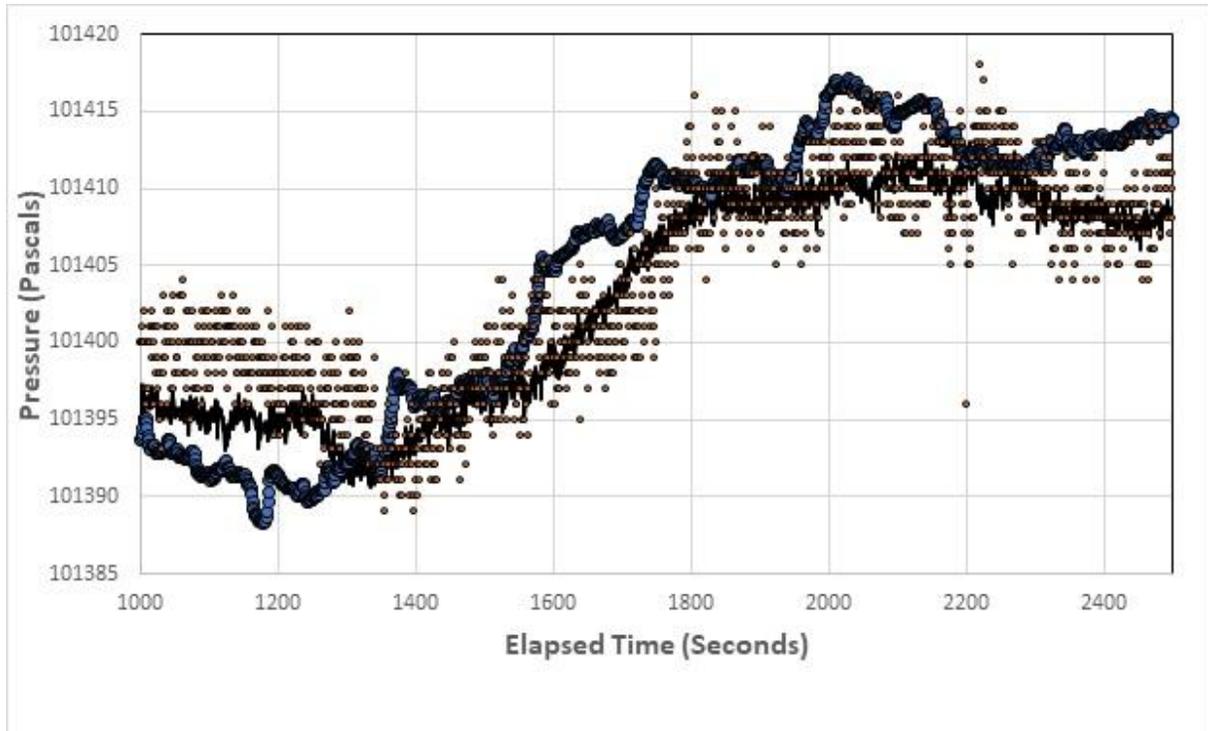


Figure 26. Data for pressure measurements during a storm.

The outdoor variations during times of increasing cloudiness show an overall rise in pressure by 0.15 hPa, and are comparable for the three phones with the iPhone (small dots) Samsung Note 5 (large dots) and the Samsung Galaxy 8s (black line) showing similar variations over time though with significantly different dispersions due to jitter and digitization.

NOAA characterizes barometric changes according to their character:

NOAA Weather Service Pressure Terminology

- Falling (or rising) slowly: 10 - 150 Pa in 3 hours
- Falling (or rising): 160 - 350 Pa in 3 hours
- Falling (or rising) quickly: 360 - 600 Pa in 3 hours
- Falling (or rising) very rapidly: > 600 Pa in 3 hours

This data is from the *Weather Handbook* by Alan Watts (2005).

Pressure Change and Wind Speed

- 100 Pa/hour: likely Force 6
- 200 Pa/hour: likely Force 7 - 8
- 300 Pa/hour: likely gale exceeding Force 8

Table 5 Pressure changes and meteorological conditions.

Pressure change	Character	Conditions
1016 to 1023	Steady	Fair and slight temperature changes for a few days
1016 to 1023	Rising rapidly	Fair followed in 2 days by warmer temps and rain
1016 to 1023	Falling rapidly	Warmer and rain within 24 hours
1023 or above	Falling rapidly	Warmer and rain within 36 hours
1023 or above	Variable	No early changes
1016 or below	Falling slowly	Rain in 18 hours continues for a few days
1016 or below	Falling rapidly	Rain and high winds followed within 2 days by cold/clear
1016 or below	Rising	Clearing and cold within 12 hours
1009 or below	Falling rapidly	Severe storm of wind and rain in 24 hours
1009 or below	Rising rapidly	Clearing and colder

Here is a series of measurements starting from a fall snow storm in Washington DC at 8:00 am (running time 0 minutes), November 15, and then clearing to blue skies the next day by 5:00 am (running time 900 minutes), November 15-16, 2018. The *Physics Toolbox* app was used on a Samsung Galaxy 8s, and was set up to take continuous readings at its minimum rate of 5 samples/sec in sessions of 8-hour duration. The battery drain was 5% per hour.

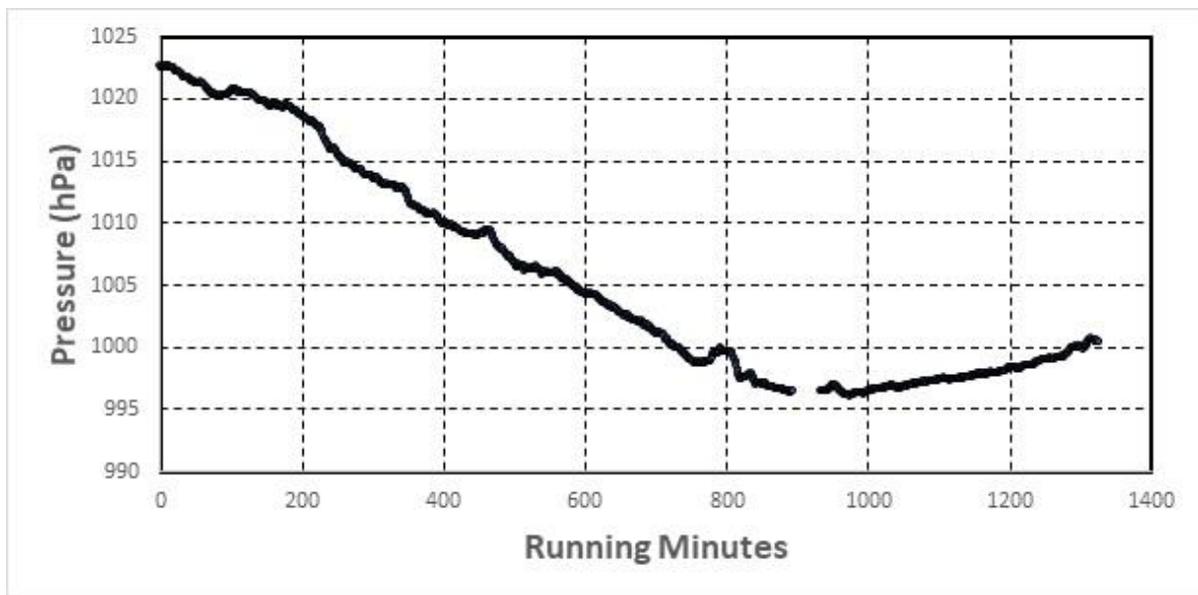


Figure 27. Pressure data during a storm.

Experiment 3 – A Simple Pressure Thermometer.

Gas pressure is proportional to temperature.

Place a smartphone with the *Physics Toolbox* ‘Barometer’ app operating and taking data, inside a sealed container. The container was a 2-quart pyrex casserole with the lid sealed by using peanut butter!

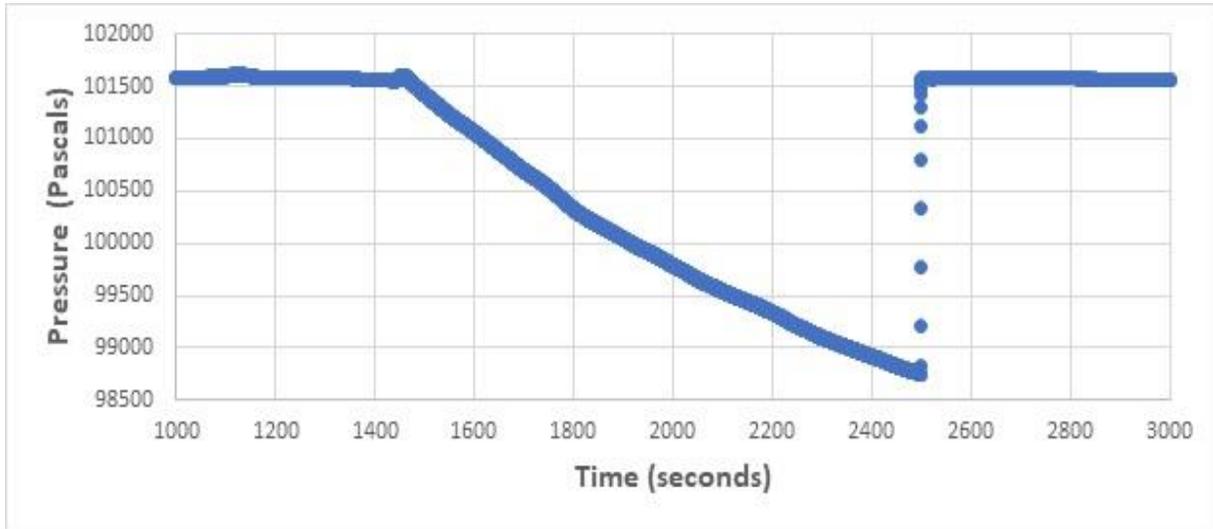


Figure 28. Pressure data for calibration.

The resulting graph shows room temperature operation at 67.4 F measured by the external OmniTemp app on a separate phone. When brought outdoors the temperature reached a stable 47.6 F at which point the barometer indicated a change from 101591 Pa to 99445 Pa, a change by 2146 Pa or 21.46 hPa. The barometer reading did not stabilize because at 2500 seconds, the ‘peanut butter’ seal failed and the pressure quickly returned to the ambient air pressure of 101591 also measured indoors.

A mason jar with an air-tight seal was purchased and both the barometer app running on a Samsung Galaxy 8s and the Omni Temp app running on an iPhone 6s were placed inside the jar and sealed with the rubber sealing ring. This sensor system was operated indoors at 70.8 F and 101351 Pa and then brought outdoors for several measurement cycles. The indoor and outdoor measurements were allowed to reach their equilibrium values each time, which took approximately 20 minutes.

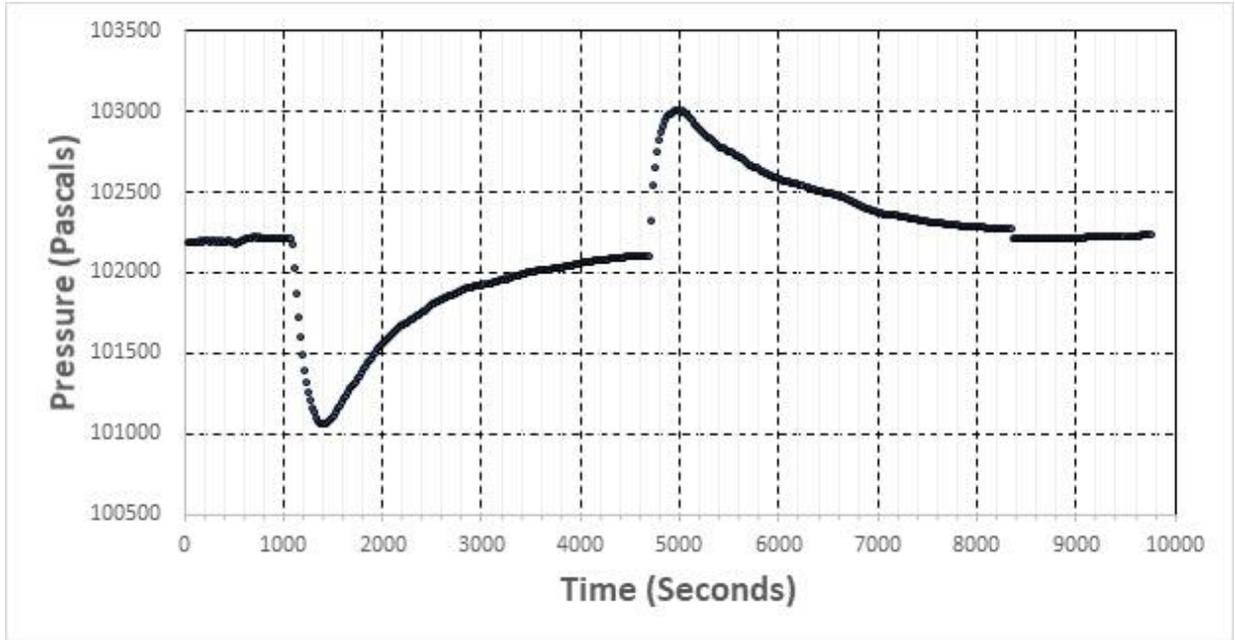


Figure 29. Pressure data for calibration.

During the first 1000 seconds the Galaxy 8s was inside the sealed mason jar indoors where the temperature was 67.6° F. It was brought outdoors where the temperature was 29.6° F at 1100 seconds and left outdoors until 4700 seconds, at which time it was brought indoors and allowed to sit sealed until 6400 seconds, at which time the jar was opened up to the ambient indoor pressure and temperature. The initial pressure decline from 102175 to 101064 of 1111 Pa for a temperature change of 67.6-29.6 = 38° F implied a rate of 29 Pa/degree F. A repeat of this experiment gave 20 Pa/degree F.

With proper calibration, this experiment could be used to demonstrate that air pressure is proportional to temperature via $P = nkT$.

Position-Distance

Virtually all Apps that display geographic location and distance do so by using the Global Positioning System satellite constellation – a marvel of 20th-Century science and engineering. Smartphones can also triangulate your location using use cell tower information. Because GPS ‘chips’ are very power-hungry, the cell tower method is often used for many applications and is quite a bit faster. Anyway, the result of this military GPS system is a commercial location system for smartphones accurate to about 4.9 m (16 ft.) radius under open sky. However, their accuracy worsens near buildings, bridges, and trees. For ‘augmented’ systems used in aviation and in the military, which use dual-frequency transmission, position errors as small as a few centimeters have been routinely achieved. The GPS satellites carry 500-watt transmitters and orbit at a distance of 21,000 km. By the time it reaches ground, the signal strength is about 4×10^{-14} watts/m². This isn’t very much power, but it is enough that modern GPS receivers-on-a-chip can detect it!

The basic idea is that at any moment, four GPS satellites are above your horizon. These satellites transmit a radio signal on two frequencies – your smartphone receives one of these. By using the signal from the fourth satellite, accurate signal timing between the GPS satellites and your smartphone can be accomplished, and from this the distances between your smartphone and the three satellites can be determined. Any change in the arrival times of the signals due to anomalous ionospheric propagation, space weather conditions, or ‘multipathing’ due to buildings and trees in your vicinity, can interfere with the distance calculations and give you a bad location estimate. Also, because clocks run slow in gravitational fields, general relativity has to be used to calculate all the gravitational corrections. Without ‘GR’, position calculations would be in error by hundreds of meters.

App Descriptions

- **A to B (Android; iOS)** - AtoB A simple utility app to measure distance between A to B. This app provides you with a GPS measure between two points **in a direct line** and the distance of that path you used to get there. It's a pretty simple tool that gives you what most running apps can't and without all the unimportant data measures. The app isn't limited to only one purpose, but it could be used to measure up a football field, the distance of a holed golf shot or any other distance measure between two points. **GPS-based distance.**
- **FollowMee** - GPS Location Tracker tracks location of an iOS device. Installing the App to a device that you want to track, it quietly records its locations (GPS, WiFi, or Cellular) periodically and uploads to a secured server. To monitor location of your tracked device, you simply open our web site (<http://www.FollowMee.com>) in any browser (either desktop or mobile). Using this app, you can track your children's movement daily, follow whereabouts of your family members or employees. In additions, you can locate your lost or stolen device. Once the app starts, it runs quietly in the background and requires no

human interaction. It continues to run even after the tracked phone reboots. It will restart itself if terminated by a user. It only stops when the stop button is pressed. Default track interval is set to 10 minutes. You can adjust it from 1 minute to 12 hours. Weekday and weekend track schedules are available. You can use MDS to push app settings to the devices. Track history costs \$6.99 extra. **GPS-based distance**

- **Cyclemeter (Android, iOS)** - Cyclemeter turns your phone into a great cycling computer. Costs \$9.99 for advertisement-free Elite version. Provides charts and maps of route taken, times and elevation changes that can be exported. **GPS-based distances and tracking.**

- **Distance- Find my Distance (Android; iOS)** – This app lets you measure the point-to-point (great-circle) distance between two or more waypoints on a map or measure the surface area defined by a polygon having three or more points as its corners. It uses online maps so it needs an internet connection. You can choose between (kilo)meters, feet/miles and nautical miles. The options menu also lets you choose to go to the Save/Restore view. **Map-based distance**

Experiment 1: How accurate are the distance meters?

There are many possible scales over which we can test these apps from tens of meters to thousands of kilometers. A straight East-West 100-foot path was established with unobstructed access to the sky. The only app that worked in this way was *AtoB*. I made four measurements and got 32m, 29m, 30m, 31m on an East-West course. Average 30.5m +/- 1.3 m. 100-feet = 30.5 meters, so this App gives exact distance upon averaging.

Cyclemeter was tested by driving in a car from A to B. It used about 13% of battery power (GPS) for 43 minutes of operation, and used 509 kby of data. *AtoB* gives two values: a straight line crow-flies estimate and an estimate for curvy paths. I list both. The curvy path seems to give the best estimates.

Table 6 Comparison of distance measures

Course	AtoB (meters)		Cyclemeter (path track)
	Straight	Curvy	
100-foot tape-measure	30.5 m	30.5m	
Home - Grosvenor	2.126 km	3.048 km	3.02 km
Grosvenor – Chevy Chase	4.332 km	6.406 km	6.31 km

Compass Bearing

Most 'compass' apps use the smartphone magnetometer to determine the direction to Magnetic North, then use the smartphone orientation information to determine in which direction you are pointing so that the magnetic bearing can be calculated. By supplementing this with GPS information, the correction to True Geographic North can be deduced. Similar calculations apply in the Southern Hemisphere. The accuracy of determining your azimuth degrees is limited by the accuracy of the smartphone 3-axis magnetometer within the horizontal plane. Most app suppliers disavow these instruments for scientific or high-accuracy orienteering work and recommend you use high-end compass technology with better magnetic sensitivity. Everyone acknowledges that environmental factors can influence the compass so it is best to use it outdoors and far away from obvious sources of magnetic noise like powerlines, generators and transformers.

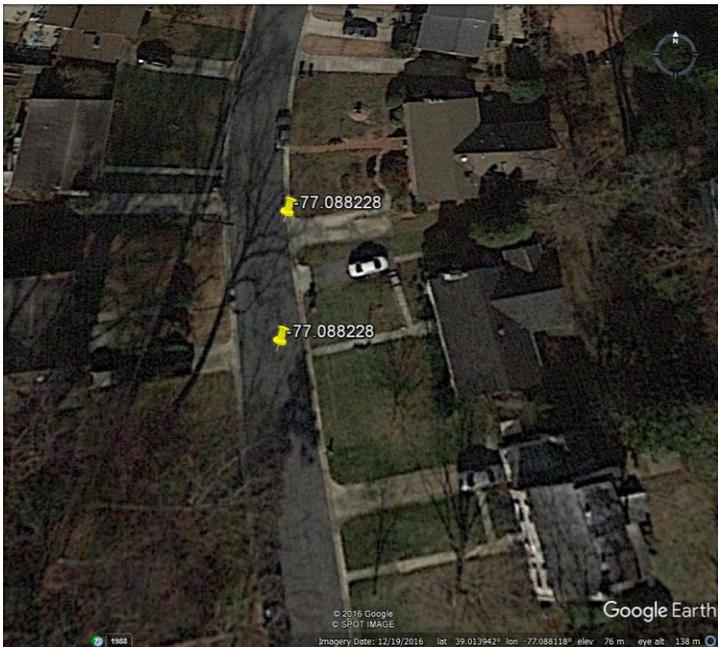
App Descriptions

- **Compass Commander (\$1.89 - Android; iOS)** - This application provides a map-oriented by a compass superimposed. It seems to make perfect a march along a certain direction marked by a compass. There are the direction (including magnetic declination), longitude, latitude, altitude and speed. The application also allows you to take advantage of geolocation services to get information on the current position and send this information via SMS to a number in the phone book (for those recipients who do not have a smartphone - grandparents and / or parents). It is possible send your position via SMS automatically setting up a time frequency. You can also find a location by address, coordinate or from and SMS received and get direction and distance from actual position. It is also possible run navigator with this target. Please note: SMS messages are a paid service: see the rates of your mobile operator.
- **Compass X (\$0.99 - Android; iOS)** – Multifunctional compass application including way point, wind direction, weather and stopwatch in a single app. - Tag up to eight different locations / destinations and Compass X shows you the heading and distance to them. - Map integration and overlay - see your heading over a map. - Heading and direction information. - Shows Sun and Moon information including a direction and their rise and set times - Shows current Moon phase - Stop watch with lap and split times. Uses GPS to set local lat/long
- **GPSCompass Basic** - GPS Compass is a small but complete navigation tool, it leverages on both magnetic compass and GPS to give you always the most accurate information available. GPS Compass shows you distance and direction to geographical location

(waypoint) you can define in many ways. GPS Compass calculates relevant points position by bearing and triangulation then saves the points as waypoints. You can see the waypoints on a map or you can use the map to define waypoints for navigation. The Basic version is limited in the number of waypoints you can define, it allows you up to 30 points. GPS Compass is the basis of the shortcoming Regatta application that adds a lot of feature for sail race navigation. A pdf version of the user manual can be found here: www.embedia.it/files/gps_compass/man_en_1_0.pdf

Experiment 1: Comparison with Analog Compass.

Self-tests. These are tests that compare the directions given for magnetic north by all apps irrespective of where they were located. The idea is to see whether the same environmental conditions lead to the same calculation error. **Absolute tests** compare the Apps bearing for



Magnetic North with that of a high-quality magnetic compass. The tests were conducted indoors and outdoors to sample various electromagnetic environments. The magnetic declination of my house (39.013904° , -77.088222°) is -10.8205°

Figure 30. Location of reference longitude near Author's home.

These two points define a line of fixed geographic longitude at -77.088228 as 0° true north. The magnetic declination is -10.82° , so the magnetic bearing should be true bearing minus magnetic declination. If you point the compass along the line of longitude, it should be a true bearing of $0/360$ but a magnetic

bearing of -10.82° , or 10.82° west.

These can determine true north to within about 5° , and correct magnetic north to within 3° but there is a systematic 3° offset east. *GPS CompassBasic* (\$0.99) is the best and *Compass X* next best (free)

Table 7 Comparison of compass bearings.

App Name	Cost	True	Magnetic
<i>Compass</i>	Free		358° NNW +/-2°
<i>CompassCommander</i>	Free	4°NNE +/- 3°	
<i>Compass X</i>	Free	5° NNE +/-25°	352° NNW +/- 3°
<i>GPSCompassBasic</i>	\$0.99	1° NNE	351° NNW +/- 2°
Calibration		0°	349° NNW
Average		2.5° NNE	354 NNW
RMS		+/- 2°	

Experiment 2: Comparison with Magnetometer Direction

The compass apps all use the smartphone magnetometer so let's see what the magnetometer is doing: *Teslameter 11th*: Inside test of magnetometer rotated 90° on tabletop.

App=*Teslameter 11th* reading from the screen

Position 1: $B_x = -5.3$ $B_y = +22.5$ $B_z = -57.5$ $|B| = 61.7$ $|B_{xy}| = 22.5$

Theta1 = $\text{atan}(-22.5/5.3) = 103^\circ$

Position 2: $B_x = -19.9$ $B_y = -3.3$ $B_z = -51.5$ $|B| = 55.3$ $|B_{xy}| = 20.1$

Theta2 = $\text{atan}(-3.3/-19.9) = 189.4^\circ$

The angle difference is $\text{Theta2} - \text{Theta1} = 86.4^\circ$ but should be 90°

App = *Tesla Recorder* with data logger in csv file

Position 1 $B_x = -6.2$ $B_y = 22.0$ $B_z = -52.2$

Theta1 = $\text{atan}(22.0/-6.2) = 105.7^\circ$

Position 2 $B_x = -22.1$ $B_y = -6.4$ $B_z = -45.9$

Theta1 = $\text{atan}(-6.4/-22.1) = 196.1^\circ$

The angle difference is $\text{Theta2} - \text{Theta1} = 90.4^\circ$ as expected. The data, logging in CSV file is clearly more accurate!

Compass App= *Compass Commander*

Bearing 1 = 358° rotate 90 to Bearing 2 = 87° E. Difference is 89°.

Conclusion: The relative bearings and magnetometer readings seem to be good to a degree or so, but the absolute bearings seem to be off by at least several degrees. This result has been confirmed by other evaluators.

Other Evaluations:

Six iPhones tested and they can't agree on Magnetic North. By Jim Galbraith Oct 18, 2013 in PCWorld.

"Compass app in both iOS 6 and iOS 7 reported screwy results across a wide range of iPhones— from the iPhone 4 to the 5s and 5c. We recalibrated each phone multiple times, and often found that the results changed wildly from one test to the next. To test the accuracy of the various phone compasses, we purchased a \$15 Suunto A-10 recreation compass and took an armful of phones out in front of our office building, away from the electromagnetic interference within. We set the Suunto compass to north and placed it on top of a map of San Francisco. We used the map's grid lines to line up the phones with the Suunto. We tested five types of phones: an iPhone 5s, an iPhone 5c, an iPhone 5, an iPhone 4s, and an LG G2. All of the iPhones except the 4s were running iOS 7 (the 4s was running iOS 6). We ran the test on three different iPhone 5 models, and found the worst deviation there. One model was not accurate at all—15 degrees away from 0 at best, and 28 degrees at worst. That's pointing starkly northeast! Another iPhone 5 did very well, just 5 or 6 degrees from 0, while a third iPhone 5 fluctuated between a respectable 6 degrees away from north and a truly misleading 21 degrees away. Just to give an Android phone a whirl, we replicated our tests with an LG G2 using Compass Pro from Gabenative, and found that the Android phone's readings were the closest to our Suunto, merely 3 to 4 degrees off." (<http://www.pcworld.com/article/2055380/six-iphones-tested-and-they-cant-agree-on-true-north.html>)

Should you trust your digital compass? By Darrel Wilkinson (2013)

"Here we are testing the ability to measure magnetic north, so any readings must reflect that. The analogue compasses used are "working" compasses so have had the baseplate correction set to zero. The iPhone compass was also set to measure [magnetic north](#). I must say that I was alarmed at the apparent error in the iPhone. The mean difference in reading was 4.8 degrees (s.d. 9.2) and 3.2 degree (s.d. 6.8) for compass 1 and compass 2 respectively. Interestingly, however, there seems to be a pattern in the data. Both compasses seem to show a shift in positive to negative/less positive error right around 90-110 degrees. While this pattern is more symmetrical for compass 1, both seem to have a similar negative excursion over approximately 70-80 degrees. So what do these errors indicate? Is this error acceptable? Of course the answer is no. Sure, the

internal magnetometer in the iPhone or any other device which uses a similar piece of hardware is probably good enough for giving you a bearing on a city street, but any remotely consequential use of that information would surely lead to undesirable results. (https://wilkinsondarren.wordpress.com/2013/06/24/19-update-should-you-trust-your-digital-compass/)

Sound Intensity

Clearly all smartphones have a microphone to enable talking. These apps take advantage of the microphone to measure ambient background sound levels. Because microphones are originally engineered to accommodate the normal ranges of speech, they are limited in the range of loudest and weakest sounds they can detect and measure spanning a range from about 60 dB to 120 dB.

Sound levels are measured in units of decibels (dB) which like the magnitude units used by astronomers to describe star brightness, is a logarithmic unit of sound power through a surface. $dB = 10\log_{10}(P1/P0)$ where $P0$ is a reference power level of 1 picoWatt (10^{-12} watts). An increase of 3 dB equals a power increase by a factor of 2.0, so an increase by 6 dB or 3+3 dB equals 2x2 or 4. In terms of actual watts, $P1$, a 60dB sound leads to a sound power of 10^{-6} watts and 120dB yields 1 watt.

Professional evaluations of these apps using variable intensity white-noise generators show that over their dynamic ranges they can be accurate to about +/- 2.0 dB using test sources of known amplitude. However, some professional evaluations also suggest that the use of smartphone apps in real-world situations is fraught with inaccuracy, and that well-trained ears would be better indicators than some of the software/hardware that was evaluated. Note, 3 db is also the smallest light intensity change the human eye can discern in terms of the brightness of stars in the sky! You can get compact, external microphones that clip into your headphone jack, but some evaluations suggest that they didn't make a huge difference in the fidelity or sensitivity of the recordings.

App Descriptions

- **DecibelMeter (Android; iOS)** – Simply launch the app and start reading the sound levels that surround you every day! Want to know just how loud this club is? Just how loud is this lecture anyhow? How good is the sound isolation in this room? Decibel Meter (Pro) can help you on these case! Download Decibel Meter (Pro) and measure the sounds around you right now! Decibel Meter (Pro) also tells you decibel exposure time guidelines. It tells you how loud is too loud. It tells you what's Noise induced hearing loss and how to avoid it. The iPhone's built in microphone is sensitive from 0 to approximately 110 decibels. Why aren't you getting a 0 decibel reading in a quiet room? A decibel is an actual measurement of sound pressure and aside from a vacuum chamber or out in space 0 dB is not really possible to read. An average quiet room will be in the 40-50 dB range. - Plotted history of the Average values. - Record and export the data to email for further analysis.(Pro Version)
- **Decibel Level (Android; iOS)** included annoying popup advertisements. Gives peak, average and current readings at slow-enough rate to follow.

-
- **DeciBel 10th (Android; iOS)** Decibel 10th" is one of very few noise/sound meter apps on the market having highly reliable and pre-calibrated measurements. Time weightings (Response Time): SLOW (500 milliseconds) and FAST (200 milliseconds)- Trimming calibration from -15dB to 15dB- Standard measurement range from 30 to 130 dB(A) Although all devices are pre-calibrated, custom calibration is suggested for serious purposes requiring higher precision and accuracy. You will need a real external device or calibrated sound meter as a reference, then adjust the trimming calibration until the reading matches with the reference. Provides nice linear plot in real time with export as CSV file. Gives average and max reading.
 - **SPLnFFT Noise Meter (\$3.99)** Our highest rated iPad-based app is the SPLnFFT app. It received our highest possible rating –It costs only \$3.99. During the four tests we conducted, this was the only app to absolutely nail the results each and every time. It passed all four tests with flying colors. In fact, this app’s results were so close to the results obtained using the \$2,000 noise meter that the tiny differences between the two are probably due to sampling error. We do have one major concern about this app – it will only measure noise up to a maximum of 94 decibels! For review see: <http://www.safetyawakenings.com/safety-app-of-the-week-42/>

Additional Resources:

Calibrating smartphone sound sensors.

Odenwald, 2019, “Smartphone Sensors for Citizen Science Applications:II”, Journal of Citizen Science, (Submitted).

Noise level chart: <http://www.noisehelp.com/noise-level-chart.html>

Experiment 1: Creating a Sound Level Library

Smartphone was placed on flat surface face up with microphone exposed. None of these free apps allow for storing and sending data via a spreadsheet or text file.

Table 8 Sound level chart for common situations

Source	DecibelMeter +/-3 dB	Decibel Level +/-4 dB	Decibel Sound Meter +/-4dB	Decibel-10 th +/-2 dB
According to Developer				30 is minimum
Summit of Mt Greylock	63	65	38	39
Basement	58	62	35	45
Quiet Room	58	59	30	43
Quiet Room	63	62	55	47
Greenbelt Park meadow	70	60	59	57
Goddard outside	72	76	45	58
Toyota Corolla inside	68	75	48	56
Outside house with leaves	70	75	35	59
Outside house no leaves	76	73	50	60
Room with TV	73	73	70	68
Toyota Corolla outside	75	76	55	65
Freeway by Cedar Lane	84	78	55	69
Queen Concert Audience	80	80	65	75
Jet plane at cruising altitude	92	88	70	86
Electric Lawn Mower	89	86	70	86
Jack Hammer @ 10 meters	89	67	73	86
Queen Rock Concert Music	95	85	75	109
According to Developer				130 is maximum

'Freeway' – was a measurement taken near the Cedar Lane overpass for 495 in Kensington, Maryland. 'Lawn Mower' – was taken 1 meter from an idling electric lawn mower. 'Rock Concert' – Was taken at the Queen Concert in Washington DC on July 31, 2017 – Audience is with no band playing, Music is during song 'We will rock you' 'Outside house no leaves' taken in front of house with no leaves on trees in late-winter April 4. 'Mt Greylock Summit' in the fog was absolutely quiet to my ears. Much more so than any other location sampled. Here is the trace of the data from

Decibel 10th. 'Toyota Corolla', idling: outside, hood up, at bumper. Inside doors windows closed. 'Greenbelt Park meadow' – The sampling spot was 1 km from nearest house and roadway (Greenbelt Road and Rt 295). Noticably not as quiet as summit of Mt Greylock.

Given that the dB scale is logarithmic, the variation in the measured values from app to app is significant. A change of 3 dB equals a factor of 2.5 times in sound energy flux. The app *Decibel-10th* seems to be the superior app in this test run. An absolute sound measurement has to be made with professional equipment to determine reliability for absolute measurements.

Experiment 2: Finding the Zero-Level.

How do the apps respond when there is no sound energy applied? In other words, where is the Zero-level in dB of the sensor? To determine this, the app must be placed in a container in which no external sound energy penetrates. Ideally, an evacuated pressure chamber would be used, but these are expensive.

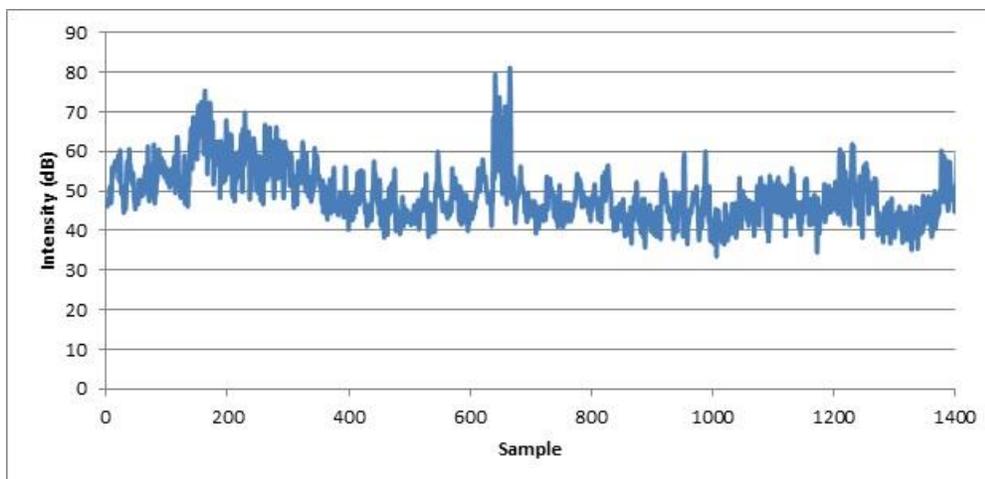


Figure 31. Sound level near zero intensity in basement.

A closed box in a quiet environment might suffice, such as from the scale above a basement area with no discernable sounds from A/C, fan, heating or other units in operation. When I did this, the result was a reading with *Decibel 10th* that was 45 dB. Between my basement and Mt Greylock I have not seen a reading smaller than 35 dB. The box set-up does not seem to make a difference in isolating the sensor from the noise-generating environment. Without the bell-jar vacuum test I can only assume that for practical purposes, the 35 dB value is close to the minimum possible reading. There may be an internal electrical issue that creates in the audio circuit a baseline minimum sound level.

Note from *Decibel 10* at Apple Store: *Please do not expect a quiet room reading will be 0 dB(A). The range 30-130 dB(A) is the standard usable range and an average quiet room will be about 30 dB(A).*

Evaluation of smartphone sound measurement applications

Chucri A. Kardousb) and Peter B. Shaw <http://asa.scitation.org/doi/full/10.1121/1.4865269>

<http://www.safetyawakenings.com/safety-app-of-the-week-42/>

https://www.acoustics.asn.au/conference_proceedings/INTERNOISE2014/papers/p565.pdf

<https://blogs.cdc.gov/niosh-science-blog/2014/04/09/sound-apps/>

NIOSH researchers have just published a follow-up study: Evaluation of smartphone sound measurement applications (apps) using external microphones to examine the performance of sound measurement apps using external microphones. The study showed sound level measurements to be within ± 1 dB of the reference system using the same test setup and apps from the initial study. This follow-up study suggests that using external, calibrated, microphones greatly improves the overall accuracy and precision of smartphone sound measurements, and removes much of the variability and limitations associated with the built-in smartphone microphones.

<https://www.cdc.gov/niosh/topics/noise/app.html>

Experiment 3: Smallest Detectable Difference

What is the smallest sound difference you can discern? Does the dB difference change as the sound gets louder? In astronomy, the smallest brightness difference between two stars you can easily discern is called 1-magnitude and corresponds to a brightness factor of 2.51 times. This equals about 3 decibels (3 dB). Does this apply to sounds too, or is the factor different?

Other Experiment Suggestions

- When the sun sets, does the 'outdoors' background noise get quieter?
- How different is the background noise when the ground is covered with snow versus bare?
- When leaves are gone in the winter, is the landscape quieter than in summer?

Acceleration

Smartphones need to know their orientation in space in order to adjust their displays to keep up with the motion of the user such as dynamically changing from portrait to landscape mode. Although general device accelerations occur along all three geophysical axis, gravity is only present along the local vertical axis (nadir-zenith). By using a simple rotation matrix, the body axis of the smartphone (X,Y,Z) and the local geophysical axis (x,y,z) can be related to each other, and the orientation of the smartphone in space can be determined. Because there is no physical way to discriminate between an acceleration caused by gravitational forces and an acceleration caused by other forces, a combination of outputs from the acceleration, and the gyroscope or the magnetometer sensors, are combined in a software process called sensor fusion, to identify the component of the smartphone acceleration due to gravity. Because gravity is a constant acceleration acting on the smartphone regardless of the other time-dependent changes, a low-pass filtering technique is implemented in software to filter-out the gravity signal. There is no separate, physical ‘gravity sensor’ within a smartphone, only a virtual gravity sensor created from sensor fusion and low-pass filtering.

The acceleration sensor is a state-of-the-art, micro electro-mechanical (MEMs) device fabricated using bulk silicon microfabrication techniques. A silicon substrate is etched to create two types of acceleration sensors. The first type measures accelerations within the plane of the silicon substrate and is used for creating the X and Y sensors. The device is an interleaved, comb-like device in which one set of tines is anchored to the smartphone case and a second set is attached to a cantilevered ‘proof mass’ whose tines flex in response to the movement in one-dimension. The movement of one set of tines relative to the other changes the capacitance between them, and this is sensed and converted to a change in voltage that will be inversely proportional to the displacement. For movement in the vertical plane to measure the Z-acceleration component, a different device geometry is used. The proof mass is cantilevered over capacitive sensors along the smartphone Body-Z axis. Flexing in the proof mass location along the Z-axis is detected and converted into voltage variations. Accelerometers have ranges up to ± 16 gs, however many smartphone designs such as the iPhone 4 limit the scale to only ± 2 gs.

The analog voltage signals from the accelerometer devices are passed to dedicated analog-to-digital converters, one for each axis, which sample the voltages up to 100 times each second. These samples are digitized from 12 to 16-bit resolution as for example the Bosch Sensortec BMA-423 (12-bits) used for wearable technology, or the BMA 280 (16-bit) used in the iPhone 6s. The measurement sensitivity of MEMs accelerometers is determined by the ADC resolution and the $\pm g$ range of the analog output. For a smartphone typically set to a range of ± 2 gs with a 14-bit ADC, if 0-g is set at the middle of the ADC range, then with 1-bit allocated to the sign of the number, the resolution is $1g = 13\text{-bits}$ (e.g. 8191 levels) for a resolution (Least Significant Bit) of 0.122 milliG/LSB or with $1g = 980 \text{ cm/sec}^2$, $1 \text{ LSB} = 0.12 \text{ cm/sec}^2$. A 12-bit ADC gives 0.488 milliG/LSB or 0.48 cm/sec^2 .

App Descriptions.

- **Sensor Kinetics Pro** (Android and iOS: David Feinstein) exports accelerometer and other supported sensor data as .csv files. Local acceleration is measured to 0.01 m/sec^2 at a fixed 30 Hz cadence and provides a real-time graphical display of all three acceleration components in the Body Frame system.
- **Physics Toolbox Sensor Suite** (Android and iOS; Vieyra Software) is a massive collection of 20 different sensor functions all in one app. All sensor data can be recorded by pressing a button. Data can be saved in a .csv file, and transferred via e-mail or shared in Google Drive or Dropbox. Clicking on the (i) icon within each mode provides information about the sensor name, vendor, and current data collection rate, in addition to a description of what kind of data is collected by the sensor, its physical operating principle, and links to additional resources, including webpages and videos that illustrate how it works. The Android model allows the display cadence to be adjustable from 420 Hz to 5 Hz. The iOS model operates in a fixed mode with a cadence of about 200 Hz.
- **MRE Acceleration Meter** (Android: MRE Programming) after calibration by gently shaking it, its Gravity Calculator mode calculates your local gravitational acceleration using your GPS or manually-entered coordinates. It uses the World Geodetic System 1984 which models Earth as a smooth ellipsoid and from your coordinates calculates a model-based local acceleration. The app also has an Acceleration Meter mode which after shaking gently to calibrate provides a digital and graphical display of your local acceleration vector components including a measure of the local gravitational acceleration using the smartphone's 'gravity sensor'. The digital display shows the local vertical component along the smartphone's current Z axis to four-digits in m/sec^2 .
- **Accelerometer: g-force meter** (Android: Miodne Projects) provides a digital display to 0.01 'g-units' (e.g. 0.1 m/sec^2) of the average, maximum and minimum readings along with a graphical display showing the elapsed observation time and the acceleration to 0.01 g's.
- **Accelerometer Meter** (Android: Keuwlsoft) provides a display to 0.01 m/sec^2 and the sampling cadence can be changed with a slider from slow to fast from 0.5 to 0.03 seconds. The exported data logging can be saved to a .csv file.
- **Accelerometer** (iOS: DreamArc) real-time display with data exporting. The measurement cadence can be changed with a slider from 1 to 30 Hz. The units are in measured and displayed to 0.001 gs. The app also allows for exported .csv files.
- **Mobile Science** (iOS: Indiana University) provides acceleration readings to 0.01 gs, with adjustable cadence up to 10 samples/sec, and mailable output as a .csv file. The 'calibrate' feature has to be disabled so that Earth's g is not automatically subtracted from the data. The .csv file reports accelerations to 0.00001 gs (0.0001 m/sec^2).

-
- **SparkVue** (iOS: PASCO) allows for data logging and .csv emailing. The acceleration display offers measurements adjustable to 0.001 m/sec^2 through a number formatting function. Cadences can be adjusted from one sample every 24 hours to to 1000 Hz

Additional Resources:

Calibrating a smartphone sensor:

Odenwald, 2018, Smartphone Sensors for Citizen Science Applications: I", Journal of Citizen Science (In Press).

Accelerometers:

What they are and how they work: <https://www.livescience.com/40102-accelerometers.html>

Can be fooled by sound waves: <https://spectrum.ieee.org/tech-talk/telecom/security/smartphone-accelerometers-can-be-fooled-by-sound-waves>

Vibration characterization:

[https://www.researchgate.net/post/Are smartphone accelerometers useful for vibration characterization in the lab](https://www.researchgate.net/post/Are_smartphone_accelerometers_useful_for_vibration_characterization_in_the_lab)

Beginners Guide: <https://www.dimensionengineering.com/info/accelerometers>

Experiment 1: Comparison of Static Acceleration

How well does your smartphone perform if you just place it on a leveled tabletop? The expectation is that the accelerations in the horizontal directions will be exactly zero but the vertical acceleration will be purely due to the acceleration of gravity near Earth's surface which is 9.8 m/sec^2 but depends slightly on your latitude according to Table 9 below based on a mathematical model (IGF1980) and measured data (BGI). The difference is due to the centrifugal acceleration effect which is highest at the equator and zero at the poles and differs by 0.05 m/sec^2 being this much higher at the poles where centrifugal force is zero. Only at the poles will your accelerometer yield the pure gravitational acceleration due to gravity.

Table 9 Observed vs predicted g-values

Latitude	BGI	IGF1980
80	-9.830	-9.831
60	-9.820	-9.819
40	-9.800	-9.802
20	-9.787	-9.786
0	-9.778	-9.780

- Step 1) Select any of the accelerometer apps, such as Physics Toolkit and open it.
- Step 2) Place the smartphone on a leveled table top, preferably a granite kitchen counter.
- Step 3) Follow the app directions to start recording data .
- Step 4) Accumulate about 100 samples of data and stop recording.
- Step 5) Follow the directions to email the .csv file to yourself.
- Step 6) Download the .csv file to your computer and open it using MS Excel.
- Step 7) From the X, Y and Z values, create a new column that calculates $G = (X^2+Y^2+Z^2)^{1/2}$
- Step 8) Plot the G values on a graph
- Step 9) Select the section of the data that shows no significant changes (flat distribution)
- Step 10) Copy and paste the selected data into a new column
- Step 11) Calculate the average value $\langle G \rangle$ of the selected N values for G.
- Step 12) In a separate column for each data point subtract the average value and square the result to calculate the single-point variance
- Step 13) Sum all the variances and divide by N-1. Take the square root of the result: σ
- Step 14) Your measured value is $\langle G \rangle \pm \sigma$

How did your measured value compare to the expectation for your latitude from the above table? Smartphones are capable of measuring local gravity to $\pm 0.01 \text{ m/sec}^2$.

Experiment 2: Measuring Controlled Acceleration with a Turntable

For circular motion, the equation of motion of the center of the iPhone is given by

$$X = R\cos(\omega T) \quad Y = R\sin(\omega T)$$

$$V_x = -R\omega \sin(\omega T) \quad V_y = R\omega \cos(\omega T)$$

$$A_x = -R\omega^2 \cos(\omega T) \quad A_y = -R\omega^2 \sin(\omega T)$$

The speed of the center is just $V = R\omega$

The magnitude of the acceleration is just $A = R\omega^2$

$$A = V^2/R = (\text{RPM}/30 \times 2\pi R)^2/R = 0.0035 \text{ RPM}^2 R$$

Turntable set up:

RPM consistency: I used a Technics SL-220 turntable with strobe synchronization and two standard speeds 33 and 45 rpm that could be adjusted with the strobe to get an exact rpm. To test the rpm and verify its consistency I placed a small magnet on the platter, and using a smartphone attached to a fixed stand, I ran the magnetometer app from *Sensor Kinetics* while the turntable was in motion. Several seconds of data were gathered and the resulting .csv file was exported and analyzed. X axis is sample number and y-axis is the Bz value in μTeslas .

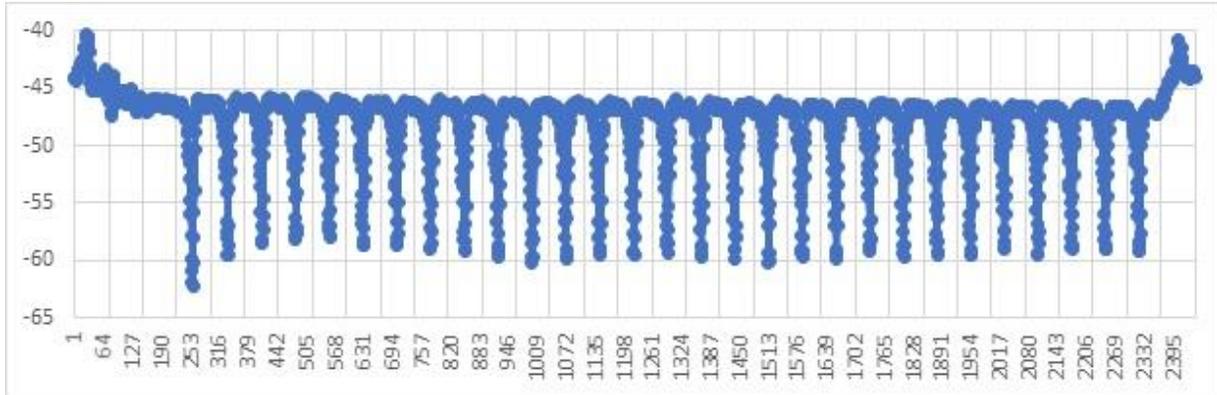


Figure 32. Acceleration data during turntable rotation.

At 33 RPM, the average sample interval between peaks is 73.7 ± 0.86 . In units of time, each rotation has a period of 1.84 ± 0.02 seconds for an RPM of $32.6 \pm 1\%$. At 45 RPM the relevant measured period is 1.44 ± 0.03 seconds and the RPM is $41.7 \pm 2.2\%$

Leveling: Unless the turntable is perfectly level on a plane perpendicular to the local direction of gravity, some of the gravity acceleration will be projected onto the tilted plane producing a slight modulation of the A_x and A_y values. The turn table was leveled by removing the vibration pads from the four feet of the turntable housing at each corner. A *SeeLevel* app was used to place shims under each foot so that the 2-d bubble level showed the minimum deviation. Because of the slight wobble in the platter as it rotates under a load, the best leveling that could be achieved was to $<1.2^\circ$. This means that the data will have a periodic acceleration term contributed by local gravity with maximum amplitude of about $\delta A = 9.8 \times \sin(1.2^\circ) = 0.20 \text{ m/sec}^2$ or 20.3 cm/sec^2 .

Stationary measurement of noise and offsets.

With the iPhone 6s placed on the motionless turntable, the Sensor Kinetics accelerometer was used to measure the local gravity and projection of the local acceleration along the z-axis onto the x and y plane.

A data series of 200 samples (10/sec) yielded $\langle A_x \rangle = 0.11 \pm 0.013 \text{ m/s}^2$, $\langle A_y \rangle = -0.066 \pm 0.011 \text{ m/s}^2$, and the acceleration of gravity $\langle A_z \rangle = -9.86 \pm 0.05 \text{ m/sec}^2$. So in cm/sec^2 we have $\langle A_x \rangle = 11 \pm 1.3 \text{ cm/sec}^2$ and $\langle A_y \rangle = -6.6 \pm 1.1 \text{ cm/sec}^2$

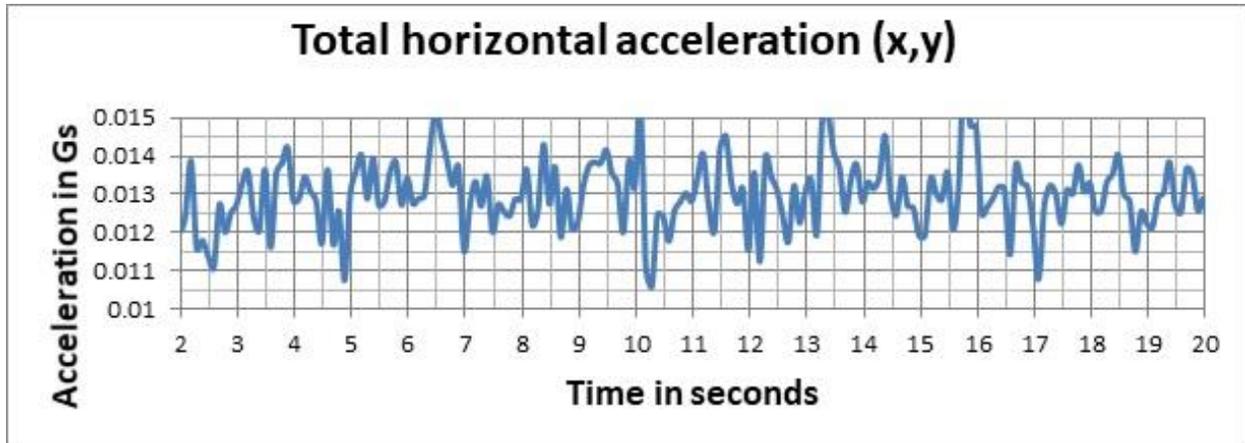


Figure 33. Acceleration data for horizontal motion.

The horizontal-plane gravity component is $A_{xy} = (A_x^2 + A_y^2)^{1/2} = 0.125 \text{ m/s}^2$. This implies a tilt angle of $\arccos(0.125/9.8) = 89.86^\circ$ or an inclination of 0.14° from the normal axis. From a previous study of detecting earth's gravity, smartphone accelerometers have a maximum ADC measurement precision of $\sigma = \pm 0.01 \text{ m/sec}^2$ or $\pm 1 \text{ cm/sec}^2$.

Rotation Measurements.

The smartphone was placed so that the long axis (Y) was parallel to the radial direction from the spindle center to the edge of the platter. The figures below show data taken at the turntable's 33 and 45 RPM settings in terms of the total A_{xy} acceleration.

$$A = V^2/R = (\text{RPM}/30 \times 2\pi R)^2/R = 0.0035 \text{ RPM}^2 R$$

$$A = \frac{V^2}{R} = \frac{1}{R} \left(\frac{2\pi R \times \text{RPM}}{60} \right)^2 = 0.011 R \times \text{RPM}^2$$

For 33 RPM and $R=0.08 \text{ m}$ for the center of the smartphone on the platter, we get $A = 1.0 \text{ m/sec}^2$ and for 45 RPM we get 1.8 m/sec^2

The ratio of the measured 45/33 accelerations should be the predicted ratio based on $A = R \omega^2 = (45/33)^2 = 1.86$. Using the actual turntable RPMs determined above, the ratio should actually be $(41.7/32.6)^2 = 1.63$.

Sensor Kinetics. This app gives an accelerometer and a gyroscope readout in three axis in fully SI units of meters/sec² and radians/sec along each axis. File can be emailed.

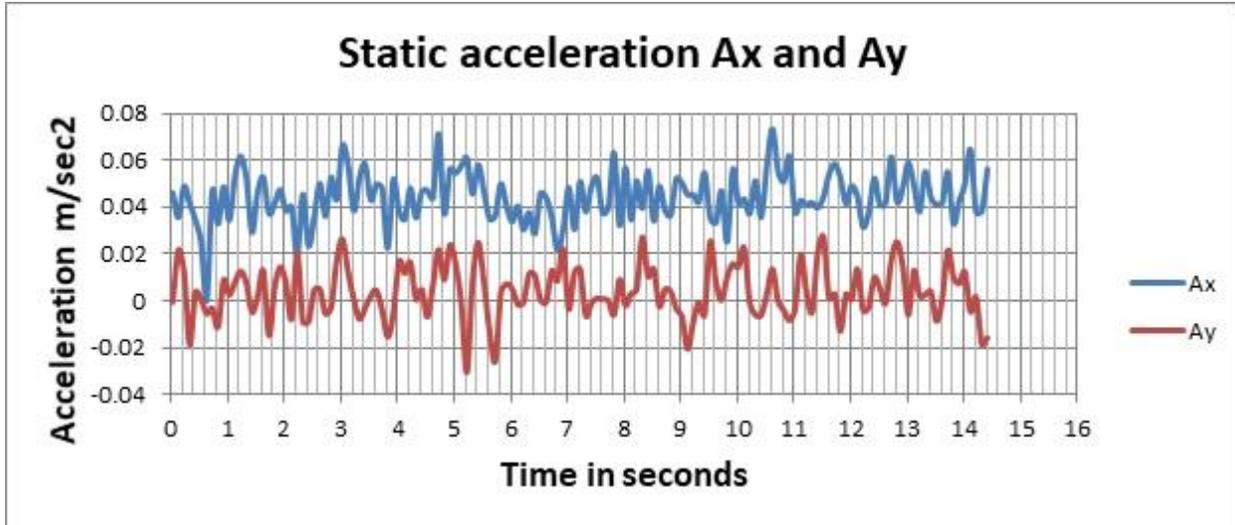


Figure 34 Acceleration data.

$$\langle A_x \rangle = 0.044 \pm 0.011 \text{ m/sec}^2 \quad \langle A_y \rangle = 0.0035 \pm 0.011 \text{ m/sec}^2$$

Gravity: $\langle A_z \rangle = -9.811 \pm 0.009 \text{ m/sec}^2$

So along the long y-axis of the phone there is essentially no net acceleration for a stationary phone, but along the shorter 'width' x-axis there seems to be a net $+0.044 \text{ m/sec}^2$ shift at 4-sigma. Now let's do the spin test at 45 RPM.

Why does the graph look like the one in Figure 35? There are two things going on. The sinusoidal jiggle is caused by the slight projection of the normal gravity vector onto the XY plane of the smartphone due to the fact that the turntable was not perfectly leveled, as we discussed in a previous section. You will note that the juggles are about 90-degrees out of phase because $A_x = A \cos(\omega t)$ and $A_y = A \sin(\omega t)$.

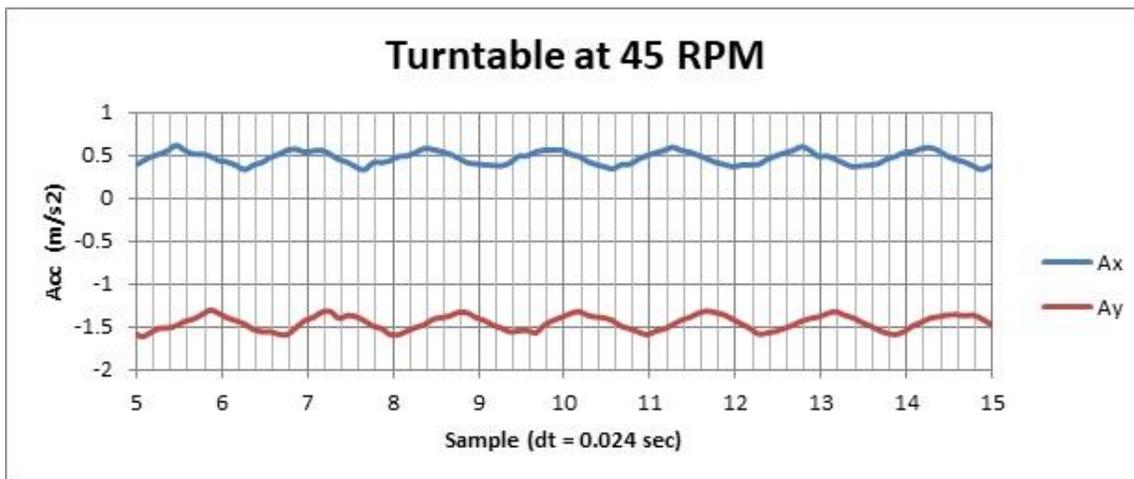


Figure 35. Acceleration data for x and y directions.

But what accounts for the two values $\langle A_x \rangle = 0.4 \text{ m/s}^2$ and $\langle A_y \rangle = -1.6 \text{ m/s}^2$? These values are much larger than the spill-over of the local gravitational acceleration into the tilted horizontal plane of the turntable. If you draw a line from the spindle hub to the location of the accelerometer sensor, this defines the value for R and the centripetal acceleration $A=R\omega^2$ at the sensor. This acceleration is along this radial direction. The values for A_x and A_y in the smartphone coordinate system are the projections of A along the x and y axis of the smartphone such that $A = (A_x^2 + A_y^2)^{1/2}$. If you orient the smartphone such that its X -axis is parallel to R , $A_x=A$ and $A_y=0$. We will explore this orientation issue and how to use it in Experiment 3.

Experiment 3: The location of the acceleration sensor.

Depending on the model of the smartphone, the accelerometer sensor, which is only about 1 millimeter in size, is located somewhere within the body of the smartphone, which you could discover by disassembling the smartphone, or by using a turntable.

First orient the smartphone so that its long axis (Y) is located parallel to the radius vector from the hub to the outside edge. Figure 36 is the plot of Absolute magnitude $|A| = (A_x^2 + A_y^2)^{1/2}$

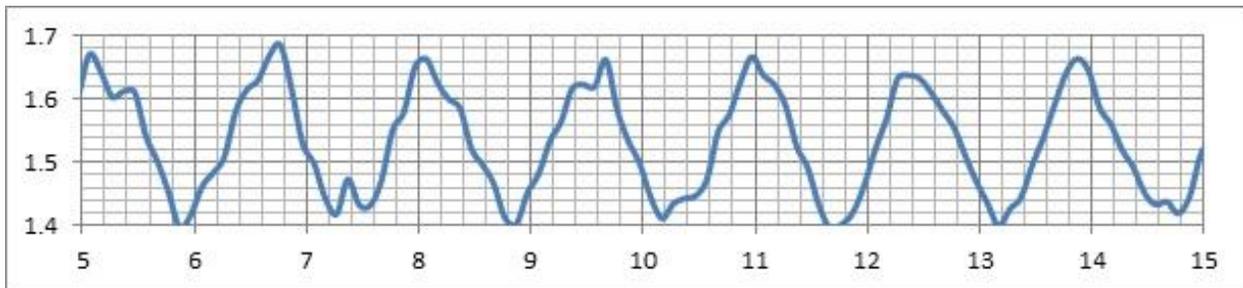


Figure 36. Acceleration data for total horizontal motion.

The resting average is $\langle A_x \rangle = 0.093 \text{ m/sec}^2$ and $\langle A_y \rangle = -0.077 \text{ m/sec}^2$. Subtract these offsets from the plotted data, and calculate the corrected absolute magnitude $|A| = (A_x^2 + A_y^2)^{1/2}$

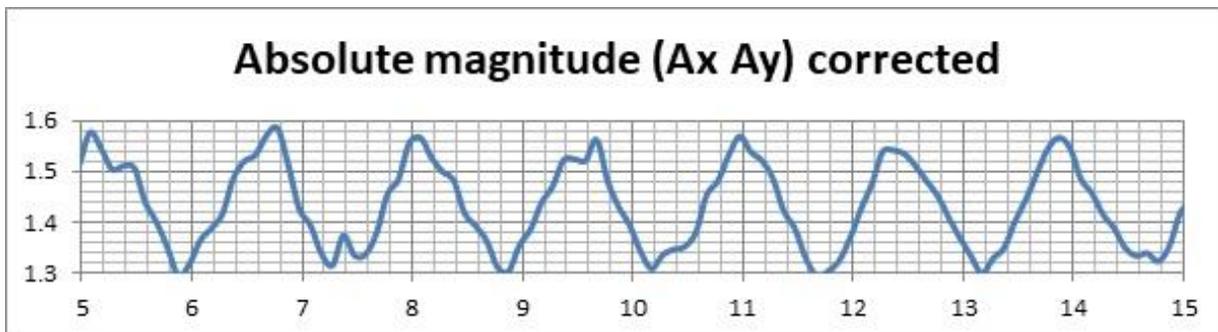


Figure 37. Acceleration data for x and y directions.

The amplitude of the modulation, $\pm 0.15 \text{ m/s}^2$ corresponds to a turntable tilt of $\text{acos}(1.4/9.8) = 89.7^\circ$ or 0.3° from the normal direction, which is well within the leveling tolerances. The offset

$|A| = 1.4 \text{ m/s}^2$ should be equal to the centrifugal acceleration of the smartphone sensor located at a radial distance of R from the platter hub. From $A = 0.011 \text{ RPM}^2 R$, and the actual RPM of 41.7, we can deduce that the acceleration sensor is at $R = 0.07$ meters from the center of the spindle. If we draw an arc across the smartphone face with this radius from the center of the platter, the accelerometer has to lay along this line. If we rotate the smartphone 90° so that the X-axis is now parallel to the radius vector and repeat this measurement series to get the new $|A|$ we derive a second radius, which we draw across the face of the smartphone. Where the two curves intersect, that is where the sensor chip is located.

For additional discussions of this method see *The Physics Teacher*, Vol. 54, April 2016, and the online article at <http://scienceblogs.com/dotphysics/2010/04/27/where-is-the-accelerometer-in/>.

The Gyroscope

In addition to an accelerometer, smartphones have a sensor that measures the amount of rotation around each of the three smartphone body axis. The iPhone 6s that was used in this experiment is a InvenSense MP67B 6-axis Gyroscope and Accelerometer Combo.

InvenSense MP67B 6-Axis MEMS IMU in iPhone 6 & 6 Plus

*InvenSense's first design win in Apple iPhone
InvenSense's Second Generation 6-Axis Devices for Consumer Applications*

With 74.5 million units of iPhone sold during the last quarter 2014, InvenSense made a very good deal by replacing STMicroelectronics as the supplier of the gyroscope for the iPhone 6 and 6 Plus.

The MP67B is a custom version of InvenSense 6-Axis device (3-Axis gyroscope + 3-Axis accelerometer) made for Apple. Compared to InvenSense's standard 6-Axis device MPU-6500, specific modifications have been realized. The main ones consist in the package modification with the use of a LGA substrate compared to a QFN leadframe, and in the ASIC which has been redesigned.

The MP67B uses a different design than InvenSense's first generation 6-axis device with a new design of the 3-axis gyroscope which now uses a single structure vibratory compared to three different structures for the previous generation of gyros. This new design results in a shrink of 40% of the 3-axis gyro area. The second benefit of this new design is that Nasiri process has been changed: cavities which were traditionally etched in the ASIC to allow MEMS structures moving are no longer used, thus resulting in cost reduction.

The report includes a complete comparison with InvenSense standard 6-Axis (MPU-6500) and with InvenSense first generation 6-Axis device.



TABLE OF CONTENTS

Glossary	Manufacturing Process Flow
Overview/Introduction, InvenSense Company Profile	<ul style="list-style-type: none"> ASIC Front-End Process MEMS Process Flow Wafer Fabrication Unit Packaging Process Flow & Assembly Unit
Physical Analysis	Cost Analysis
<ul style="list-style-type: none"> Package <ul style="list-style-type: none"> Package Views & Dimensions Package Opening Wire bonding Process Package Cross-Section Die <ul style="list-style-type: none"> View, Dimensions & Marking MEMS Removed MEMS Sensing Area MEMS Cap ASIC Delayering & Process Die Cross-Section: ASIC Die Cross-Section: Sensor Die Cross-Section: Sealing Die Cross-Section: Cap 	<ul style="list-style-type: none"> Yields Hypotheses ASIC Front-End Cost MEMS Front-End Cost MEMS Front-End Cost per process steps Total Front-end Cost Back-End : Probe Test & Dicing Wafer & Die Cost Back-End : Packaging Cost Back-End : Packaging Cost per Process Steps Back-End : Final Test & Calibration Cost Component Cost
Comparison with InvenSense MPU-6500	Estimated Price Analysis
	<ul style="list-style-type: none"> InvenSense Financial Ratios Component Price

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Performed by
SYSTEMPlus
CONSULTING

This gyroscope uses a tuning fork vibration sensor in which a pair of prongs are made to flex at their natural frequency, and will resist turning, so will fall behind the plane of the rest of the unit if the phone is turned.

Figure 38. Example of gyroscope technical information.

Typical smartphone gyroscopic sensors have a digitization level of 0.001 radians/sec. As a comparison, the rotation of Earth produces a 2π radian change every 24 hours at the equator, which is a rate of 0.00007 radians/sec and so is unmeasurable by smartphones.

Experiment 1: Simple controlled rotations using a turntable.

The turntable used in the experiments involving the accelerometer sensor can be used to apply a known rotation rate to each of the gyroscopic sensors. I used a Technics SL-220 turntable with strobe synchronization and two standard speeds 33 and 45 rpm that could be adjusted with the strobe to get an exact rpm, or at least to minimize the wobble sound in music.

To test the rpm and verify its consistency I placed a small magnet on the platter, and using a smartphone attached to a fixed stand, I ran the magnetometer app from *Sensor Kinetics* while the turntable was in motion. Several seconds of data were gathered and the resulting .csv file was exported and analyzed. X axis is sample number and y-axis is the Bz value in μ Teslas.

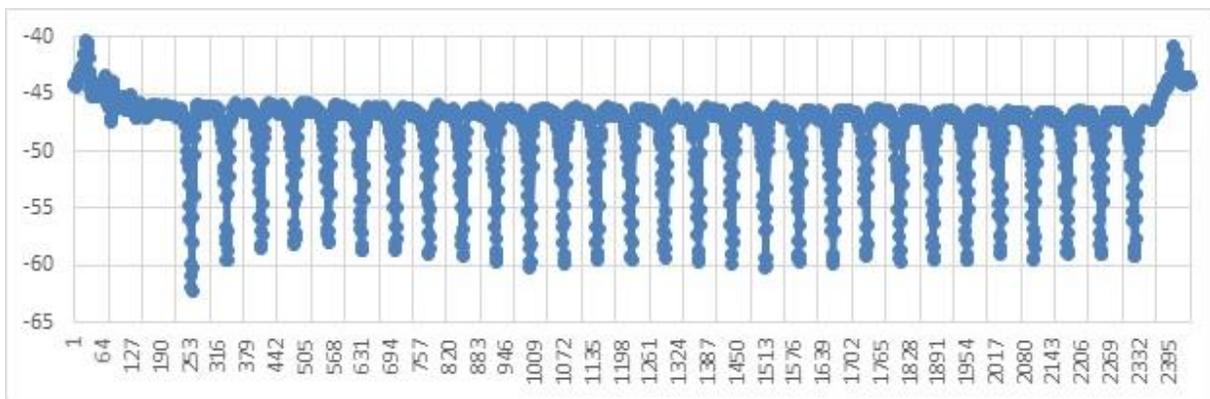


Figure 39. Magnetometer data for turntable rotation.

At 33 RPM, the average sample interval between peaks is 73.7 ± 0.86 . In units of time, each rotation has a period of 1.84 ± 0.02 seconds for an RPM of $32.6 \pm 1\%$. The rotation rate is just $2\pi/\text{period}$ or $\omega(33) = 3.4$ radians/sec. At 45 RPM the relevant measured period is 1.44 ± 0.03 seconds and the RPM is $41.7 \pm 2.2\%$. The rotation rate is then $\omega(45) = 4.36$ radians/sec.

The smartphone Z-axis will be perpendicular to the turntable platter when it is leveled properly, so the experiment will determine the rotation rate in the XY plane around the Z-axis.

Step 1) Place the smartphone on the turntable platter and open the *Sensor Kinetics* Gyroscope app.

Step 2) With the turntable off, record several seconds of data to determine the static noise properties of the gyroscope sensor. Typical, plotted data should look like Figure 40.

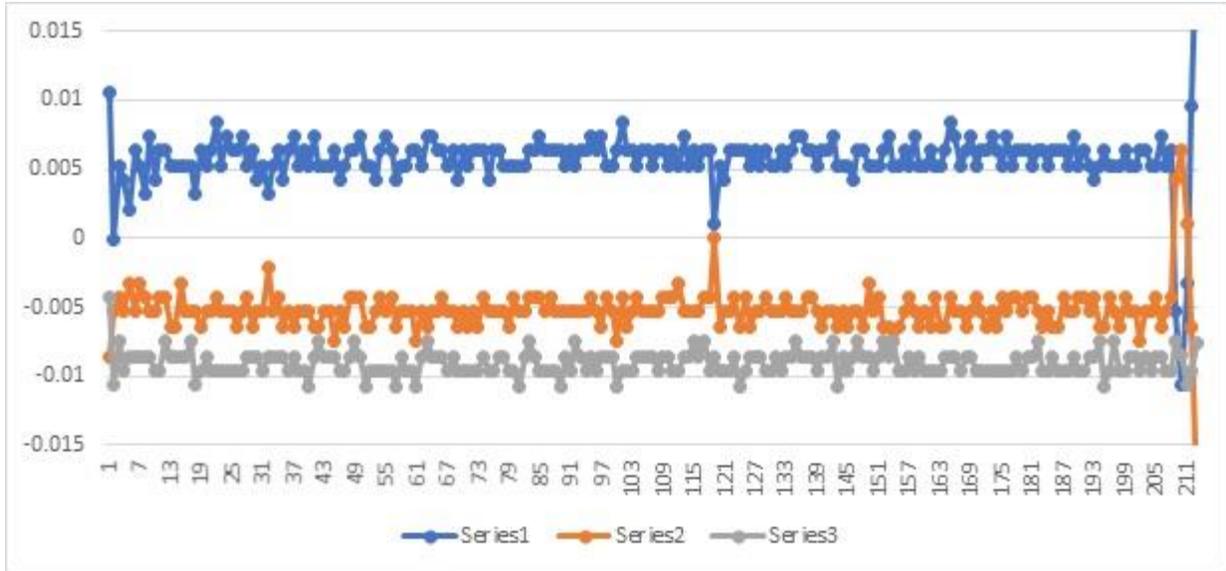


Figure 40. Gyroscope sensor data.

The ADC digitization levels are clearly visible in the step-like value changes. The steps have a value of 0.001 radians/sec. The average values for the stationary smartphone are

$$\langle \omega_x \rangle = +0.0058 \pm 0.002 \text{ radians/sec}$$

$$\langle \omega_y \rangle = -0.0052 \pm 0.0018 \text{ radians/sec}$$

$$\langle \omega_z \rangle = -0.009 \pm 0.001 \text{ radians/sec.}$$

These are the systematic offsets that have to be subtracted from the sensor on the rotating turntable to get the corrected values for the rotation rates.

Step 3) Restart the data-taking for the gyroscope app and turn on the turntable at the 45 rpm rate. Take data for a few seconds then switch to the 33 RPM rate. The data should look like Figure 41.

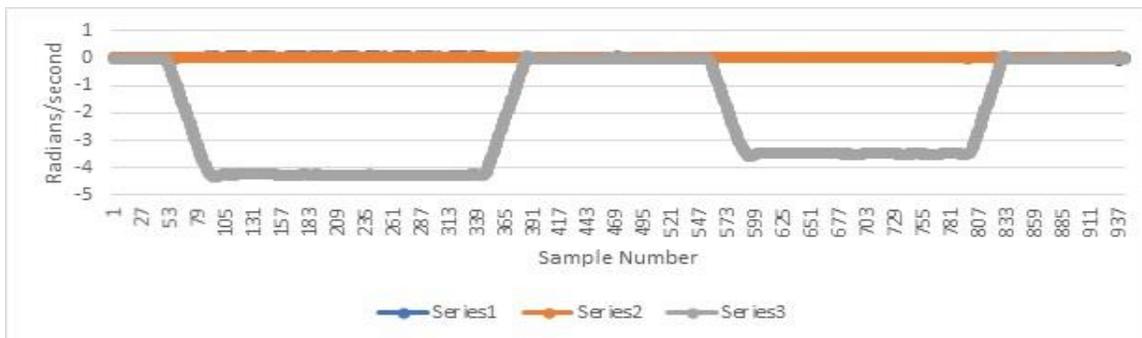


Figure 41. Gyroscope sensor data.

Step 4) Note that only the Z-axis values are non-zero as expected. Now calculate the averages and standard deviations for the two rotation speeds to get

$$\langle wx \rangle = +0.050 \pm 0.0021 \text{ r/s}$$

$$\langle wy \rangle = -0.0006 \pm 0.0017 \text{ r/s}$$

$$\langle wz \rangle = -4.291 \pm 0.046 \text{ r/s}$$

Now subtract the offsets from Step 2 to get

$$\langle wx \rangle = +0.044 \pm 0.0021 \text{ r/s}$$

$$\langle wy \rangle = -0.0046 \pm 0.0017 \text{ r/s}$$

$$\langle wz \rangle = -4.282 \pm 0.046 \text{ r/s}$$

The z-axis rpm is the only one that is relevant and is similar to the predicted rate of 4.36 radians/sec for an rpm of 41.7 based on the actual rpm of the turntable. The rpm equivalent to the measured rotation rate of 4.28 radians/sec is 40.9, which is within the range of the platter speeds adjusted by the turntable stroboscope.

Step 5) Perform a similar procedure to determine the rotation rate at 33 RPM.

If we plot the 45RPM data for ω_z at a higher resolution we get the plot in Figure 42.

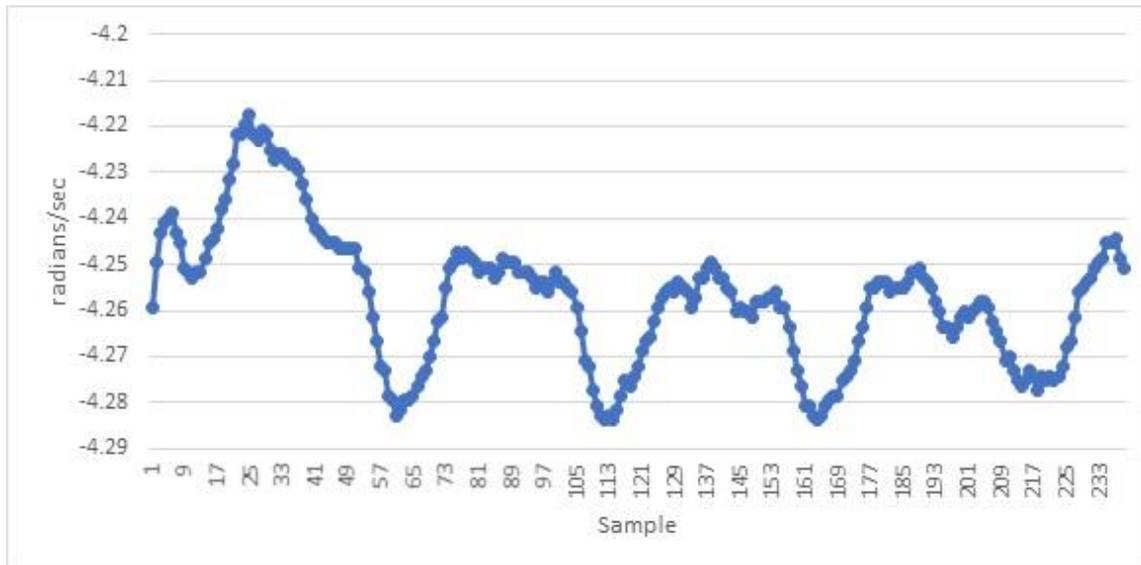


Figure 42. Gyroscope data showing variations.

This shows that the turntable speed is not constant but varies between -4.29 and -4.22 with periodic changes every 50 samples (1.4 seconds). The range of RPM values is 40.9 to 40.3, which could be detectable by the ear as a slight pitch change in music by about 1.3%.

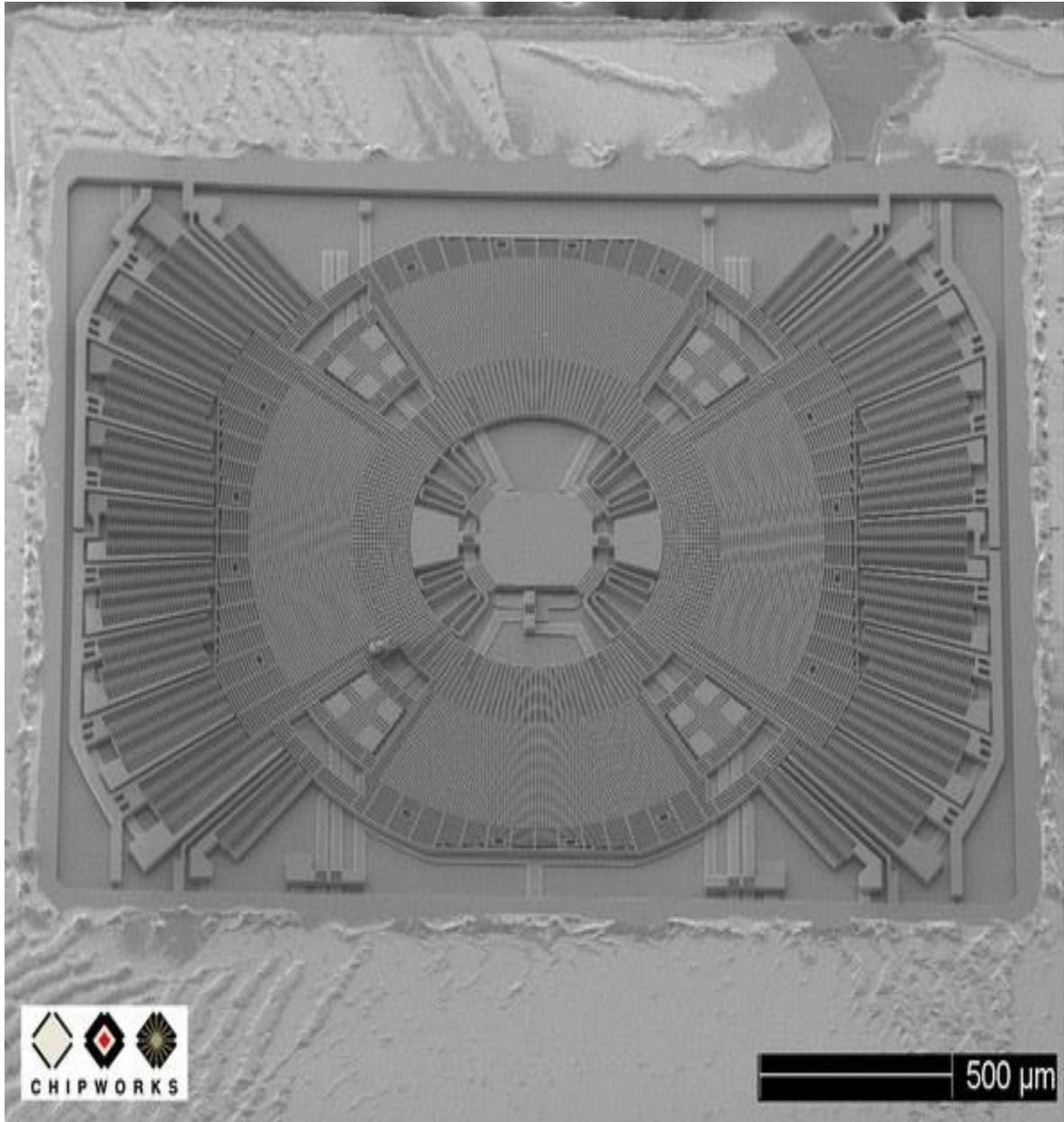


Figure 43. A MEMS gyroscope sensor.

Light Intensity

Smartphones use light meters to set the exposure speed and the ISO ‘film speed’ of the digital camera. These metering systems are created by ‘spotting’ the pixels in a portion of the camera array chip to measure the incident light that falls on them. This information in the ‘spot metering’ mode is used to set the exposure speed of the camera. The f-stop of a smartphone camera is fixed usually at f/2 (fast) or f/2.4 (slow) or so, unlike in digital cameras. The roughly half-stop difference in cameras corresponds to the faster cameras seeing 50% more light, and so they perform better under low light level conditions.

Smartphones also use ambient light sensors are stand-alone devices to determine how bright the display needs to be for comfortable viewing. These sensors are not very directional unlike the camera light metering system. Most smartphone ambient light sensors are photodiodes whose current flow varies with the amount of applied light.

The following apps measure light intensity in terms of illuminance in units of Lux. Physically, illuminance is related to the flux of radiant energy per square meter, and is the product of the emission spectrum and the normalized spectral response function of the human eye, and summed over all wavelengths to get the total energy flux. Because each wavelength is weighted by how well the human retina can detect it, there is no single conversion factor between lux and W/m^2 , and it is not possible to make a conversion unless one knows the spectral composition of the light. Mathematically

$$Lux = \frac{\int_0^\infty B(\lambda,T)N(\lambda)d\lambda}{\int_0^\infty N(\lambda)d\lambda}$$

where $B(\lambda,T)$ is the blackbody function in units of spectral energy density $watts/m^2/steradian/meter$, and N is a weighting factor between 0 and 1.0 such that at the peak of the human eye response at a wavelength of 555 nm we have $N(555) = 1.0$. As an example, 0.0029 watts of green light ($\lambda = 510$ nm, $V(\lambda) = 0.5$) will provide an illuminance of 1 Lux, while only .0015 watts of yellow light ($\lambda = 555$ nm, $V(\lambda) = 1$) provide 1 Lux, and it takes 0.015 watts of red light ($\lambda = 650$ nm, $V(\lambda) = 0.1$) to provide 1 Lux.

Because the flux of sunlight during noon daylight conditions is about 100,000 Lux, and this corresponds to a solar insolation of about 1000 $watts/m^2$, we can estimate that 1 lux = 100 $watts/m^2$. A 60-watt incandescent light bulb might generate 850 lumens. But since 1 lux = 1

LUX RATING CHART			
	Light Level (LUX)	Foot Candles (FC)	
Day Time	Sunlight	107,527	10,000
	Daylight	10,752.70	1,000
	Overcast Day	1,075.30	100
	Very Dark Day	107.53	10
	Twilight	10.75	1
Night Time	Deep Twilight	1.08	0.1
	Full Moon	0.108	0.01
	Quarter Moon	0.0108	0.001
	Starlight	0.0011	0.0001
	Overcast Night	0.0001	0.00001

lumen/1 m², if the bulb is at the center of a sphere with a surface area of 16 square meters, you have 850/16 = 53 lux. Generally a 60-watt bulb does not produce 60 watts of luminous energy but significantly less, so the conversion factor above does not apply.

App Descriptions

- **LuxMeter Galactica** – (\$1.99) If you professionally deal with lighting or simply want to measure the illumination of the certain place, and you do not have a special device, you can use this application to make light measuring, combine the values obtained (lux) with a photo of your place of measurement, add your comments and save your image in photo album or send via e-mail. This app is used for evaluative light measuring and is not a substitute for specialized device.
- **Light Meter (free) by Butta**. A camera view allow you to view the area you want to measure. you can choose between the front and rear camera. Additionally the shutter speed, ISO and focal are displayed.
- **Light Meter (1.99) by Polyanskiy** - Exclusive accuracy of measurements. Calibrated with professional high-precision illuminometer. We have made this Light Meter for that you could save on purchasing of professional illuminometers which cost hundreds of dollars. At the same time we love beautiful design and we like to surprise, therefore, having acquired our illuminometer you also receive the graceful and extremely convenient device with intuitive interface.
- **Light/Lux Meter (free) by Giudicelli** – Displays Lux, f/stop, speed, ISO.
- **Light Meter (\$1.99) by Polyanskaya** - Calibrated with professional Illuminometer. - Realtime measurements - One-time measurement - Average and Max values - Indoor and outdoor modes - Front and rear cam - Lux and Foot candle measurements.

Many apps use the camera's own exposure setting, so can vary by nearly a stop at times from what a meter might see. iOS devices provide a "brightness value" in their EXIF data stream, and that value appears to track external meters very closely. Apps that use the brightness value, or that do image analysis on captured pix to compensate for the camera's ETTR behavior, should track an external meter to within a tenth of a stop.

The 'camera view' meters are clearly the easiest to use since you have a direct image of the exact field being metered. The lux values registered for these camera-based apps are nearly identical across the entire dynamic range of the light levels. The 'spot' systems provide no indication of the location in the field being metered and although they are roughly consistent in measured values with each other, the values are significantly different from the lux values of the camera-metering systems. Although the spot systems claim to measure lux units, the values of the measurements are dramatically different than the camera-view apps at the high light levels.

Additional Resources:

Calibrating a smartphone sensor:

Odenwald, 2019, Smartphone Sensors for Citizen Science Applications: II”,
Journal of Citizen Science (In Press).

Experiment 1: Exploring Lumens and Lux.

Unlike magnetism measured in teslas, temperature measured in degrees, or sound intensity measured in decibels (dB), light has a variety of units associated with it depending on the specific circumstances. Also, these units can be represented either in SI (lumens, lux etc) or English measures (candelas, foot-candels, etc), which further complicates the discussion. In addition to the plethora of measuring units, there is also the added complication that because light is an electromagnetic (EM) phenomenon, it comes in various wavelengths, so the units are generally wavelength-dependent. Moreover, the measuring device or sensor also has different responses to the applied EM energy depending on the wavelength. In all of the discussions we will use the SI units of lumens and lux to indicate light intensity, and discuss other units as needed.

Light Energy

Light is a form of energy measured in units of Joules, and the energy carried by an individual particle (photon) of light is described exactly by the equation

$$E = \frac{h c}{\lambda}$$

where h is Planck’s Constant, which has a value of 6.6×10^{-34} Joule-seconds, c is the speed of light (3.0×10^8 m/s), and λ is the wavelength of light in meters. For example, a photon of light near the center of the visible spectrum has a wavelength of 500 nano-meters (nm), and so will carry an energy of 4.0×10^{-19} Joules. Chemical reactions that involve light require about 1.6×10^{-19} Joules, so ordinary visible light is more than adequate to drive many chemical reactions in nature, especially photosynthesis.

Light Luminosity.

The flow of light energy per unit time is called its luminosity and is measured in Jules/second (watts). When the filament in a lamp is heated, the combination of the light generated over the entire surface of the filament, and through the entire surface of the bulb is called its luminosity. If

the bulb remains on for 100 seconds, the total energy it has radiated in that time from its filament is just 100 times its luminosity and is measured in Joules. This energy has to be supplied by chemical reactions or by electricity, or in the case of the sun by thermonuclear fusion of hydrogen nuclei. For ordinary sources of light, the unit called Lumens is used to represent the emitted watts of a given light source. This unit is wavelength-dependent. For light emitted at 555 nm, where the human eye is most sensitive, 1 watt = 685 lumens. Here is a list of common light sources and their lumens-equivalent. Note: luminosity is a fixed feature of a light source and has nothing to do with the Inverse-square law.

Table 10 examples of common light intensities

Source	Luminosity
Candle	13
Flashlight	15-200
40-watt bulb	450
60-watt bulb	800
100-watt bulb	1600
Car headlight	1500
Mercury-vapor lamp	60,000
Stadium lighting	250,000
Searchlight	13,700,000

Light Brightness

The flow of light energy through a surface of a given area is called the flux. It is measured in watts/meter². This quantity is significant because it directly relates to the brightness of a light source. This is also the quantity that is directly governed by the Inverse-Square Law.

Brightness is measured by the unit called lux such that 1 watt/m² = 1 lux. From our previous discussion of luminosity, we also see that 1 lux = 1 lumen/ 1 m². Because light is emitted in all directions from a light source, geometrically we envision the light at the center of a sphere whose surface area is just $S = 4\pi R^2$. For example, a 100-lumen bulb emits a flux of $100/4\pi R^2$ into the surface of a sphere of radius R in meters. If R =1 meter, then the brightness of this source will be $100/4\pi = 8.0$ watts/m² or 8.0 lumens. This also means we can work this relationship backwards. For example, if we measure a light source that reads 10 lux at a distance of R=3 meters, its luminosity is 1130 lumens.

$$\text{Lumens} = (\text{Lux}) \times (4\pi R^2)$$

Experiment 2: Light Flux and Electrical Power

When consumers want to install a solar-electric system on their roofs, they often call-in professional installers to assess whether various locations on their property will provide enough sunlight to let solar panels function properly. You can make this initial assessment yourself before you call-in the professionals, just to see if you are at least in the right ball-park! A comparison of a professional hand-held sunlight watt meter and a smartphone running a light metering app, *Galactica*, led to the following table of values:

Table 11. Lux vs power

Galactica (lux)	DT-3701 (w/m ²)
138	0.9
1700	10
3462	18
6700	23
8000	55
15000	108
20,000	230
33,000	361
41,000	458
55,000	460
60,000	488
70,000	603

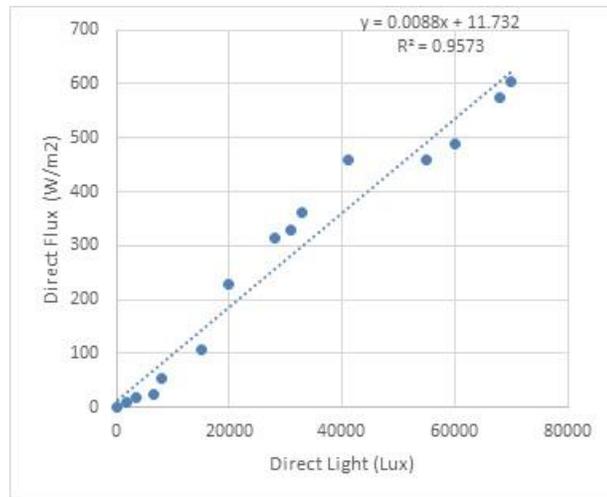


Figure 44. Graph of surface power and light intensity

The linear fit to the data in Figure 44 gives a slope of $0.0088 \text{ (w/m}^2\text{)/Lux}$ or $113 \text{ watts/m}^2\text{/lux}$.

- Step 1) Place a white sheet of paper on the surface to be tested.
- Step 2) Measure the reflected light from the paper using a camera-metering app to get the intensity measurement R.
- Step 3) The white paper only reflects 60% of the incident flux, so the incident light $I = R/0.6$

- Step 4) Multiply I in lux by 113 to get the incident watts/m². The maximum at ground level with the paper laid flat on the ground is about 1000 watts/m².

Experiment 3: A Catalog of Every-Day Light Levels

The illuminated world and our eye's sensitivity to it, range across many orders of magnitude from bright noon-day sunlight in the summer, to dim shapes discerned by fully dark-adapted eyes at night. Select one of the metering apps and use it to measure light levels. Indicate whether the light is direct (D) or reflected (R).

Table 12. Representative intensity measurements

Source	Type	Spot-Hex	Galactica	Extech
Noon-Jan 29 th	R	382,500	177,000	69,900
Noon-October	R			
August sunlight – 4:00 PM	R	45,200	177,000	66,500
August twilight – 6:00 PM	R	12,100	45,600	17,000
60 watt bulb @ 18"	D	311	552	386
Daylight Livingroom	D	71	55	58
60 watt bulb @ 72"	D	31	46	39
Dim Room –newspaper unreadable	R	2	17	10
Twilight 30m after sunset	D	0	7	1.3

Experiment 4: The World of Dim Lighting

LUX RATING CHART			
	Light Level (LUX)	Foot Candles (FC)	
Day Time	Sunlight	107,527	10,000
	Daylight	10,752.70	1,000
	Overcast Day	1,075.30	100
	Very Dark Day	107.53	10
	Twilight	10.75	1
Night Time	Deep Twilight	1.08	0.1
	Full Moon	0.108	0.01
	Quarter Moon	0.0108	0.001
	Starlight	0.0011	0.0001
	Overcast Night	0.0001	0.00001

The sun is a great source of variable lighting throughout the day, so for a simple study of faint light levels we can explore the brightness of the sky as the sun sets or rises. The measurements are made by using the camera-imaging *LUX light meter*, whose lux scale has a zero-level when the camera lens is fully closed by covering it. The *Galactica* meter has a minimum meter level of 7 lux when the camera lens is covered and is not likely to be useful for faint metering.

Beginning at sunset (or sunrise) begin making measurements of the brightness of the sky directly overhead (zenith). To gauge how often to measure, wait until the zenith sky brightness decreases below 1000 lux at sunset at the start of twilight. Ideally you want to have at least 10 measurements during twilight. Astronomical twilight ends about 45 minutes after sunset, which is when the first stars become visible

A sample of these measurements might look like the data in Table 13, created on October 30, 2018. Local sunset occurred at 6:09 PM. Astronomical twilight ends 30 minutes after sunset when the sun is 18 degrees below the horizon. This is also the start of ‘dusk’. There were no stars visible by the author at 6:38 pm.

Table 13. Twilight lighting measurements

Time	<i>Galactica</i>	<i>LUX</i>	Notes
5:19 pm	7,100 lux	13,600	Still comfortably bright outside.
5:38 pm	6,300	5,800	
5:58 pm	2,200	2,300	Street lights turn on
6:08 pm	736	765	
6:20 pm	218	136	Noticeably dark
6:28 pm	34	20	Can read newspaper with difficulty
6:31 pm	14	9	Crickets start chirping!
6:38 pm	7	0	Sill can see faint blue sky and read large font
6:38 pm			End of astronomical twilight – first stars -Dusk

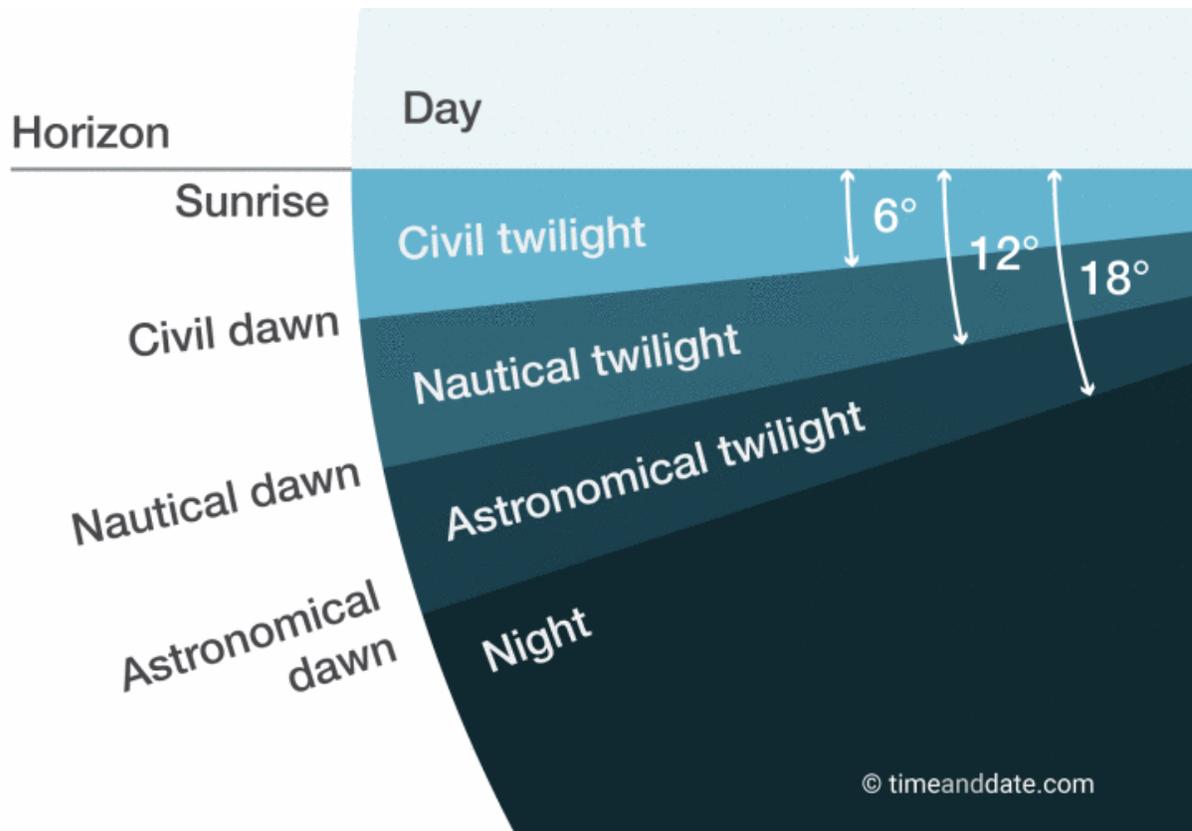


Figure 45. Diagram of sunset lighting conditions by name.

Astronomical twilight occurs when the Sun is between 12 degrees and 18 degrees below the horizon.

Astronomical dusk is the instant when the geographical center of the Sun is at 18 degrees below the horizon. After this point, the sky is no longer illuminated. This is the beginning of night time.

Experiment 5: Reflectivity and Albedo

The albedo of an object or surface is a measure of what percentage of light will be reflected from it for a given incident lighting. For example, a perfectly-reflecting mirror will have an albedo of 100% ($A=1.0$) so that if 100 watts of visible light energy fall on its surface, 100 watts will be reflected. The most perfect black surface, on the other hand, has an albedo of 0% ($A=0$) and will reflect no visible light energy at all. Instead, it absorbs 100% of the light energy, so if you want to figure out how hot a surface will get in direct sunlight, the amount of absorbed light energy is just $E = 100\% - \text{Reflected}$, or in terms of A , we have $E = \text{Incident Energy} \times (1-A)$.

Reflectivity is an important factor in professional photography because although the light source (sun, lamps) may have one set of colors, the surface of clothing and a subject's skin have

different albedos at different wavelengths so to select the right exposure speed and f/ stop is a complicated process depending on what features of the scene you want to emphasize and get the proper color balance.

To calculate albedo, you need to measure with a light meter the amount of reflected light in Lux, and the amount of light that is falling on the surface in Lux so $\text{Albedo} = (\text{Reflected}/\text{Direct})$. If 10,000 lux falls on a surface and 6,000 lux is reflected, the albedo is 0.6 or 60%. This is about the same as a sheet of ordinary white laser printer paper. Fresh snow has an albedo of 80%, while fresh asphalt has an albedo of 10% or 0.1.

Table 14. Albedos for various objects of interest.

Surface	Direct(lux)	Reflected(lux)	Albedo
Street Asphalt (greyish)	257,000	16,100	0.06
Grass lawn in shade	16,600	431	0.03
White laser printing paper	11,350	7,000	0.62

Note: When using sunlight, use a piece of white paper for which the albedo is already known (i.e. 0.62). Place the paper next to the surface you want to test. The direct sunlight striking the test surface will be the measured, reflected light from the white paper divided by 0.62.

Experiment 6: Atmospheric Aerosols and Haze

Sunlight passes through the atmosphere and scatters off of dust and water droplets to give various levels of ‘blue sky’. A crisp fall afternoon generally has an intense blue sky, while a hot summer afternoon tends to have a milkier blur sky due to the amount of moisture carried in the air.

One way of gauging atmospheric transparency is to measure the sky. This needs to be done in a specific way so that measurements on different days can be compared. First, from basic geometry, the thickness of the atmosphere in the overhead direction (zenith) is the minimum possible thickness compared to other angles above the horizon. This minimum distance intercepts what is called the ‘airmass’ of extinction. At different slant distances the amount of airmass increases. This means looking directly overhead you should see the deepest-blue sky, and as you sight towards other angles approaching the horizon, the airmass increases and the milkiness of the sky should increase towards the horizon.

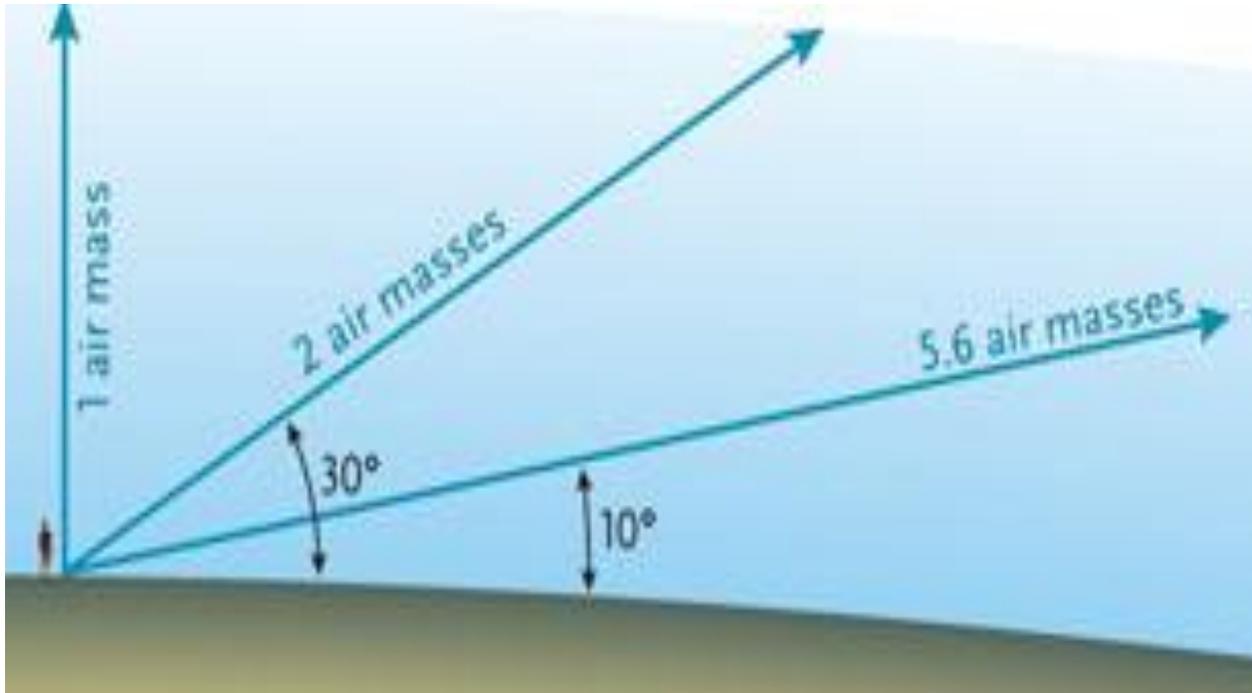


Figure 46. Geometry of air mass. The number of airmasses is given by $A = 1/\sin(\text{altitude})$.

On several fall days in October, a series of light meter readings were made of the sky brightness at 9:30 am from the zenith to the horizon shown in Table 15. The app *Galactica* was used.

Table 15 Sky brightness measurements. 55% humidity, light milky blue sky.

Elevation	Sun angle	Intensity
24	21	17,500 lux
60	21	15,700
80	21	12,600
90 Zenith	21	11,600

The appearance of the sky correlated with the change in zenith airmass, with milkier skies being more reflective and brighter.

If we can determine the brightness of the sunlight at the top of the atmosphere, we can convert the zenith brightness into a direct physical measure of the amount of aerosol in the atmosphere. We know that 1367 watts/m^2 is the flux of sunlight, and from our conversion factor in Experiment 3 we get $1367 \times 113 = 154,500 \text{ lux}$. The relationship between the brightness of the zenith atmosphere and the 'optical depth' τ is given by

$$I(\text{obs}) = (154.5 \text{ kLux}) e^{(-\tau/\sin(\theta))}$$

So that for I(obs) in column 4 and θ from column 3 we get τ in column 5. The average zenith optical depth was $\langle\tau\rangle=1.10$

Table 16. Derived sky opacities

	Time	Sun angle	Zenith Brightness	τ
55% humidity	9:30 am	21	11,600	0.92
	10:30 am	28	16,100	1.05
	11:30 am	32	19,600	1.09
Very high thin clouds	12:30 am	37	18,600	1.27
Temp = 58F	1:30 pm	37	18,400	1.27
	2:30 pm	33	19,500	1.12
	3:30 pm	26	16,000	1.00

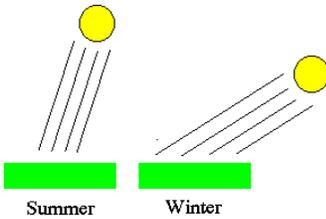
The optical depth is related to the physical path length L in kilometers, and the opacity coefficient, k , for the aerosol according to $\tau = kL$, but k is also related to $k=\mu\rho$ where μ is the absorption cross section per gram for water droplets, and ρ is the density in grams/cm³. This means that the average density of water droplets is given by $\rho = \tau/(\mu L)$. For water droplets, the absorption cross section in the visible band is about $\sigma = 8 \times 10^{-24}$ cm²/molecule and $m(\text{H}_2\text{O}) = 3 \times 10^{-23}$ gm/molecule so $\mu = 0.26$ cm²/gm, and the atmospheric path length through the troposphere where most of the water is located is $L=30$ km or 3×10^6 cm, so we get $\rho = 1.1/(0.26 \times 3 \times 10^6) = 1.4 \times 10^{-6}$ gm/cm³. In terms of grams/m³ which is used by meteorologists we get $\rho = 1.4$ grams/m³. Typical values can range from 0 for very dry air to 30 gm/m³ for saturated air (100% humidity) at 30° C (86° F), so our estimate is in the right ball-park! The main sources of uncertainty are in the values for L and σ .

Additional Resources:

Scattering cross section for water droplets. https://www.researchgate.net/figure/Self-continuum-absorption-cross-sections-of-water-vapour-measured-near-3007-and-5000-cm_tbl1_324556578

H. G. Houghton and W. R. Chalker, "The Scattering Cross Section of Water Drops in Air for Visible Light," J. Opt. Soc. Am. **39**, 955-957 (1949)

Experiment 7: Detecting the seasonal insolation change.



The Lux unit is a measure of light intensity in terms of energy per square meter. We know that for a fixed light source brightness, the light intensity on a surface increases as the inclination angle of the source above the plane of the surface increases. This explains how the high- inclination summer light rays from the sun lead to a brighter daylight (higher lux at noon) than the slanted rays of the winter months (lower lux at noon). We can predict these seasonal changes for any latitude on Earth. First a bit of astronomy!

Figure 47. Geometry of sun angle illumination

In the Clinometer section, I created a solar elevation model for the entire year, which predicted the angle of the sun above the local horizon at any latitude and day of the year. Select your latitude α and day of the year T .

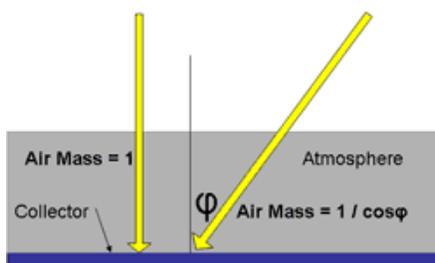
$$\delta = -23.5 \sin(2\pi(T-79)/365).$$

$$\sin(\text{elev}) = \sin(\alpha) \sin(\delta) + \cos(\alpha)\cos(\delta) \cos(\text{HA})$$

sunset and sunrise occur when the hour angle is $\cos(\text{HA}) = -\tan(\alpha)\tan(\delta)$

Given the solar elevation angle, we can now compute the surface intensity of the light on a horizontal surface in watts/meter² based on a solar power at the top of the atmosphere of 1360 w/m² and $I = 1360 \sin(\text{elevation})$, with average atmospheric absorption added, this is reduced to about 1050 w/m², so at ground level we have $I = 1050\sin(\text{elevation})$. Converting from this to light radiance units, Lux, is not easy because the visible light we see and measure is not the same as the total intensity of all electromagnetic radiation used in the 1050 w/m² value. What should be valid, however, is that the measured brightness in Lux should be proportional to $\sin(\text{elevation})$. One last correction is the atmospheric airmass correction.

Airmass and opacity.



Energy Absorption by the Atmosphere

Figure 48. Geometry of airmass and opacity.

The amount of light diminution depends on the transparency of the atmosphere that the light passes through. Astronomers typically include a photometric correction for ‘airmass’ in their measurement of stellar brightness. If each cubic meter of the atmosphere has a gas/dust/aerosol density that is uniform, the total extinction is given by $I/I_0 = e^{(-kx)}$ where x is the path length through the atmosphere in kilometers and k is the opacity coefficient per kilometer. For a plane-

parallel atmosphere, the path length is related to the elevation angle of the source by $x = L/\cos(\text{zenith angle}) = L/\sin(\text{elevation})$ where L is the absorption of the air directly overhead. The solar insolation at the top of the atmosphere is 1360, but on the ground we have only 1050 w/m^2 , so the extinction of sunlight through a clear atmosphere is $1050/1360 = 0.772$. The natural log of this is -0.259 . So we can change our attenuation formula so that measurements are in terms of the elevation angle: $I(\theta) = 1360e^{(-.259/\sin(\theta))}$.

To check, we see that at an elevation angle of 90° at the zenith, we get $i(\text{zenith})=1360e^{(-.259)} = 1050$. At 45° we have $I(45)=1360e^{(-0.366)} = 943 \text{ w/m}^2$. So if the flux of sunlight is 185,000 lux at high noon, it will be only about $185000e^{(-0.366)} = 128,000 \text{ Lux}$ when the sun is at an elevation of 45° in the mid-afternoon. At this point, the only other significant correction is for the degree of transparency of a cloud-free sky. This can vary from bright blue (high transparency) to faint milky white (lower transparency) and this correction has to be judged on the day of observation. We can even consider measuring this using the light meter by making observations at high noon under a variety of clear-sky conditions.

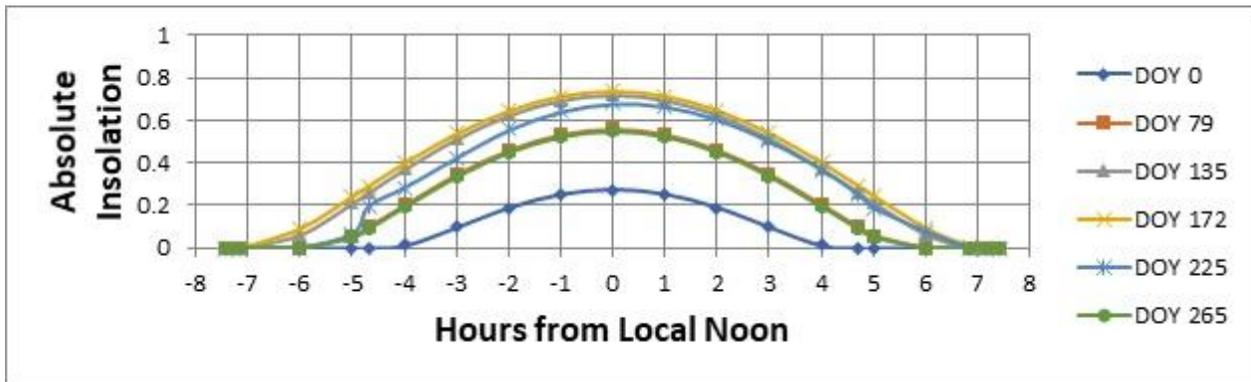


Figure 49. Graph of modeled change in solar insolation with time.

So, with the elevation angle and airmass corrections included, the Lux measurements we make on any DOY should follow (and with a single scale factor be proportional to) the appropriate curve for the DOY shown above. At Noon on the summer solstice, DOY 172 the sun is at an elevation of 75 degrees and the insolation is 97% of the maximum. If the sun were directly overhead at noon it would be 100%. A difference of 15 degrees in the \sin function at these angles only changes the insolation by 3%.

We can now create predicted plots of the solar brightness for a given day and compare it against what the light meter says. The hourly change should be accurately detected and a scale factor can be computed. For this we need a perfectly cloudless day with unobstructed views of the low-angle sky (no forests or buildings on the local horizon).

I placed a white sheet of heavy-stock paper on a horizontal surface and directed the front camera to the paper so that the paper fully covered the camera FOV. Used *Galactica*, and *Lux LightMeter*, I have indicated in red the digits that are rapidly changing during the real-time measurement. For the *Galactica* measurements I used the *Theodolite* app. For *Light Meter* I used the *SeeLevel* app. *Theodolite* seemed more reliable and gave a photo of the measurement FOV.

The *Galactica* showed you the FOV camera image so you knew what area was being metered but *LightMeter* did not give any indication of what it was metering, and the lux measurement seemed way off.

Table 17 Light measurements

EDT	HA	<i>Galactica</i>	<i>Lux LightMeter</i>
8:00	-5		
9:00	-4	70.7k / 21°	8,628 / 22°
10:00	-3	118k / 30°	249,231 / 32°
11:00	-2	202k / 41°	496,693 / 40°
12:00	-1	283k / 49°	1,003,724/50°
1:00	0	283k / 51°	1,002,707 / 51°
2:00	+1	283k / 50°	1,003,232/ 50°
3:00	+2	202k / 48°	681,143/ 42°
4:00	+3	88.4k / 35°	380,887 / 38°
5:00	+4	83.2k / 28°	70,592/28°
6:00	+5	13.0k / 16°	12,850/15°
7:00	+6	6.7k / 4°	5,822 / 5°

Note that after spring equinox we switch to Daylight Savings Time, so true Standard time (HA ‘time’) is 1 hour earlier.

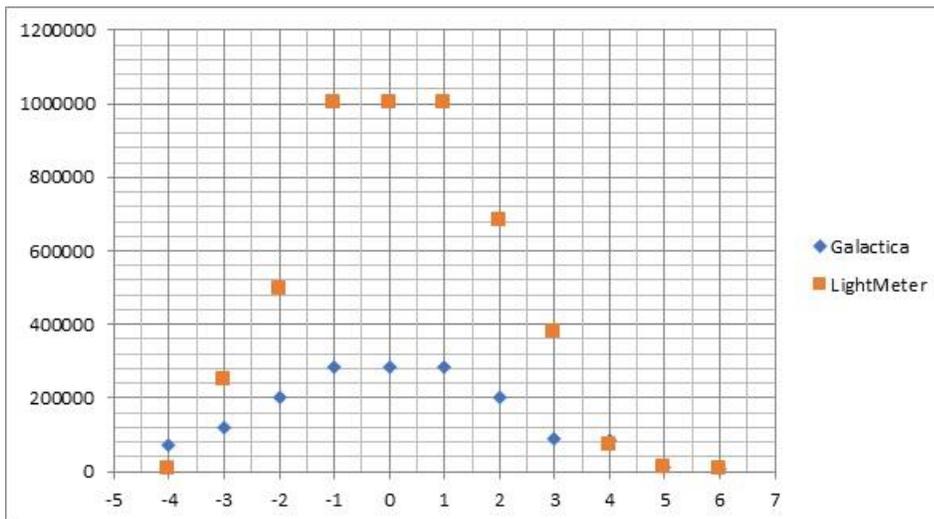


Figure 50. Comparison of metering changes during daytime.

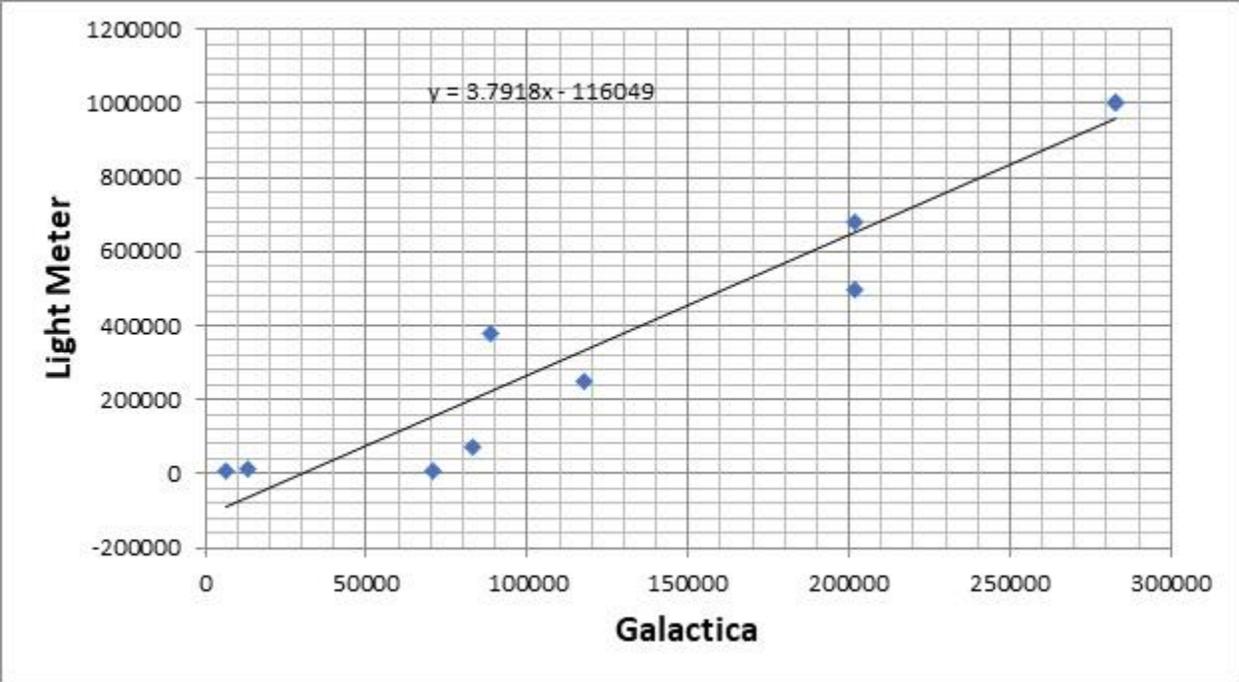


Figure 51. Comparison of two metering apps and best fit curve.

Color Temperature

The basic idea is that the emission of the sun follows a black body curve in wavelength. By measuring the brightness of the sun at two different wavelengths you can determine which black body curve matches the sun. Each curve is for a unique temperature, so these wavelength versus brightness measurements lead to a unique prediction for the solar temperature.

To predict how bright the sun will be for a specific sensor, we have to multiply the sun's emission curve by the spectral response of the sensor. The product of these two, when summed up over the spectral range of the sensor, gives the amount of light the sensor can detect. Here's the math.

$B(T, \lambda)$ = the sun's full blackbody curve at a temperature T . It gives the amount of light emitted by the sun's surface at each wavelength, λ , and is in units of watts/meter²/steradians/micron. Where the width of the spectrum sampled at a wavelength λ is given in units of microns, and the emission from this solar surface covers an angular area measured in steradians.

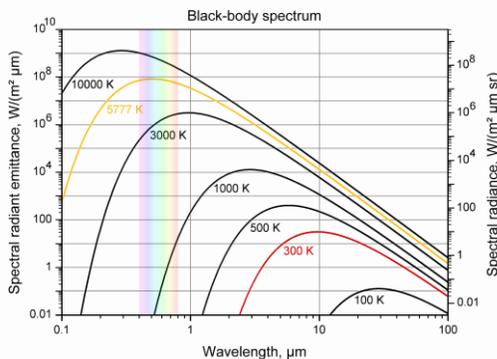


Figure 52. Example of black body curves (Credit Wikipedia)

This figure shows the blackbody curves for different temperatures. If you had one sensor that measured the brightness at 1 micron and another that measured it at 0.1 microns, they would show predictably different brightness at each temperature. Mathematically, $B(T, \lambda)$ is defined by Planck's Law of Radiation given by:

Table 1.7.1 Planck's law of radiation

spectral radiance of black body $B\lambda$ is given as follows.

$$B\lambda = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{\exp(hc/k\lambda T) - 1}$$

- $B\lambda$: black body spectral radiance ($W \cdot m^{-2} \cdot sr^{-1} \cdot \mu m^{-1}$)
- T : absolute temperature of Black body (K)
- λ : wavelength (μm)
- c : velocity of light 2.998×10^8 ($m \cdot s^{-1}$)
- h : plank's constant 6.626×10^{-34} (J·s)
- k : Boltzmann's constant 1.380×10^{-23} (J·K⁻¹)

If you had a perfect sensor, you could measure exactly the value of $B(T,\lambda)$ at every wavelength, however, light sensors such as the human eye or the pixels in your camera, do not give you $B(T,\lambda)$ but another measurement we can call $C(T,\lambda) = R(\lambda) B(T,\lambda)$. For example, if the sensor only detects half of the light photons it received at a wavelength of 650 nm, then $R(650) = 0.5$ and $C(650) = 0.5 \times B(T,650)$.

$R(\lambda)$ is a function that gives the response of the sensor to light at a wavelength of λ . It ranges from 0 for no response to 1.0 at full (100%) response. The amount of light detected by the sensor is given by the fundamental equation of photometry:

$$f(T) = \frac{\int_0^{\infty} B(T, \lambda) R(\lambda) d\lambda}{\int_0^{\infty} R(\lambda) d\lambda}$$

R is a dimensionless weighting function and B has the units of watts/m²/steradian. If the source of the energy is isotropic then the flux $F(\text{watts/m}^2) = 4\pi f$. If the flux is emitted in only a limited direction such as a laser beam within an emission angle of $\Delta\Omega$ steradians, then $F = \Delta\Omega f$. Since Lux is also a measure of light energy (lumens) per meter², $F(T)$ is proportional to the predicted Lux measured with the sensor.

Suppose you add a filter to limit the range of wavelengths entering the sensor. The formula then becomes

$$f(T) = \frac{\int_0^{\infty} B(T, \lambda) R(\lambda) w(\lambda) d\lambda}{\int_0^{\infty} R(\lambda) w(\lambda) d\lambda}$$

where $w(\lambda)$ is the spectral response of the filter. The following figure shows how these three functions, B , R and W work together to define how many photons will be detected by your sensor.

On the next page is the spectrum of the 10,000-kelvin star Vega (black line) and the response functions (W) of five astronomical filters. The R-band filter transmits almost 50% of the photons from Vega's spectrum in its wavelength range. Although the G0-band only detects 1/3 of the photons, the spectrum of Vega has twice as many photons in this wavelength range than is covered by the R filter, so the flux from Vega in the G filter will actually be higher than in the R filter.

By carefully measuring the flux ratios between the R and G bands, you can relate this ratio called the color temperature to the surface temperature of Vega so long as the spectrum is a black body spectrum. For other kinds of light sources, or where extinction is not constant in wavelength between the bands, this color temperature does not equal the surface temperature and another method has to be used. We can use a smartphone light meter measuring in lux units to photometrically measure the brightness of an object in three carefully-selected filters covering the R, G and B bands. The ratio of these band intensities can then be compared to predicted ratios for

specific Planck black body curves over a range of temperatures to determine the closest match. If the source is an incandescent object similar to a black body, the derived color temperature from the filters will closely match its actual thermal temperature.

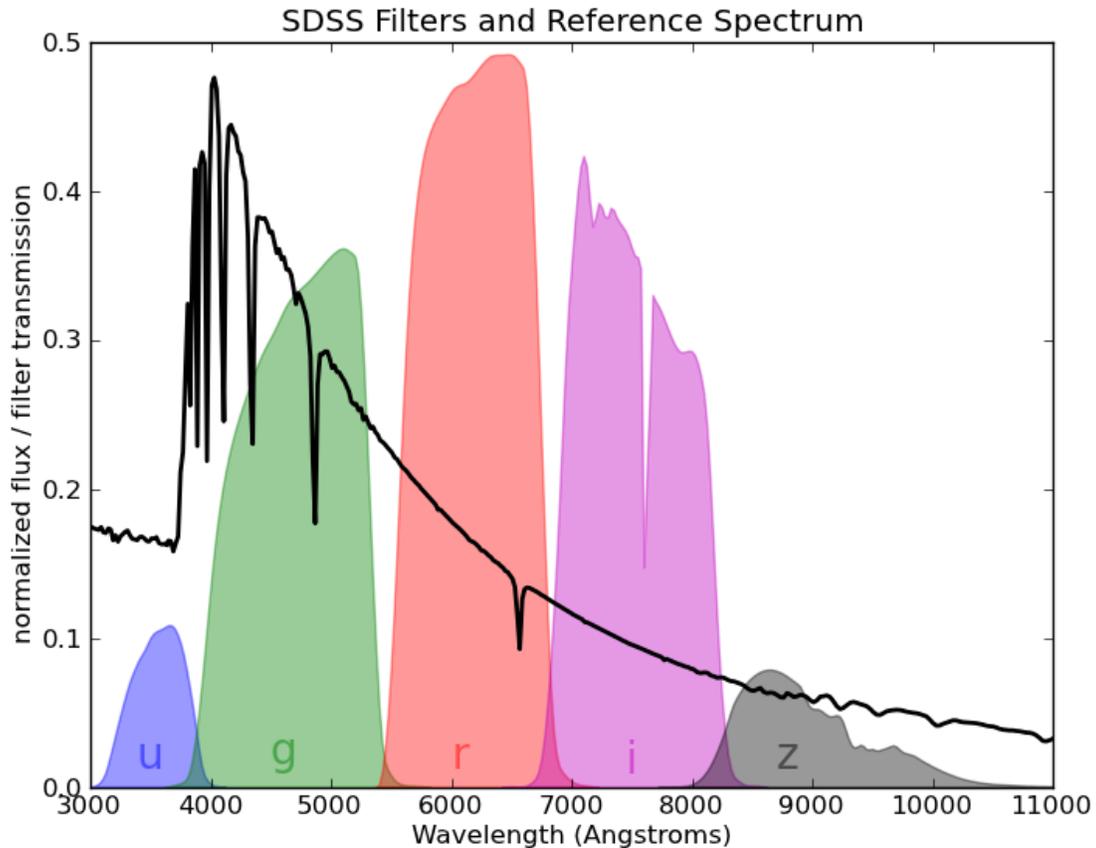


Figure 53. Black body curve (black line) viewed through five different filters.

To convert a smartphone into a color temperature ‘spectrophotometer’ we need to select a set of filters to place over the lens to narrow the response of the CCD chip to a small range of wavelengths. For this we select a set of four filters that are easily available and low-cost. The response functions for these filters are shown below.

Blue filter: Wratten #47 (about \$30.00) is considered superior, but still leaks significant light beyond 800 nm so the photometry is corrupted by far-red emission. Blue filter will need an IR blocking filter to remove light beyond 700nm.

IR blocking filter (about \$37.50) - Edmund scientific 55-235, 12.5 mm x 12.5mm cutoff at 650 nm.

Green Filter: Wratten #58 (about \$19.95)

Red Filter Wratten #25 (About \$40.00)

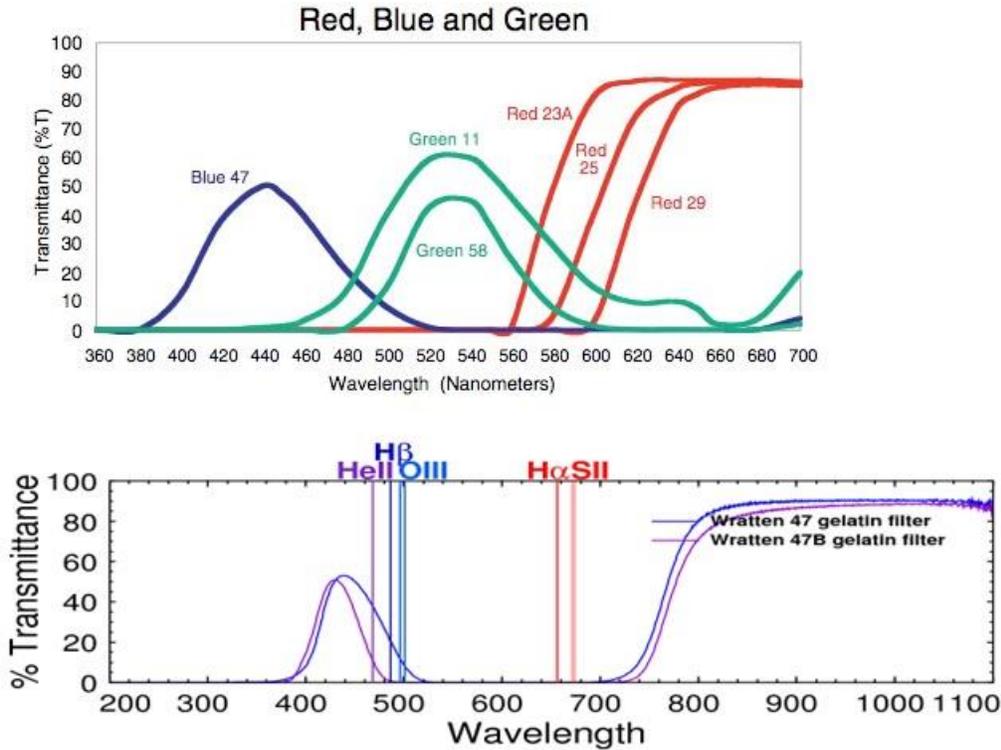


Figure 54. Filter response curves and their wavelength sensitivity.

In the plot for the Wratten 47/47B filter, notice the large response to radiation in the infrared at wavelengths longer than 700 nm. Without the IR Blocking filter, this energy would be added to the blue-filtered light intensity and compromise the accuracy of the color temperature measurement by showing more blue light, which incorrectly implies a hotter source temperature.

Apps and color meters:

A variety of apps designed for photographers provide information on the color temperature of the light entering the camera so that exposure settings can be adjusted to convert the warmer blue-tones revealed by tungsten lighting to a more pleasing color balance.

- **Photometer** (iOS, \$0.99 by lostview GmbH) – digital light meter, color meter features.
- **ColorAssist Lite** (iOS, Android, Free, by FTLapps) - provides RGB values for selectable areas in an image from 1x1 pixels up to 50x50 pixels in real-time.
- **Cine Meter II** (iOS, \$4.99m by Adam Wilt) – provides color temperature in kelvins, and a spectrum display.
- **Lightspectrum Pro** (iOS, Android, \$1.99 by AM PowerSoftware) – provides color temperature and a light source spectrum display.

Additional Resources:

Smartphones as UV monitors:

http://epublications.uef.fi/pub/urn_nbn_fi_uef-20141156/urn_nbn_fi_uef-20141156.pdf

Smartphone RAW files for digital photometry:

<https://www.chipworks.com/about-chipworks/overview/blog/inside-iphone-6-and-iphone-6-plus-part-3>

Experiment 1: App-based Spectrophometers: How good are they?

To compare how these apps performed, we obtained a professional color meter: Sekonic C-7000 Spectromaster (\$2,200). This instrument not only measures the color temperature of the light source between 380 and 780 nm, but displays a spectrum of the light at a resolution of 11 nm.

The tests were performed on an iPhone 6s. For RGB values we used the camera image-based *Photometer* and *ColorAssist* apps, with RGB values in 2^8 steps from black (0) to white (256) units. I tested common incandescent, LED and compact fluorescent (CFL) bulbs, representing three different spectra. Soft white (SW) and daylight (D) categories are also indicated when manufacturer provides them. The sunlight measurement was taken of the noon-time zenith sky brightness with cloudless conditions in early-November (temperature, humidity). The results are shown in Table 18.

Table 18 Color values for different light sources

Source	RGB values		Color Temperature (kelvins)		
	Photometer (±5)	ColorAssist (±5)	Sekonic (±2)	CineMeter II (±10)	LightSpectrum (±10)
40-watt LED D	234,238,224	220,225,205	5020	4760	4955
40-watt incandescent SW	248,168,92	235,158,86	2660	2730	4868
40-watt LED SW	247,184,92	230,164,81	2690	2850	4875
60-watt CFL (15w) SW	245,175,74	253,182,86	2545	2785	4911
Candle flame	251,141,73	250,179,97	1800	1675	3199
Sunlight 9:00am zenith	44,109,191	53,115,203	2450	100,000	5437
Sunlight Noon zenith	48, 103, 185	75, 138, 218	13700	5000	5276

Experiment 2: Creating a catalog of common spectra

Several apps and professional metering systems such as the *Sekonic C-7000* provide a graph of the spectrum of the incident light. A comparison of these spectra can determine how accurate smartphone systems are compared to professional, and more expensive, systems. For this comparison, we use the *LightSpectrum Pro* app on an iPhone 6s, and compare with the Sekonic spectrum, which we take to be the most accurate non-smartphone system. As a test we select a 40-watt soft-white LED and a 40-watt incandescent lamp. The resulting spectra are shown below. Sekonic on the left, *LightSpectrum* on the right.

Incandescent:



Soft-white LED

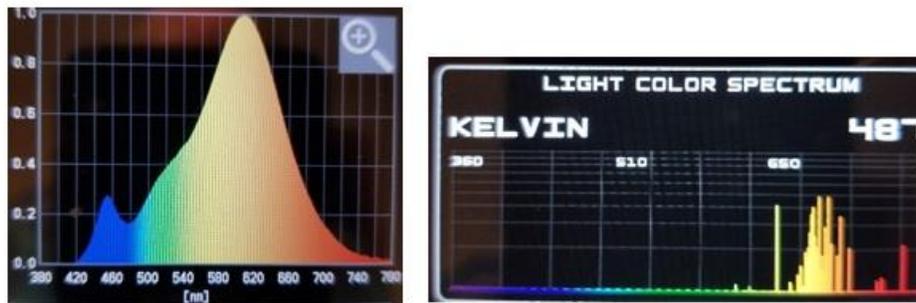


Figure 55. Examples of four different light source spectra.

The spectra are significantly different. This calls into question what operating principles are used to resolve the light spectra by each of the two technologies, along with the operating principle that results in a color temperature measurement.

For normal astronomical multi-channel spectrophotometers, each wavelength band in the spectrum is measured through a series of very narrow-band filters each designed to peak at the wavelength being measured. Alternatively, with diffraction gratings or with scanning etalons in fourier transform spectroscopy other methods are available. When these calibrated measurements are then plotted according to the center wavelength of the band-pass, you get a spectral curve spanning the visual range of wavelengths. These narrow-band scanning systems are generally very expensive.

Smartphones only have wide-band RGB filters and so they should only be able to make a crude 3-band measurement at low resolution (100 nm wide) of the source spectrum. The *LightSpectrum* app displays spectra of light sources at resolutions of $50\text{nm}/15 = 3\text{ nm}$! This is not plausible by using software alone. The developer of this app could not be reached to ask how their resolution is achieved. The Sekonic light meter also offers a spectrum to a resolution of about 10 nm. The Sekonic corporation was contacted through their website with a similar question.

Additional resources:

Light sources with known spectra. <http://www.asdi.com/getmedia/05be9959-ebac-4251-a1ce-1537b5e7b743/SPECTRAL-SIGNATURES-OF-NIGHTTIME-LIGHTS.pdf.aspx>

Popular Mechanics: <http://www.popularmechanics.com/technology/gadgets/reviews/g164/incandescent-vs-compact-fluorescent-vs-led-ultimate-light-bulb-test/?slide=15>

Aquarium owners want the bluest (>10,000 k) light for healthy fish. <https://reefs.com/2012/06/04/diy-reef-lighting/>

Comparison of the Electromagnetic Spectra of Common Light Sources: A General Chemistry Laboratory Exercise, <http://pubs.acs.org/doi/abs/10.1021/ed200658y>

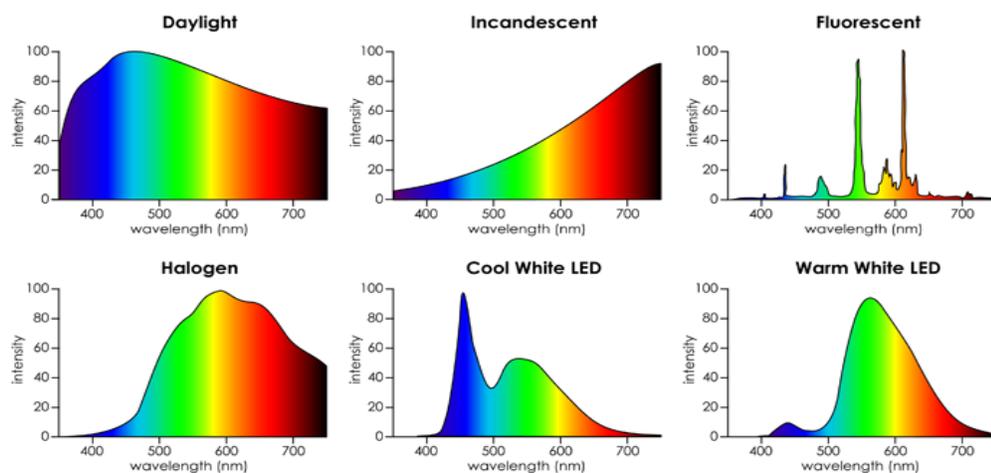


Figure 56. Examples of common light source spectra.

Time

Smartphones require an internal clock to run logic circuitry among many other apps and functions. Inside the phone there is usually a vibrating quartz crystal oscillator that generates a stream of ‘square-wave’ pulses, which are then counted by another system to form the clock signal. These clock circuits are kept alive by an independent CMOS battery even when the smartphone is turned off or its main battery is depleted. Commonly used oscillators are either crystal oscillators (XO), temperature controlled XOs (TCXO), or voltage controlled TCXOs (VCTCXO). XOs are usually specified with an accuracy of ± 10 ppm to ± 100 ppm, TCXOs are more accurate at ± 1 ppm and are often used to drive audio components. Typical drifts in parts-per-million (PPM) are shown in column 3 of Table 19 adapted from Guggenberger et al (2014).

Table 19. Representative drift rates for different smartphones

Device	Type	Drift (ppm)
Samsung Galaxy Note 10.1	Tablet	17
Samsung Galaxy SII	Phone	274
Samsung Galaxy Spica	Phone	15
Acer Iconia A200	Tablet	13
Apple iPad 2 Wi-Fi	Tablet	14
LG Nexus 4	Phone	7
Asus Nexus 7 2012 Wi-Fi	Tablet	3

The drift rate refers to how rapidly a timing error will accumulate to move the clock ahead (+) or behind (-) the actual time. For example, a day consists of 86,400 seconds, so if the drift rate is -30 ppm, the clock will lose $86400 \times (30/100000) = 2.5$ seconds each day. After a month, this same clock will fall 1.3 minutes behind an actual, calibrated time standard. Practically, this means if your colleagues used their smartphones to set a meeting alarm for next month at exactly 3:00 pm, the alarms on each phone might start chiming several minutes before the actual meeting start time.

The clocks are also affected by the smartphone temperature, and this can cause drifts to be not only significant and time-dependent, but almost impossible to allow for, under real-world conditions.

App Descriptions

The following apps allow for sub-second interval displays.

- **Official US Time** (iOS) by David Baumgarten - The app accesses the NIST, USNO, and time.gov. iPhone/iPod Touch/iPad has a direct link to the official U.S. time. A live clock will tell you what time it is in each of the six main U.S. time zones. No way to record information for later use.
- **Emerald Time** (Android, iOS) by Emerald Sequoia LLC - uses Network Time Protocol to display the current time of day as defined by the international standard atomic clocks. Its accuracy is limited only by network latency and is usually accurate to less than +/-100 milliseconds. Includes Leap Second display. No way to record data for later use.

-
- **Timestamp Camera Basic** (Android, iOS) by Yubin Chen – Allows you to add the current time and location when capturing images. This is the only app that can record video with the time watermark accurate to one millisecond (0.001 second). Photo and video are saved to the smartphone Camera Roll with various time stamp formats including decimal milliseconds (ms).

Additional Resources:

Calibrating a smartphone sensor:

Odenwald, S., 2019. Smartphone Sensors for Citizen Science Applications: Radioactivity and Magnetism. *Citizen Science: Theory and Practice*, 4(1), p.18. DOI: <http://doi.org/10.5334/cstp.158>

How to use smartphone timers

<https://www.howtogeek.com/244996/how-to-use-the-alarm-timer-and-stopwatch-on-android/>

Classroom use:

<http://www.govtech.com/education/news/cellphones-in-classrooms-part-2.html>

<https://www.goconqr.com/en/examtime/blog/40-uses-for-smartphones-in-school/>

Experiment 1: Measuring your smartphone drift rate.

We want to compare the drift rates between platforms and the apps that are running on them. To do this we have to disable the automatic synchronization that smartphones can do by comparing their internal clocks to the time standard present on the WiFi or cellular carrier signal.

- Step 1 - For the iPhone, open ‘Settings’ and ‘General’, then ‘Date & Time’, and turn off the ‘Set Automatically’. For Samsung, open ‘apps’ and ‘Settings’, ‘Date & Time’, and switch off ‘Automatic Date and Time’.

To record the start and stop times, we use the *TimeStamp* app on both platforms. The <https://time.is/> service compares your device’s current clock time with the local atomic clock time. Initially with the automatic setting on, this service was opened on each platform and for the iPhone reported that its time was exact and differed from atomic time by +12ms with a synchronization error of ± 20 ms. For the smartphone its current time was 0.8 seconds behind atomic time with a synchronization error of ± 23 ms. With both platforms running *time.is* their clocks indicated the same current time to within the 1-second display accuracy. The resource *time.is* could not be used for subsequent millisecond studies because it only reported time to the nearest second.

- Step 2 - The *TimeStamp* app was opened on each platform and under the ‘clock’ icon in the lower right corner of the screen, the ‘format’ feature was selected. The format options

were scrolled until one appeared that showed a 24-hour time and a decimal-seconds display e.g. 22:20:36.5.

- Step 3 - Next, the online website *currentmillis.com* was opened on a laptop, and with the Automatic Setting on each smartphone disabled, a pair of photos were taken of the laptop screen showing the current UNIX/atomic time to the nearest millisecond.
- Step 4 - The drift rate is calculated by plotting the difference between the UNIX time and smartphone time in seconds against the running time in seconds since the start of the test. A linear regression line was then fitted to the data. The slope of the line was then multiplied by 1 million to obtain the drift rate in ppm.

Table 20. Drift rates for a variety of platforms

Smartphone	Trial	Duration (hrs)	Drift (sec)	PPM
iPhone 6s	1	50.6	-2.0	11
	2	21.0	-2.1	28
	3	16.1	-1.4	24
	4	25.0	-1.8	20
Samsung Note5	1	50.7	-3.9	21
	2	22.1	-2.3	29
	3	16.1	-1.9	33
	4	26.3	-3.2	34
iPad	1	50.6	-2.0	10
	2	22.0	-2.4	30
	3	13.9	-1.0	20
	4	24.5	-2.6	29
Analog battery wall clock:				
Inexpensive clockworks		100	3.5	80
Expensive clockworks		100	0.13	3

The performance of the apps is complicated to evaluate. As Figure 57 shows, the drift rates are not monotonically increasing, but experience sudden glitches after about one-day of operation. Similar behavior is seen for both smartphone platforms. This type of behavior within networked

systems is well-understood and has been extensively discussed, for example, by Benndorf and Haenselman (2016) and Mani et al. (2016).

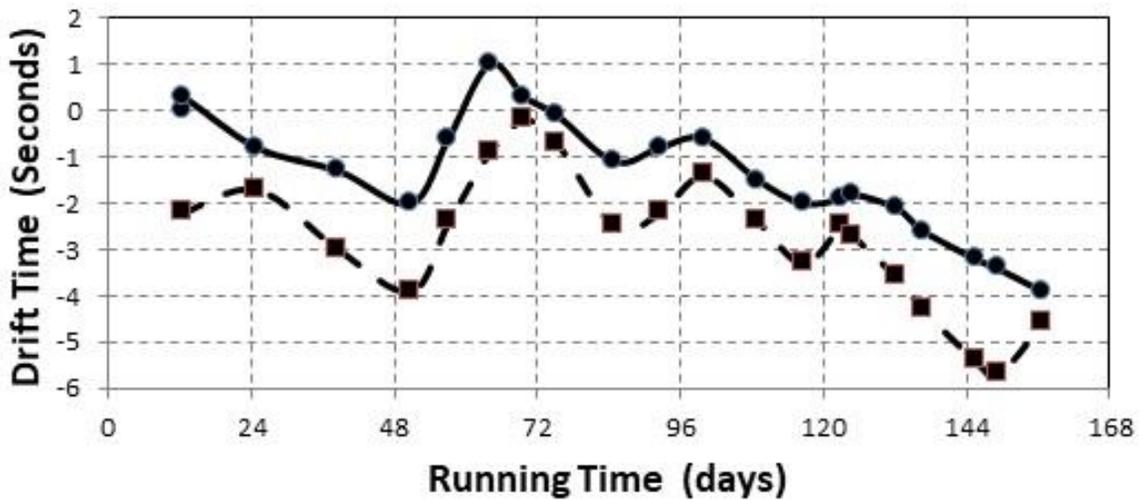


Figure 57 Difference between UNIX time, and iPhone (solid line) and Samsung (dashed line) times in seconds displayed by *TimeStamp*.

All systems that employ crystal oscillator-based clocks are subject to long-term drifts caused by processor utilization, temperature and humidity variations. There are also higher-frequency ‘jitter’ effects at the 10-50 millisecond timescale caused by delays in the network connections between the distant NTP server and the application. These, however, do not explain the much longer-term discontinuities seen in Figure 57.

A study by Buchholtz and Tjaden(2006) polled thousands of servers for their current time and found a variety of behaviors. Of particular interest were servers that displayed sudden jumps in time by 10 to 40 seconds at regular intervals. This behavior was credited to the server checking a reference time on a regular basis, but there was no explanation for why the clock offsets under constant drift were not all the same, i.e nulled to zero, after synchronization. This situation is analogous to what is seen in Figure 5 where, apparently, the smartphones may try to synch to a non-existent reference (NTP has been disabled), but does not explain either the amplitude of the drifts or the irregular intervals between constant-drift states recorded in Table 10.

The bottom line is that smartphones should not be run independently of periodic synchronization with an internet-based time service at least every hour to avoid drifts of up to 0.1 seconds. This may not be practical if the user is ‘out in the field’ with no cellular access, in which case the artifacts shown in Figure 57 will play a significant role in measuring events spanning long time intervals. Assuming from Table 20 a drift rate of 25 ppm, after ten hours your time interval will be off by one second for the typical platforms in this study. Moreover, the large transitions from 1 to 3 seconds between the uniform drift periods can occur at unpredictable times during a measurement series.

Experiment 2: Absolute Time Comparison

What time is it, really? The issue of ‘time transfer’ and synchronization is a very complex process involving a distributed network of clocks. At the speed of light, a signal (radio or optical) travels 300 kilometers in a millisecond, 3 kilometers in a microsecond, and in a nanosecond such signals travel a mere 30 centimeters. These kinds of lengths can easily build up if

While it is easy to synchronize clocks to the nearest minute to compute labor compensation, a variety of industries require time synchronization to milliseconds (stock market: <https://www.technologyreview.com/s/416805/trading-shares-in-milliseconds/>), microseconds (computer packet switching and video) and even nanoseconds (GPS, and optical switching times in telecommunications). The National Institute of Standards and Technology (NIST) Physical Measurement Laboratory provides the official ‘atomic standard time’ for the United States. While the actual atomic clocks are accurate to a nanosecond, the technical requirement for a global coordinated universal time (UTC) is that the time standard must be accurate to better than one microsecond.

The NIST servers listen for a NTP request, and respond by sending a data packet in the NTP format: JJJJ YR-MO-DA HH:MM:SS TT L H msADV UTC(NIST) OTM

The data packet includes a 64-bit timestamp containing the time in UTC seconds since January 1, 1900 with a resolution of 0.200 nanoseconds. TT, L and H indicate the current national time mode (standard or daylight-savings), whether a Leap Second was added, and the health of the NIST server. ‘msADV’ is the number of milliseconds that NIST advances the time code to partially compensate for network delays. The advance is currently set to 50ms. ‘UTC(NIST)’ is contained in every time code. It indicates that you are receiving Coordinated Universal Time (UTC) from the National Institute of Standards and Technology (NIST). ‘OTM’ (on-time marker) is an asterisk (*). The time values sent by the time code refer to the arrival time of the OTM. In other words, if the time code says it is 12:45:45, this means it is 12:45:45 when the OTM arrives. NIST transmits over one billion clock updates every day to laptops, computers and other systems. It also communicates with over a half-billion internet-based devices every day using its Network Time Protocol (NTP). Typical synchronization accuracy is between 1 and 30 milliseconds, but this varies with the speed of the internet path between client and server.

Most system clocks are set to an absolute time-base using a protocol called NITZ that relies on a connection with your carrier, which is in turn synched to NIST to ensure that the time stays in sync. By checking every ten minutes or a few hours, your smartphone time is synched with local atomic time with NTP, and the drifts can remain very small, and generally below 1ms. This checking is enabled from your ‘Settings’ tool under ‘Date&Time’, and can be switched on or off. If ‘off’ your smartphone time based entirely upon its internal clock will slowly drift away from the actual local time as we showed in the previous section. According to a study conducted at James Madison University, more than 25% of Web servers are off by 10 seconds or more. Apps such as

ClockSync for Android and iOS phones by Sergey Baranov will synchronize your phone to atomic time automatically every 10 seconds and claims millisecond accuracy.

As a result, it is easy to get your smartphone time synched to the second to within ± 100 ms when an NTP-based server is available via WiFi or cellular connection, however even under these conditions, it is virtually impossible to achieve reliable sub-second time coordination due to the various time-delays in the cell phone and internet systems that amount to 10ms or more. It typically takes this long for the time signals to propagate from the location of the atomic clock to the end-user. This is also near the resolution limit of most computer operating systems. Also, there are very few inexpensive instruments or apps that actually report the current time to 0.001 seconds in their displays, which makes it difficult to compare the absolute times for mobile devices at this resolution.

Experiment 3: Synodic Rotation Period of Earth by Gnomon Shadow.

The time it takes the sun to travel once around the sky from, say, Noon on November 10 to Noon on November 11 is called the solar day. It is actually a compound period composed of the rotation period of earth and the time it takes earth to travel 1 degree along its orbit.

You can measure the length of this solar day by using TimeStamp to photograph the shadow of a gnomon as it arrives at a mark on a piece of paper. Return the next day at the same time to photograph the shadow's return to the same spot marked the previous day.

For best results, use a piece of tape placed perpendicular to the shadow on the first day and mark the center of the gnomon shadow, taking a quick photograph of this shadow/mark with Timestamp. Leave the gnomon stick exactly where it is and do not move it for the next 24 hours.

The next day arrive a few minutes early and get ready to photograph the gnomon shadow at the exact moment the shadows center line arrives at the mark you made the previous day. The photographs below show what such a pair would look like and their time stamps.



Figure 58. Change in gnomon shadow between two consecutive days at exactly noon.

The time stamp indicates that on November 10 the time was 12:12:04.048 and on November 11 it was 12:12:15.285. The difference between these time stamps is 24h 00m 11.2 seconds. This is the duration of the solar day on November 10 (DOY=314). If you make this same pair of measurements at different times of the year you will notice that this solar day interval is not fixed. That is because Earth's orbit is not a circle and it travels at faster speeds near perihelion (ca December) than at aphelion (June). If we take the average of these intervals we get the Mean Solar Day, which is the length of day by the sun if the speed of Earth were its average value each day of the year. The difference between the Mean Solar Day (24 hours or 86,400 seconds) and the daily variations is called the Equation of Time shown in Figure 59.

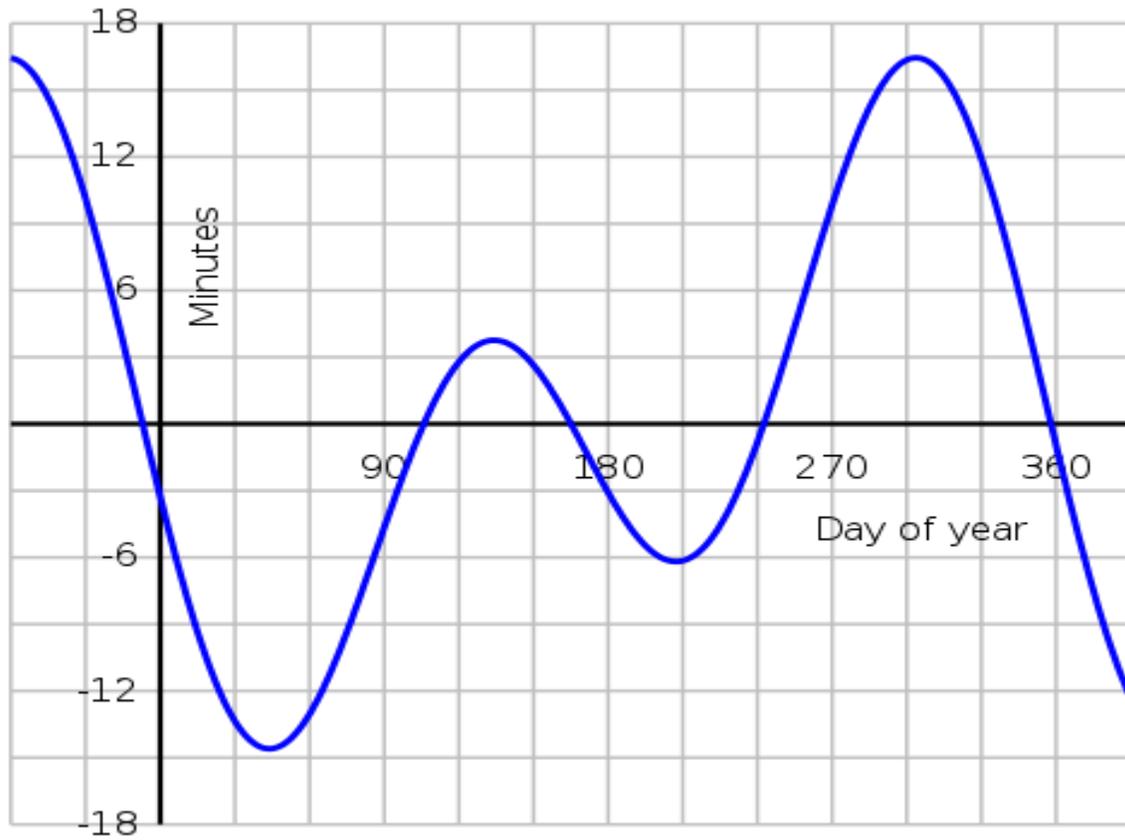


Figure 59. Sun time versus Noon Equation of Time for a full year.

Experiment 4: Sidereal Rotation Period of Earth by the Stars.

The app *Timestamp* (iOS) will let you take a picture and record on the image the current time to the nearest millisecond. A similar app is available on Android phones.

The sidereal period of Earth is the actual time it takes for earth to complete a full rotation of 360° about its axis. It is measured by determining how long it takes a star to return to the same location in the sky as viewed from the surface of Earth. Because both the sun and moon have motions across the sky of $1^\circ/\text{day}$ and $15^\circ/\text{day}$ respectively, we cannot use them for this purpose.

To measure this period, astronomers use a fixed zenith-pointing telescope with a reticle in the eyepiece to make a very accurate timing of this interval. They pick a single bright star and note the time when it aligns exactly with the north-south oriented line on the reticle. They then come back the next night and wait for the star to return to the eyepiece field of view and again line up with the reticle marking. The time difference will then be the sidereal day duration.

As a substitute for this, we have to use a less complicated and therefore less accurate approach.

-
- Step 1) On a clear night or morning, select a bright star and from your location note when it makes contact with some feature (power pole, etc). Take a photo of this moment with *TimeStamp*.
 - Step 2) At the same time the next night return to exactly the same spot and wait for the same star to make contact with the same feature. Take a photo of this moment with *TimeStamp*. See the two photos below taken by an iPhone 6s camera.
 - Step 3) The difference between the times noted by the *TimeStamp* images should be close to the length of Earth's sidereal day.

Experiment 5: Sidereal Rotation Period of Earth by Pendulum.

The app *Timestamp* (iOS) will let you take a picture and record on the image the current time to the nearest millisecond. A similar app is available on Android phones.

A pendulum oscillates within a plane in inertial space, and Earth rotates around this plane over the course of a sidereal day. This was discovered by Jean Foucault in 1851. By measuring how far this plane rotates we can determine Earth's sidereal day length. For this we need an operating pendulum at a science museum. A list of these can be found here

https://en.wikipedia.org/wiki/List_of_Foucault_pendulums#United_States

Alternately you can build one yourself. If you are interested in building your own, you might like to go to your library and read two articles from Scientific American magazine: pages 115-124 of the June 1958 issue and pages 136-139 of the February 1964 issue. You can also use a webcam view of a Foucault pendulum in Germany

<https://www.webcamtaxi.com/en/germany/lower-saxony/foucault-pendulum.html>

The relationship between the pendulum's period and sidereal day depends on your latitude according to $Pendulum\ Period = Sidereal\ Day / \sin(\text{latitude})$. By measuring the pendulum's angular speed and multiplying by $\sin(\text{latitude})$ you get the sidereal day period.

- Step 1) Take a picture of the pendulum with *TimeStamp*
- Step 2) Return to the view after a few hours and take a second picture.
- Step 3) Draw a line that represents the path the pendulum took through the center of the floor pattern for each picture.
- Step 4) Use a protractor to measure the angle between the two paths relative to the center point.

- Step 5) Note the times for each photograph and subtract the later time from the earlier time.
- Step 6) Convert the time difference into a decimal value for the number of seconds.
- Step 7) Divide the degree difference by the number of seconds in Step 6 to get the rate in degrees per second.
- Step 8) Divide 360° the rate in Step 7 to get the number of seconds to travel a full rotation.
- Step 9) Multiply your answer in Step 8 by $\sin(\text{latitude})$ to get Earth's sidereal day period.

Figure 60 is an example for the webcam pendulum in Osnabruck, Germany (52.28° North)

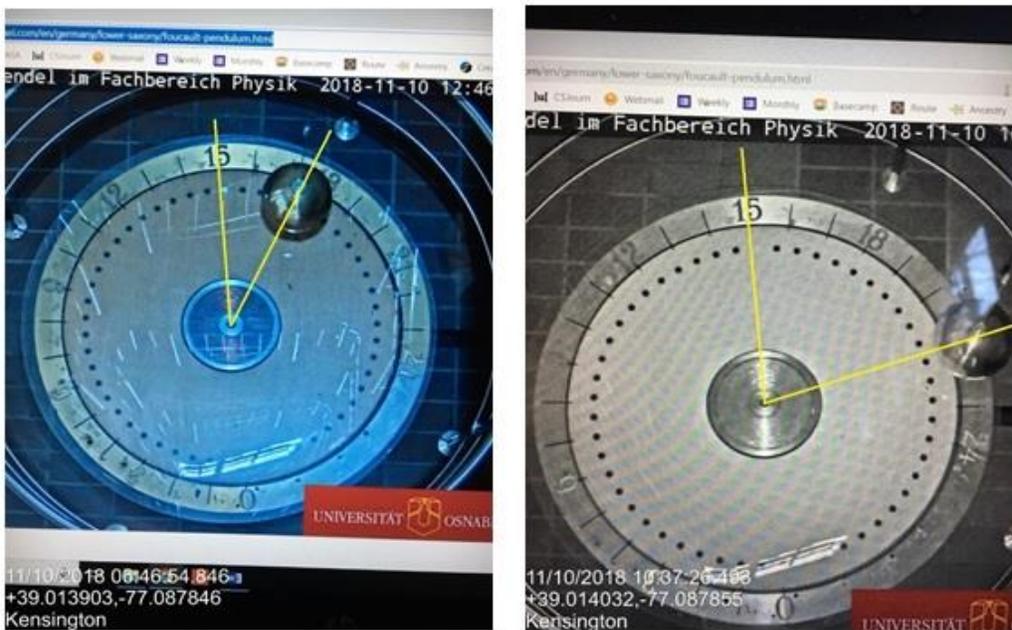


Figure 60. Photos of a pendulum showing Earth rotation. The left-hand photo taken at 6:46:54.846 (angle 30.5°) the right-hand photo taken at 10:37:26.493 (angle 76°)

The angle difference is $76 - 30.5 = 45.5^\circ$. The time difference is $38246.493 - 24414.846 = 13831.647$ seconds.

The rate is $45.5 / 13831.647 = 0.003289$ degrees/sec, so one complete orbit takes $360 / 0.003289 = 109437.207$ seconds.

The latitude correction is $\sin(52.28) = 0.787337$ so $T = 109437.207 \times 0.787337 = 86163.928$ seconds, which equals 23h 56m 3.93 seconds.

The actual sidereal period is 23h 56m 4.091 seconds, so this initial measurement is only 'off' by about 160 milliseconds!

If we make repeated pendulum measurements, we can reduce this difference considerably.

Astrophotography

Smartphone cameras are fun to use for many applications, but one that may not be too familiar involves using it to photograph the sun, moon and stars. You will not get professional-grade photos of these objects, but you will have the satisfaction of having connected with the sky in a very personal way, and who knows what ideas you may come up with!

Camera Sensors and Pixels

The basic steps in forming an image are shown in Figure 61. The light from a source passes through a lens, then passes through a filter that presents the CMOS array of pixels with a 2x2 patch containing 1 red, 2 green and 1 blue-sensitive filters called a Bayer mask. Each of these 2x2 pixel arrays represents one image 'pixel' to provide the image processor with the information to recover the color of the image pixel. The processor then combines the information from the red, green, and blue-sensitive pixels to create a color image.

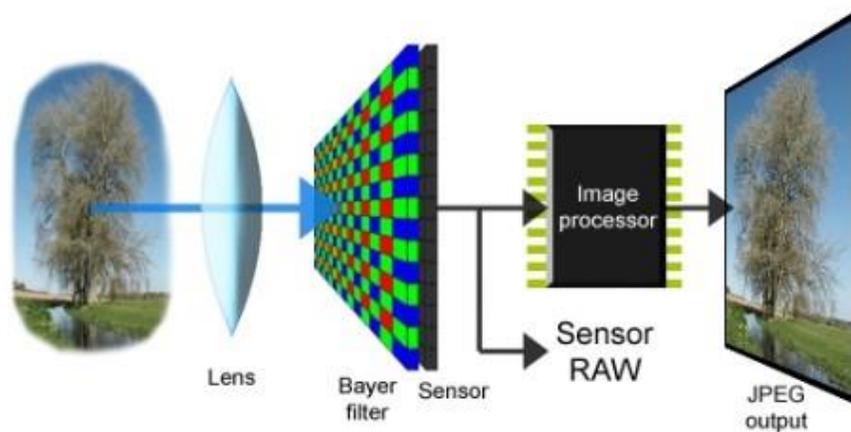
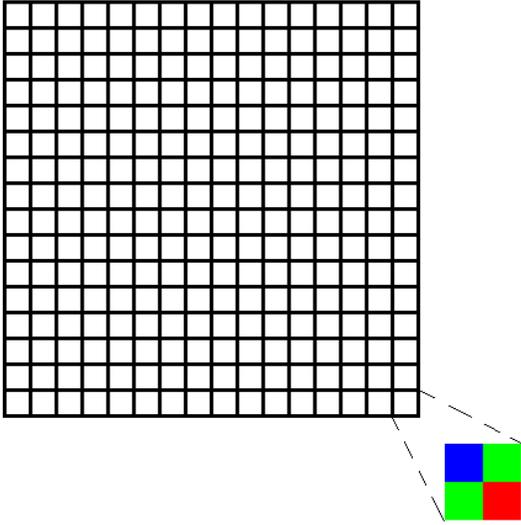


Figure 61. How a camera image sensor works.

The smartphone lens is typically about 2 millimeters in diameter, and the Bayer filter and array are both about 1/4 to 1/3-inch (3 to 5 millimeters) in size. Because the physical size of the CMOS sensor changes very little from generation to generation, the increased megapixel size of cameras has to do with the pixel sizes being made smaller and smaller. Current 13 megapixel cameras have pixels only slightly more than 1 micron across. Previous generations of 8 megapixel arrays used 1.4-micron pixels. Will pixels get smaller than this? Probably not. The wavelength of



red light is about 0.7 microns so current micron-sized pixels only allow 1 to 2 waves of light to cross their dimensions. For the shorter-wavelength blue light, we are already at 1 wavelength of light, and this is the physical limit of light sensor technology. From now on, it is the physical size of the array that will increase.

Figure 62. Location of red-green-blue pixels in a camera array.

By the way, the use of ‘megapixel’ numbers to describe arrays comes from the product of the pixel width and height of the array, but there is an ambiguity over whether you mean the actual array chip or the smaller number of image pixels after the array has been Bayer filter-binned to create the final image. For example, if the CMOS array has 4 million pixels, each with its own RGB filter, the final image will have only 1 million pixels. Fortunately, powerful software running in the camera’s CPU can actually use the Bayer color grid information to figure out or ‘de-mosaic’ the data and estimate the full color value for each pixel in the sensor array! The end result is that the camera’s resolution is better than one would expect if each 2x2 block containing all three colors represented a pixel, but not quite as good as each individual pixel. The real-world resolution limit of a Bayer array is typically around 1.5X as large as the individual pixels.

Smartphones come with two imaging chips called the front camera and back camera. The front camera faces you when you are using the smartphone display and is good for ‘selfies’. The back camera is on the opposite side of the smartphone than the main screen and is used for higher-quality images. These cameras are slightly different. For example, the iPhone 6s has a front ‘selfie’ camera with 1.2 megapixels (f/2.2) and a back camera with 12 megapixels (f/2.2 and 1.22 micron pixels). In other respects they are photometrically identical. The GOOGLE Pixel camera, meanwhile, has a 8 megapixel front camera (f/2.4 and 1.4 μm pixels) while the 12.3 megapixel back camera (f/2.0 and 1.55 μm) is a faster camera with larger pixels.

<https://www.anandtech.com/show/10753/the-google-pixel-xl-review/4>

Camera apps.

To actually use your front and back camera arrays you have to use an app that accesses the image output from the array in each of the three colors, and allows the exposure speed and ISO speed to be adjusted. The f/stop is fixed by the aperture unlike for normal digital/film cameras.

- **iOS Native Camera app** – This comes with the smartphone.
- **Camera+** (iOS, LateNite Apps, \$2.99) – includes automatic metering as well as a self-timer to delay the triggering of the exposure by 15 seconds. The manual mode lets you adjust the exposure speed (1/8000 to 30sec) and ISO (0.01 to 2000) independently using sliders on the screen. Images can be saved and emailed to be downloaded on a laptop or PC for further processing with Photoshop or etc. Outputs as jpeg, TIFF or RAW formats.
- **ProCamera** (iOS, Cocologics, \$5.99) - ProCamera gives you control comparable to an advanced DSLR camera with semi-automatic and full manual controls. Set specific values for exposure time, ISO sensitivity, and white balance (color temperature & tint), etc. Self-timers, live histogram. Optionally remove Geo-Tags and/or resize photos and videos when sharing via text message, email, or to social media
- **Camera FV-5** (Android, Free/\$3.95) is one of many camera apps for manual camera fans. It emulates various DSLR features. That includes manual shutter speed, ISO, light metering, focus, white balance, and more. It also supports JPEG, DNG (RAW), and PNG file formats. There is support for 30 languages as well.

Definitions:

Exposure speed: This is simply the amount of time that the lens remains open so that the image can be formed on the array. Speeds can vary from 1/10000 to many seconds depending on how bright the subject matter may be. For astrophotography of stars, it is important that the camera be able to take exposures of at least one second or more. For lunar and solar images, exposures will be typically much less than one second.

F/stop: Light enters the lens across its full radius, but if you restrict the radius by a factor of 2, the light intensity reaching the array will also be cut down by a factor of $2^2 = 4$. The F/stop is a measure of the radius change that you apply. F/1 means the full lens radius is

open to the light. F/16 means that only 1/16 of the lens's radius is providing light. This number is the ratio of the open radius of the lens to its focal length. For example, a 20mm radius lens with a focal length of 60 mm has an f/number of $60/20 = 3.0$ so f/3.0 is the 'fastest' speed the lens can support. If I reduced the radius by a factor of 2 to 10 mm, the lens now has f/6.0 which photographers would call a slower setting because less light $(3/6)^2 = 1/4$) gets through and so the exposure speed has to be doubled to get the same amount of illumination. Smartphone cameras have a fixed f/number of typically 2.2, but faster cameras with f/1.7 are becoming available. Opening a camera by one stop is equivalent to operating with 1/2 the exposure speed.

ISO: This is a measure of how fast the array+lens+hardware is, or how efficient it is in converting photons to electrons. A higher ISO number makes your camera more sensitive to light so that you can take pictures under poor lighting conditions. Think of it as one photon generating n electrons in each pixel, but if you double the ISO there are now 2N electrons produced per photon. For example, a setting of ISO 400 makes your camera twice as sensitive to light as ISO 200, and also means you can halve the exposure time. Electronically, it is a measure of the number of electrons generated by the incident photons of light. A practical limit is reached at very high ISO values where even very dim light completely floods each pixel with the maximum number of electrons so the pixel 'well' becomes saturated. Typical pixels in smartphone cameras can only accommodate about 1 million electrons before they are fully saturated and the pixel turns 'white'. The larger the pixel size, the larger is the well capacity which enables higher ISO operation and faint light registry with moderate exposure times.

Camera sensitivity: <https://www.ptgrey.com/white-paper/id/10912>

Photometry -<http://www.clarkvision.com/articles/digital.photons.and.qe/>

Telephoto Lens Attachment

Tripod adaptors for smartphones can be found on the Internet and cost between \$3 and \$10 brand new, but you can also find them more inexpensively at several auction sites on the Internet. A telephoto lens system is absolutely a must-have for eclipse photography with a smartphone. There are a number of zoom lenses for smartphone photographers designed solely to provide magnification without resorting to digital zoom. Most of these like the one on the left below, clip directly to the smartphone over the existing lens, and provide total magnifications of 8x for less than \$20.00, and 12x at a cost usually under \$40.00, but you will need to purchase a tripod for your smartphone to avoid shaking. Some 12x systems like the one shown to the right below include a tripod and a mounting bracket for the smartphone that is far sturdier than a clip-on system. It is worth pointing out that the magnification touted by smartphone telephoto lenses is not the optical magnification, which is customarily used by astronomers and opticians to describe an optical system. It is common in the smartphone telephoto industry to advertise the maximum product of the smartphone's digital magnification times the optical magnification of the telephoto.

It is only, however, the optical magnification that affects the resolving ‘power’ of the system as we will discuss in a later section following actual measurements of various systems.

At an advertised magnification of 12x, with a digital magnification of 3x and an optical magnification of 4x, the moon disk subtends 4 times as many pixels as a normal 1x camera view, and the optically-enhanced resolution equivalent to a low-power binocular view (e.g. 4x50) results in rather clear viewing of the moon’s dark mare features.



Figure 63. Examples of smartphone telephoto lenses and tripods.

To rival a binocular view (7x50) you need a telephoto with an optical magnification of 7x or an advertised magnification of $3 \times 7 = 21x$ or higher. Nevertheless, the properly designed and mounted ‘12x’ system provides images of the moon that are quite adequate for eclipse photography and promises to resolve some of the larger details of coronal structure provided the exposure is in the proper range to avoid coronal over-exposure and burn-out.

Binocular photography

If you have a pair of binoculars, you can try one of two strategies: you can have your friend hold the binoculars pointed at the eclipse and then position your smartphone camera lens up against the eyepiece, or you can get a binocular-smartphone adapter and a tripod. By the way, if you go for the later approach you will have a fantastic system that you can use for many other photographic opportunities like capturing photos of birds and wildlife, close-up views of sports events or other public activities.

It will be difficult to use the hand-held binocular and camera because there are so many factors that you literally have to juggle to make it work. There will be a lot of movement of the camera and binoculars that will make getting a properly-focused shot hard to take in the short time you have during totality. You will spend so much time trying to set up the shots and line things up optically that you will probably miss seeing the eclipse for yourself! The only sensible way to do this is to get a binocular attachment for your tripod, and a smartphone lens adaptor for the binoculars. With this set up, all you have to do is point the tripod/binocular at the eclipse in progress, and set the focus on your binoculars and camera once and for all. Then you just need to touch the camera exposure button when you want to take a shot, or better yet, set the camera for

video mode! Binocular tripod clamps are easy to find on the Internet and cost between \$10 to \$20 for the simple screw-type adaptor, or \$25 for the strap-type adaptor that works for nearly all types of binoculars. Remember, this is an investment that by itself will pay dividends later on when you want to observe non-eclipse subjects, so it is definitely not a one-shot investment.

Camera lens adaptors for binoculars can also be found on the Internet and cost between \$10 for one designed specifically for your camera type, or between \$50 - \$70 for a universal adaptor that fits most phone types, and that you can use for telescopes too. All you need to do is use the automatic exposure and focus camera setting and hand-point the binoculars at whatever you want to study close-up.

Experiment 1: Finding the Resolution of Your Camera

The quality of the final image is all about what kinds of detail you can see. For some types of photos, you just want to see the moon in the same field of view as the group of friends you are actually photographing. This can be done by just using your smartphone as-is and composing your photo to include your friends with the distant moon in the background sky. The amount of detail depends on the resolution of your camera system and whether the lunar disk covers ten pixels in the final image or 100+. Here's a simple and rather direct way to figure this out from the comfort of your own room. The Author used an iPhone 6s.

Set up a meter stick or a yard stick horizontally on the edge of a table or some other stable platform. From a distance of five feet, take a 1x picture of it, and email it to yourself at its full uncropped size. For example, for an iPhone 6s, this is a 2.8 megapixel file with 3024x4032 pixels. The first thing we will figure out is the field of view of the image. In this instance, a carpenter's 18-inch stick with a width of 48 millimeters was used, and the distance (5 feet) corresponds to 305 millimeters. The width of the image at this distance is 117 millimeters.

From the properties of a right-triangle, the half-width of the image corresponds to an angle from the distance of 5 feet given by the formula: $\tan \theta = (\text{width}/2)/d = (117/2)/305 = 0.192$ so $\theta = 10.86^\circ$ and the full-width is just 21.7° .

The format of the picture is 3024x4032 or 1:1.33 so the angular field-of-view is from scaling just $21.7^\circ \times 28.9^\circ$.



Figure 64. Photo of meter stick at 5-feet.

How do we figure out the optical resolution? We photograph the measuring stick with millimeter marks from various distances until we can no longer clearly discern them as distinct no matter how we digitally zoom the image. For the iPhone 6s, this happens at a distance of 7.5 feet

or 2286 millimeters. So again from the above formula we now have $w=1$ millimeter and $d = 2286$ millimeters so the half-angle is 0.0125° and the resolution is 0.025° . Now, astronomers tend to use arcseconds as a unit of angular measure because most astronomical objects are pretty small in the sky, so this becomes an angle of $0.025 \times 3600 = 90$ arcseconds, or alternatively 1.5 arcminutes. Now the resolution of the human eye is about 2 arcminute in the middle of the visible spectrum, so this camera will show a scene at slightly worse than ‘retinal’ resolution. For example, at a distance of 7.5 feet, a normal human eye would have seen the markings in the meter stick above with slightly more clarity. This difference, outdoors, is not discernable by most people given the varying responses of the human pupil to changing light conditions. It is the available, full diameter of the human eye pupil (larger at night and smaller in the day) that determines your visual acuity.

For astronomical purposes, the most important factor in any imaging system, be it a telescope or a camera, is its ability to resolve details in the objects being studied. Because objects in the sky are measured in terms of angular units, we have to figure out how well our imaging system can resolve details at a specific angular scale. The basic formula that determines the angular subtense of an object is $\theta = 206265 \text{ diameter/distance}$, where the diameter and distance are in the same units (meters, kilometers, light years) and the calculated angle, θ , will be in arcseconds. Recall that there are 360-degrees in a full circle. Each degree can be divided into 60 arcminutes, and each arcminute can be divided into 60 arcseconds and 1 radian is an angular measure equal to 206,265 arcseconds. ($360 \text{ degrees}/2\pi = 1 \text{ radian} = 57.2958 \text{ degrees} \times 3600 = 206,265 \text{ arcseconds}$). The diameter of the moon is about 1897 arcseconds or just over 0.5 degrees.

What this means is that for our example of the iPhone 6s at its native 1x resolution, its 90 arcsecond resolution will cover the diameter of the sun and moon by $1897/90 = 21$ resolution elements. That is the best you will be able to do no matter what ‘photoshop’ trickery you try to do! The way in which a smartphone’s pixels are matched to its maximum optical resolution is actually complicated by the fact that the pixels in the very small CMOS chip are greatly magnified so that they can be displayed on the screen of the smartphone. Manufacturers talk about ‘retinal resolution’ for their displays but this has little to do with the actual optical resolution of the CMOS sensor! The iPhone 6s boasts a retinal resolution, but as we have seen, the camera resolution itself is about 50% worse than the canonical definition of typical human retinal acuity.

The only way to optically improve the clarity of an image is with the smartphone attached to a telephoto, a pair of binoculars or a telescope. This makes the aperture of the camera lens much larger, and so it decreases the angular scale of the field of view, which is covered by the CMOS array pixels.

I purchased what was advertised as a 12x telephoto lens for an iPhone 6s to see how much improvement I would get. This, by the way, is not really suitable for hand-held operation. The lens clips on to the iPhone over the existing camera lens, but this attachment is very fickle and subject to slippage. You have to remove the iPhone case completely, and through considerable trial and error find the optical sweet-spot where the optical axis of the telephoto lens is aligned with the camera’s optical axis. You also need a tripod to keep the camera from jiggling as you focus and take the exposure. That said, here is an example of the same calculations above. I took the

following picture of the carpenter's yardstick as above using a tripod-mounted and focused system from a distance of 13 feet.



Figure 65. Photo of meter stick at 13 feet.

On the original image, it is just possible to make out the millimeter separations in this very difficult to focus image, even with a tripod. It could only be done by putting the camera on a 5-second timer because the mere act of pressing the button caused vibrations and blur! So $d = 3962$ millimeters, $w=1$ millimeter and so $\text{Tan } \theta = (\text{width}/2)/d = (1/2)/3962$ so $\theta = 0.0072^\circ$ and the full-width is just 0.0144° for a resolution of 52 arcseconds. This is actually no different than using the 1x camera lens!

Next, I purchased a more-expensive *Eco-Fused Universal Smartphone camera lens kit* from Amazon.com, which also offered a 12x telephoto this time supported on the camera by a sturdier mounting clamp. Sensibly, the kit comes with its own table-top tripod, which is a hint that this is not recommended as a hand-held instrument. I determined its optical resolution by going outdoors on a sunny day and placing a meter stick at a distance of 44 feet, at which point I could just make out the millimeter gradations on the original image.



Figure 66. Photo of meter stick at 44 feet.

So $d = 13411$ millimeters, $w=1$ millimeter and from $\text{Tan } \theta = (\text{width}/2)/d = (1/2)/13411$ so we have $\theta = 0.0021^\circ$ and so the full-width is just 0.0042° for a resolution of 15 arcseconds. It is common for imaging systems to use 2 pixels for each resolution element (called Nyquist sampling) so this corresponds to a digital resolution of about 7.5 arcseconds/pixel.



Figure 67. Author photo of moon through smartphone telephoto, and photo of total solar eclipse by Fred Espenak

On the left is a photo in Figure 67 of the moon taken with this ‘12x’, tripod-mounted system with my *iPhone 6s*. I again used the 5-second delay timer to eliminate jitter and took a series of 5 photos selecting best of the batch to get the best atmospheric conditions. If I compare this 6x image with the image taken by astronomer Fred Espenak on the right (effective resolution of 7.5 arcseconds/pixel) it is clear that this relatively inexpensive telephoto system for the smartphone will produce exceptional ‘close-up’ eclipse photos even for the discriminating amateur astronomer!

Experiment 2: Star Photography

The easiest kind of astrophotography you can try is simply photographing stars and constellations. On November 1, at 6 am I went outside during last-quarter lunar phase and slightly overcast thin clouds towards the southern horizon looking towards Washington DC. The stars in Orion were clearly visible to the naked eye, but no stars elsewhere fainter than about +3m because of moon and haze. I took two photos at 1sec ISO-2000 using the hand-held *iPhone 6s* camera.

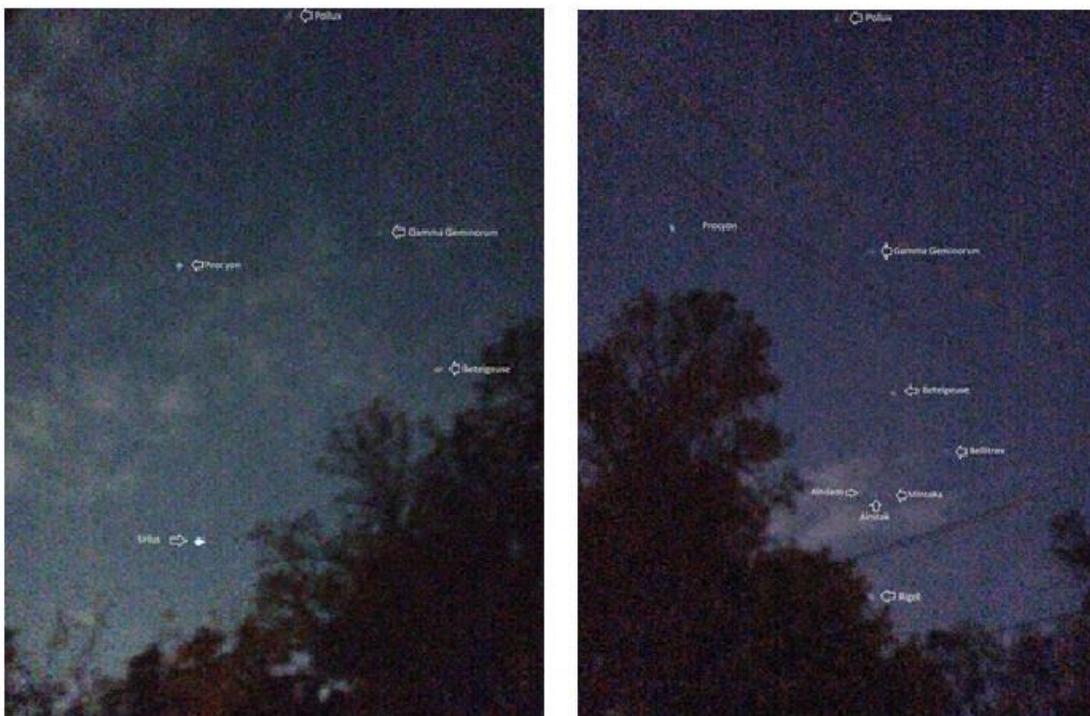


Figure 68. Star photos taken with a smartphone.

Sirius ($m=-1.4$) was dazzling. Betelgeuse ($+0.42$) was noticeably red, and the faintest star was Procyon ($+2.7$), but other stars were visible to the eye but not many more. Note the cloudiness illuminated by city lights in the above images. The stars Gamma Geminorum ($+1.4$), Bellatrix ($+1.6$) and Pollux ($+1.1$) were also visible. So under these crude, hand-held and cloudy conditions I could photograph stars down to about $m=+2.7$.

I was actually surprised that the smartphone could see stars, and so a few days later I did another photography session on November 4 at 3:00 am, and under cloud-free skies typical of ‘clear’ suburban conditions. This time, I used my Nikon D3000 DSLR camera on a tripod, and used my smartphone camera bracket to avoid shaking the smartphone during the exposure.

With the Nikon on the tripod, and a 15-second delay timer to eliminate camera shaking, I took a 30s exposure at ASA 800 with the camera’s 18mm lens set at its minimum f/4 stop.



Figure 69. Star photo taken with a Nikon camera.

The picture in Figure 69 was photoshopped slightly to darken the sky and increase the star contrast. The faintest stars had a magnitude of λ Ori with a magnitude of +3.5, but the stars in the shield π 3, π 4 and π 5 are +3.1, +3.7, +3.7. Also π 2 is faintly visible at +4.3. **The limiting magnitude seems to be +4.3.**



Figure 70. Star photo taken with a Nikon camera.

The Pleiades field was shot at 30 seconds near the zenith with far less haze than for Orion, F/4 ASA 800 with the Nikon. Based on the Hyades cluster map, σ and ρ Tauri are visible at +4.7 and +4.7. The faint Pleiades star Pleione at +5.0 was barely visible in an enlarged photograph, so the limiting magnitude of this field is about +5.0.

As a comparison, here is a 30-second exposure of the zenith sky taken under near-perfect conditions far from city lights in rural Vermont on September 15 with the Nikon set at ISO 800. The limiting magnitude for the stars in this series was +8.0.



Figure 71. Star photo taken with a Nikon camera.

The bottom line is that 30-second photos with the Nikon can indeed produce urban astrophotos down to a magnitude limit near +5.0, just slightly above the typical human limit of +6.0m.

The smartphone was then used to take a series of at ISO 800 to match the Nikon photographs, and took a series of exposures at $\frac{1}{2}$, 1, 2, 3, 4, 6 seconds. Only the $\frac{1}{2}$, 1 and 2-second images were not saturated. The 2-second image is shown in Figure 72. The comparable image with the Nikon would be 4-seconds at ASA 800.



Figure 72. Star photo taken with a smartphone.

To check on whether increasing the ISO of the smartphone made any difference, I took 1 picture with the smartphone at 2 seconds and ASA 800 (Figure 73) and 2 seconds at ASA 2000 (Figure 74) to compare.



Figure 73. Star photo taken with a smartphone. ISO800



Figure 74. Star photo taken with a smartphone. ISO2000.

Aside from the increase in the sky/array background in the ISO2000 image, the faintest stars seem to be +3.0 because the shield stars are not visible, nor is λ Ori (+3.3).

It is a common practice for astrophotographers to ‘stack’ a series of photos to increase the image sensitivity. By adding together the pixel values and averaging them pixel-by-pixel, you can achieve a \sqrt{N} decrease in the background noise if it is a Gaussian process. I took a series of 16 images of Orion with the iPhone set at 2 seconds and ISO 800. I also used the free DeepSkyStacker (<http://deepskystacker.free.fr/english/index.html>) program to load, register and ‘co-add’ the 16 images. The result is the following image.



Figure 75. Several star photos taken with a smartphone averaged together.

The sky background is cleaner as expected because of the $\sqrt{16}$ reduction in the pixel noise, but the process achieved no significant improvement in the number of stars. Limiting magnitude is still about +3.0.

For the Nikon image I used 4% star detection threshold. The pictures were 10 images at 30 seconds at ISO 800. The file was in TIFF format and it was 50 mbytes large. To convert it to jpeg for printing you open 'image' then click on 'mode' and select 8bits/channel Then click file, 'save as' and select jpeg.



Figure 76. Photo of Orion using a Nikon camera and multiple photographs.

I tried to register and stack the corresponding iPhone images with the *DeepSkyStacker* star threshold set at 6%. The result was this image:



Figure 77. Smartphone star photos with stacked images.

This stacked version in Figure 77 based on 9 images should have provided $\sqrt{9} = 3$ times more sensitivity to faint stars, equal to a full magnitude fainter than a single 2-second image, but is actually no better than a single image though the background sky is a bit cleaner, so we have reached a minimum magnitude near +4.0 for detecting stars.

The reason for this disappointing result is that the camera array is not designed to have a large dynamic range so that it can accommodate bright scenery during the daytime and the faint light from individual stars. The digitization level for the array image data is only about 12-bits or 4096 brightness levels compared to the Nikon which has a 14-bit, 16,000-level range. Also, the above images were jpegs and this further limits the dynamic range after compression to only 8-bits (256-levels). If jpeg-formatted sky images give you the photographic experience you want, then the images above are a good indicator of what to expect when using your smartphone.

An important work-around is the RAW image format, which captures the data from the array before compression. It is capable of providing the full 12-bit range of the smartphone and the 14-bit range of the Nikon.

Working with uncompressed RAW files.

- Step 1 – Open the Camera+ app
- Step 2 – Press the ‘+’ symbol near the exposure button.
- Step 3- Make sure the RAW option is highlighted in blue. If not tap the RAW symbol to highlight it.
- Step 4 – Tap the view screen to close the menu and take your photo
- Step 5 – Tap on the lower-left flower petal icon to open Lightbox
- Step 6 - Select the raw image. It will have a small ‘J’ symbol at the top left on the thumbnail.
- Step 7 – Press the ‘save’ icon for a second until the ‘Save Photo’ pop up menu appears.
- Step 8 – Select the ‘Export RAW DNG’ option. The image will now appear in your Camera Roll library.
- Step 9 - Exit the Camera+ app and open your ‘Photos’ archive app.
- Step 10 – Tap the image
- Step 11 - Tap the lower-left icon [Square with up arrow] to open your mode of sharing.
- Step 12 – Select Mail and send this image file to your mailbox
- Step 13 – Open your mailbox and save the image file to your desktop.

Experiment 3: Sun Photography

Safety Issues

There is quite a lot of discussion online about whether you can damage your smartphone camera by pointing it directly at the sun. The basic argument in favor of it being safe for the camera is that the lenses are generally very small (2 millimeters or so) and do not admit enough light. Also, cameras come equipped with UV filters that cut down on some of the visible light landing on the sensor chip. Finally, they automatically set their exposures for very short times.

Nearly every photographer that comments on this issue says it is OK if you do it very briefly such as when you are taking a scenery photo and the sun is in the picture. The argument for it not being safe is that some of the more recent smartphones use larger and faster lenses (f/1.7 to f/2.0) to get better resolution, and that can be a problem.

Most digital cameras have an Auto mode in which they will automatically reduce the exposure speed and increase the f/stop to take the photo, and this will not harm the camera. However, you will need to point the camera at the sun, and you will no doubt accidentally glimpse the full-on solar disk and THAT could damage your eyes if you prolong it.

Your most difficult challenge will be in managing your expectations! Smartphones were never designed to do sun and moon photography. The standard lenses are very small, and provide hardly any resolution at all for even the largest objects in the sky like the sun and moon. These objects are only $\frac{1}{2}$ -degree in diameter, and for a typical megapixel smartphone, their disks will only cover a few dozen pixels in your final image.



Figure 78. Example of smartphone photo of the sun.

Many of us have taken pictures of scenery on the ground at a small angle to the sun, and all that seems to happen is that the disk of the sun blooms into a distorted glare that can be attractive for some photo compositions. Here's an example from a free 'wallpaper' website. It certainly looks very pretty in the way that the image was composed, but you do not see the disk of the sun at all.

If you want to image the sun, you really want to at least see the disk of the sun! The only option is to place a filter in front of the smartphone lens to drastically cut back the light and glare. You will end up with a pitch-black sky and a small disk for the sun as big as the full moon. This will give you a sense of scale as you set up your eclipse shots. Many people have posted their attempts at doing this.



This is a typical, digitally-enlarged view of a direct photo through a smartphone. The red halo you see is not the solar corona but simply an artifact of the camera lens and a consequence of the particular ISO and exposure used by the automatic setup.

Figure 79. Example of smartphone photo of crescent moon.

You might be tempted to use the digital zoom feature of your smartphone but this is rather useless. All you will be doing is cropping your original image to a format where it magnifies the image. It will not alter the actual resolution of your shot, which is the factor that determines how much detail you will actually see. For that, you have to increase the optical resolution of your camera, which the camera industry calls the optical zoom. For digital cameras like the compact point-and-shoot models or the high-end Nikon D3000 DSLRs, they can provide optical zooms from 2x to 6x by using attached lens systems or by switching to other lenses. For example, a telephoto lens with a focal length of 300 mm will provide 6x optical zoom for a standard 50 mm camera. Some smartphones such as the iPhone 7Plus deliver 2x optical zoom. The Galaxy Zoom smartphones have a variable focal length. For example, the Galaxy S4 Zoom has a 24-240mm f/3.1-6.4 lens. The optical zoom is calculated by dividing the longest focal length value to the shortest one. So in the case of the S4 Zoom, the optical zoom is 10x. Fortunately, for all other fixed-focus smartphones, there are two rather inexpensive ways to add substantial optical zoom capability.

Be very careful using optical zooming on the sun because the concentration of the sunlight by the lens CAN damage the unfiltered camera array!!

Experiment 4: Lunar photography



Most of the ‘beauty shots’ you will see related to the moon are taken with professional digital cameras on tripods, or shot through a telescope, but with a little effort and preparation, you can make photos a lot more spectacular than this!

Lunar photography is a challenge because the camera will try to automatically adjust the exposure but most of the view will be the dark sky, so the moon’s disk will be overexposed and show no details. To get around this, most smartphones let you adjust with your finger where the focus and metering spots will be in the field. There are many smartphone apps that have greater flexibility than the one that comes with your camera, and you should consider testing as many of them as you can before the eclipse to find the right one. Several apps exist for both Android smartphones and iPhones, which claim to enhance your device’s picture-taking abilities. One of the best is Adobe *Lightroom*. Other decent low-light apps include *Camera+* and *NightCap* available for iPhone users. Like anything in life, practice makes perfect so get out there and experiment!

If you are using a smartphone, you need to make sure the image is properly focused. Don’t count on your auto-focus to do this. You have to do it manually, and this is as simple as tapping the screen and holding your finger on the moon to lock the focus. Then slide your finger up or down to darken or lighten the exposure. On iOS camera apps, tapping an object will center a box around it and show a little sun icon. This is the exposure slider. Drag it down until you see details on the moon image. Android camera apps usually have an exposure setting too, but it might take some hunting around to find it. For example, on the popular Samsung Galaxy S5, with the camera app running, tap the gear icon to expand the settings options, tap the gear again to enter camera and video settings mode, and tap again to enter the app’s master settings. The Exposure Value control is halfway down the list, and can be slid between +2 (brighter picture) and -2 (dimmer picture). Digital zoom will not work to create a magnified, clear image. Instead, go buy a \$20-\$40 dollar zoom lens attachment that will give you a total of digital zoom x optical zoom = 12x to 18x. For moon photography this will let you see a large moon disk, resolve mare features, and perhaps see a few large craters.

Consider using the delay timer set at 5 seconds so that once you press the exposure button, the camera waits 5 seconds before taking the shot. That gives your camera/tripod/clamp system plenty of time to settle down and produce vibration-free images.



Your most difficult challenge will be in managing your expectations! Smartphones were never designed to do lunar photography. The standard lenses are very small, and provide hardly any resolution at all for even the largest objects in the sky like the sun and moon.

Figure 80. Enlarged smartphone photo of full moon.

These objects are only $\frac{1}{2}$ -degree in diameter, and for a typical megapixel smartphone, their disks will only cover a few dozen pixels in your final image. For example, the photo to the left is an enlarged full moon taken by the author with an *iPhone 6s*. Cameras are designed to ‘retinal resolution’ and no more. This works out to about 2 arcminutes in angular resolution, so for a 30-arcminute moon you get about 15 pixels across its disk. When you account for image blooming, the disk can be much larger than this when its halo is included.

The photo in Figure 81 of the young crescent moon with earthshine and the bright planet Venus was taken by Chris Vaughan using a Samsung Galaxy S4 smartphone.

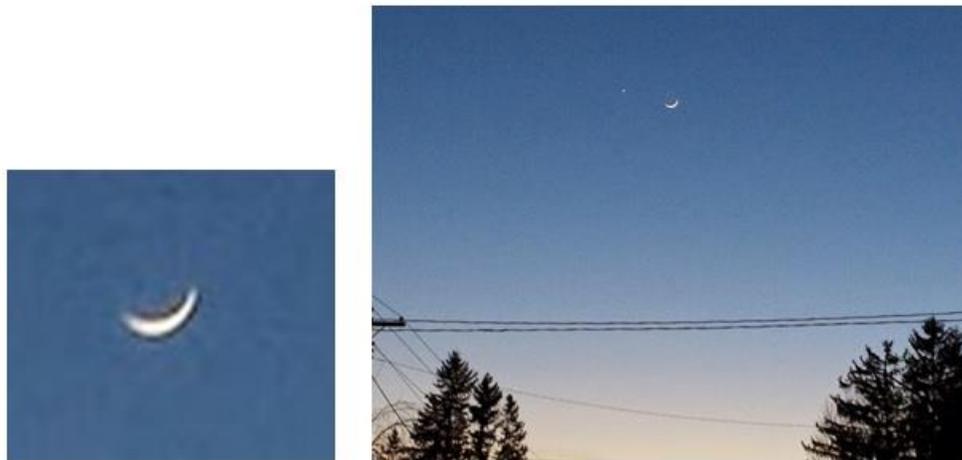


Figure 81. Examples of smartphone photo of crescent moon.

If you enlarge this image around the moon you get the result shown on the left. You can tell it is the moon, but there is no detail to be seen in the prominent lunar mare, which are easy to see with the naked eye. Several inexpensive smartphone adapters let you mount your device on a telescope or binocular such as the *iOptron Universal Smartphone Eyepiece Adapter*, the *Orion SteadyPix Universal Smartphone Adapter*, and the *TeleVue FoneMate*. These cost about \$40.00. If you own a pair of binoculars or a telescope, a simple trick is to hold the camera lens over the eyepiece to achieve greater magnification. Some of the world’s best astrophotographers have used this binocular technique with a tripod to take stunning photos of the moon using just their smartphone. Photographing the sun is a bit more of a challenge because of its brightness.

You might be tempted to use the digital zoom feature of your smartphone but this is rather useless. All you will be doing is cropping your original image to a format where it magnifies the image. It will not alter the actual resolution of your shot, which is the factor that determines how much detail you will actually see. For that, you have to increase the optical resolution of your camera, which the camera industry calls the optical zoom. For digital cameras like the compact point-and-shoot models or the high-end Nikon D3000 DSLRs, they can provide optical zooms from 2x to 6x by using attached lens systems or by switching to other lenses. For example, a telephoto lens with a focal length of 300 mm will provide 6x optical zoom for a standard 50 mm camera. Some smartphones such as the iPhone 7Plus deliver 2x optical zoom. The Galaxy Zoom smartphones have a variable focal length. For example, the Galaxy S4 Zoom has a 24-240mm f/3.1-6.4 lens. The optical zoom is calculated by dividing the longest focal length value to the shortest one, so in the case of the S4 Zoom, the optical zoom is 10x. Fortunately, for all other fixed-focus smartphones, there are two rather inexpensive ways to add substantial optical zoom capability.

Telephoto Lens Attachment

A telephoto lens system is absolutely a must-have for lunar photography with a smartphone. There are a number of zoom lenses for smartphone photographers designed solely to provide magnification without resorting to digital zoom. Most of these like the one on the left below, clip directly to the smartphone over the existing lens, and provide total magnifications of 8x for less than \$20.00, and 12x at a cost usually under \$40.00, but you will need to purchase a tripod for your smartphone to avoid shaking.

Some 12x systems like the one shown to the left include a tripod and a mounting bracket for the smartphone that is far sturdier than a clip-on system. Tripod adaptors for smartphones can be found on the Internet and cost between \$3 and \$10 brand new, but you can also find them more inexpensively at several auction sites on the Internet.

It is worth pointing out that the magnification touted by smartphone telephoto lenses is not the optical magnification, which is customarily used by astronomers and opticians to describe an optical system. It is common in the smartphone telephoto industry to advertise the maximum product of the smartphone's digital magnification times the optical magnification of the telephoto.

It is only, however, the optical magnification that affects the resolving 'power' of the system as we will discuss in a later section following actual measurements of various systems.

Figure 82. Smartphone photo of full moon using telephoto.



At an advertised magnification of 12x, with a digital magnification of 3x and an optical magnification of 4x, the moon disk subtends 4 times as many pixels as a normal 1x camera view (about 20), and the optically-enhanced resolution equivalent to a low-power binocular view (e.g. 4x50) results in rather clear viewing of the moon's dark

mare features. This image was taken by the Author with a *iPhone 6s* and a 12x telephoto, tripod-mounted.

To rival a binocular view (7x50) you need a telephoto with an optical magnification of 7x or an advertised magnification of $3 \times 7 = 21x$ or higher. Nevertheless, the properly designed and mounted '12x' system shown here provides images of the moon that are more than good enough for most amateur purposes including lunar eclipse photography, or just plain having fun and being satisfied that you did it yourself. If you want to get even fancier, you can use this telephoto plus resolved moon capability to format some beautiful photographs of cityscapes and wilderness views of the moon rising over buildings or mountain ridges.

Binocular photography

If you have a pair of binoculars, you can try one of two strategies: you can have your friend hold the binoculars pointed at the moon and then position your smartphone camera lens up against the eyepiece, or you can get a binocular-smartphone adapter and a tripod. By the way, if you go for the later approach you will have a fantastic system that you can use for many other photographic opportunities like capturing photos of birds and wildlife, close-up views of sports events or other public activities.

It will be difficult to use the hand-held binocular and camera because there are so many factors that you literally have to juggle to make it work. There will be a lot of movement of the camera and binoculars that will make getting a properly-focused shot hard to take in the short time you have during totality. You will spend so much time trying to set up the shots and line things up optically that you will probably get enormously frustrated!

The only sensible way to get good shots this way is to get a binocular attachment for your tripod, and a smartphone lens adaptor for the binoculars. With this set up, all you have to do is point the tripod/binocular at the moon, and set the focus on your binoculars and camera once and for all. Then you just need to touch the camera exposure button when you want to take a shot, or better yet, set the camera for video mode!

Binocular tripod clamps are easy to find on the Internet and cost between \$10 to \$20 for the simple screw-type adaptor, or \$25 for the strap-type adaptor that works for nearly all types of binoculars. Remember, this is an investment that by itself will pay dividends later on when you want to observe non-lunar subjects, so it is definitely not a one-shot investment.

Camera lens adaptors for binoculars can also be found on the Internet and cost between \$10 for one designed specifically for your camera type, or between \$50 - \$70 for a universal adaptor that fits most phone types, and that you can use for telescopes too. All you need to do is use the automatic exposure and focus camera setting and hand-point the binoculars at whatever you want to study close-up.

Experiment 5: Stellar Photometry with JPEG images

At some point you will have used your smartphone to take all of the really cool images of the sky and moon that you can think of. At this point you are ready for doing more than just taking ‘beauty shots’. For astronomers, we also spend a lot of time just taking the best photos we can of various astronomical objects so that we can analyze how they look and what their relationship is to other objects in the sky. The next thing we want to do is measure the brightness of various objects. This process is called photometry, and it is something you can explore with your smartphone images.

The basic idea is that the star or object emits photons of light that travel through space, pass through your telescope, and land on your image. To determine how luminous the object is we need to convert these photons received into physical units such as watts/meter². From these physical units we can determine how luminous the object is once we know its distance. So, we need a precise and repeatable way to gauge the brightness of objects that we photograph. To do this we have to count the photons we received.

Working with common jpeg image files is not a very ideal way to measure light intensity because the actual light intensity has been translated into a compressed scale of 256 levels in an 8-bit format. The advantage, however, is that it uses a very common file format and demonstrates the basic techniques. You will need a program that can pick out individual pixels and display their intensity values on the RGB 0-256 scale. One such program is *Adobe Photoshop Elements 9*, which will be used in this example.

- Step 1 - From Photoshop, open the star image.
- Step 2 – On the top menu bar select ‘Windows’. A display area on the lower-right screen should open showing a histogram of the pixel intensities in the image.
- Step 3 – On the histogram screen, in the ‘channel’ menu select ‘red’.
- Step 4 – With the cursor right click it and open a small square surrounding the star you want to measure...in this case Betelgeuse. A small window opens up and click on the check mark to select this pixel field.
- Step 5 – On the histogram screen, note the number of pixels (e.g 2205) and the mean value (e.g. R=98.9). Repeat for the green and blue channels (G= 80.7 and B= 75.3).
- Step 6 – Return to Step 4. At the top of the screen click on the ‘undo’ icon to return to the original star field. Place the cursor in an area adjacent to the star and draw a box with as close to the same number of pixels as for the star measurement. Make sure none of the star’s light is in this box.
- Step 7 – Repeat Step 5 for this background measurement in each color (e.g. R=57.7, G=52.2, B=62.1).
- Step 8 – Subtract the background mean value in Step 7 from the star’s mean value in Step 6 (e.g. R=98.9 – 57.7 = 41.2, G= 28.5 and B=13.2 for Betelgeuse). This removes the contribution of the sky background light from the measurement of the star’s brightness.
- Step 9 – In your data table, enter these values for the selected star:

Star	R	G	B
Betelgeuse	41.2	28.5	13.2

- Step 10- Return to Step 4 and repeat Steps 5-9 for at least 5 stars of different brightnesses. An example is shown below. Look up the visual magnitude of each star you used and organize the table from the brightest star (most negative apparent magnitude) to the faintest star (most positive apparent magnitude) as shown in Table 21.

Table 21 RGB values for selected stars from smartphone image

Star	Magnitude	R	G	B
Sirius	-1.46	134.5-91.9= 42.6	114.6-84.8= 29.8	141.9-79.4= 62.5
Rigel	+0.12	154.9-90.0= 64.9	141.8-70.6= 71.2	181.5-77.3= 104.2
Procyon	+0.34	69.2-54.0= 15.2	66.0-43.3= 22.7	93.2-55.4= 37.8
Betelgeuse	+0.42	98.9-57.7= 41.2	80.7-52.2= 28.5	75.3-62.1= 13.2
Aldebaran	+0.87	83.7-46.5= 37.2	80.4-47.1= 33.3	97.9-56.8= 41.1
Bellatrix	+1.64	79.0-70.2= 8.8	70.4-58.5= 11.9	86.3-64.0= 22.3
Mintaka	+2.25	82.8-78.2= 4.6	72.4-63.0= 9.4	84.1-66.3= 17.8

Very approximately, because the ‘green’ channel is very close to the center of the visual band, the stars organized by decreasing visual magnitude should also organize the green channel values in descending order as well. The magnitude scale is a logarithmic scale but the pixel values are on a linear scale, so to place them on the same scale as for apparent magnitude we use $m = -2.5\log(I) + C$. The constant term, C, established where the zero-magnitude scale fits into the pixel intensity scale. We can do this conversion for each RGB values and star to get Table 22.

Table 22. Corrected RGB magnitudes of stars

Star	Magnitude	Temperature	R+Cr	G+Cg	B+Cb
Sirius	-1.46	9,940 k	-4.1	-3.7	-4.5
Rigel	+0.12	11,000 k	-4.5	-4.6	-5.0
Procyon	+0.34	6,530 k	-3.0	-3.4	-3.9
Betelgeuse	+0.42	3,500 k	-4.0	-3.6	-2.8
Aldebaran	+0.87	3,910 k	-3.9	-3.8	-4.0
Bellatrix	+1.64	22,000 k	-2.4	-2.7	-3.4
Mintaka	+2.25	60,000 k	-1.7	-2.4	-3.1

We can graph the apparent magnitudes in the visual band with the magnitudes in the G band to get the following plot:

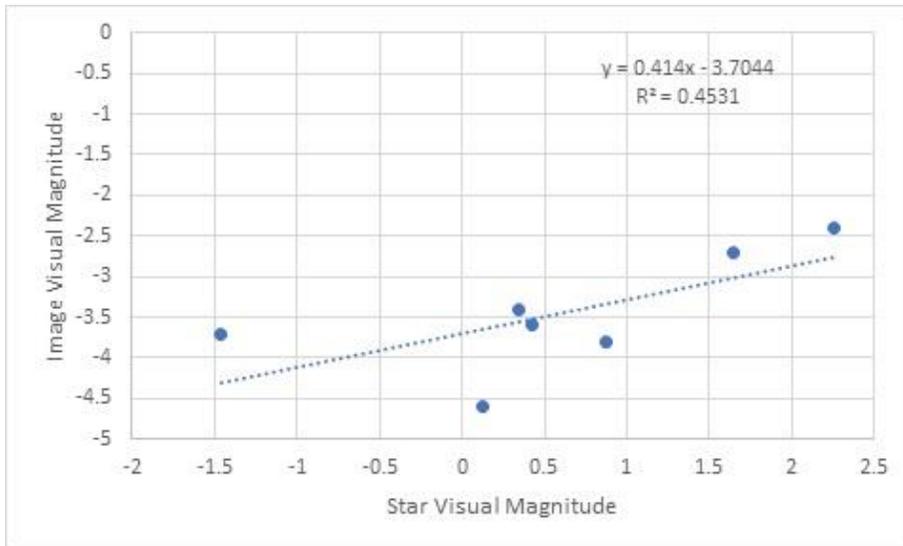


Figure 83. Plot of star magnitude and star image brightness from camera photo.

This provides a rough estimate for the brightness of the star in the image in terms of the actual visual magnitude of the star. The dispersion around this curve is about $\pm 0.5m$ so that is a practical limit to how well we can measure the brightness of a star using smartphone, Jpeg images. There are several outliers and an inspection of the star images shows that the images are irregular in shape, so combining this data with that from other images of the same stars would improve the statistics and hopefully reduce the $\pm 0.5m$ scatter. Another way to plot this data is directly in terms of the mean pixel values in Table 1 with the unique bright star Sirius eliminated so that only fainter stars are considered:

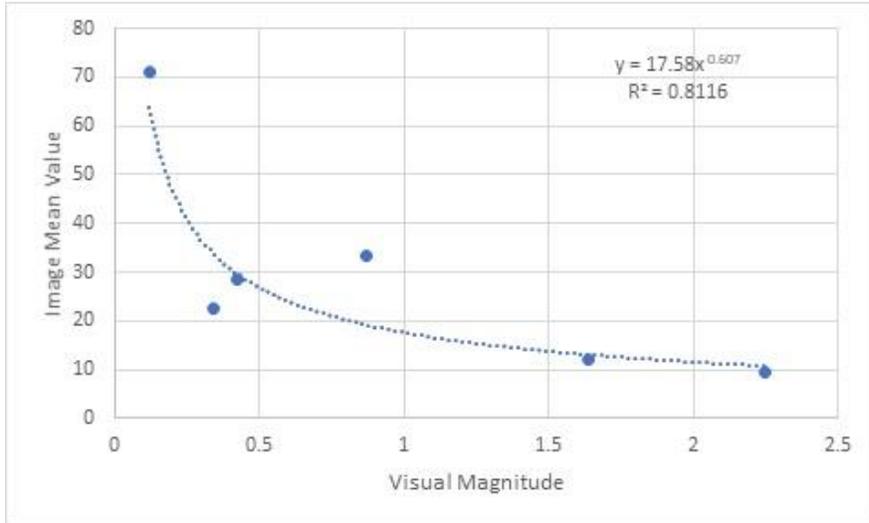


Figure 84. Plot of star image brightness and actual visual magnitude.

The fitting curve predicts that from Experiment 1 photographing stars with the smartphone, that the magnitude limit of +3.0m corresponds to about $C = 17.58(3)^{-0.6} = 9$ of the G-band intensity units due to the sky/array background. We can check to see if this is related to the noise level of the array under completely dark conditions with the lens blocked off. In the following figure is the image of the ‘dark field’ taken at 2 seconds at ISO 800 to match the star images.

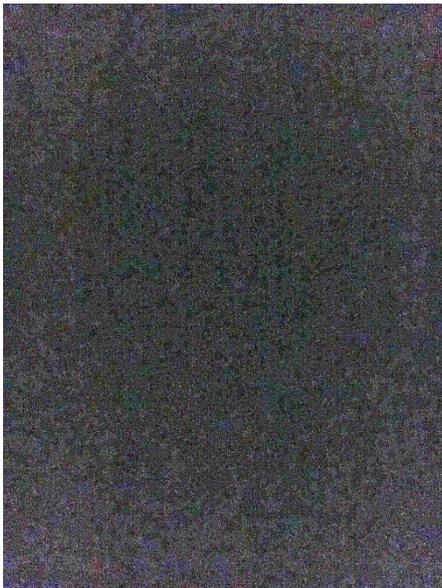


Figure 85. Smartphone image of a covered camera lens ‘dark field’.

The measurements on this dark field show that the (R,G,B) values vary from (58, 61, 59) in the center to (90, 84, 91) at the edges, so the array itself contributes a significant number of

background counts, and also skews the RGB intensities slightly, being more color-neutral at the center ($R=G=B$) and bluer at the edges ($B > R$ or G).

We can actually estimate the colors of the stars by comparing the B and G values. Hot stars will produce more blue light compared to green, while cool stars will produce more red light in the R band. The magnitude difference between the Blue and Green bands is a good measure of temperature, which is similar to the B-V index used by astronomers.

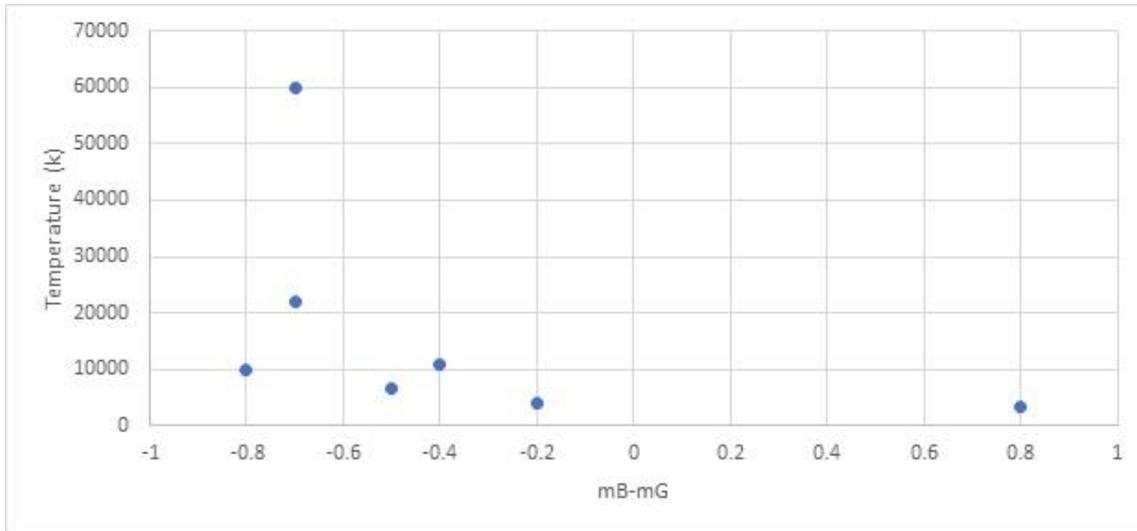


Figure 86. Temperature of a star versus smartphone magnitude differences.

So, if in our smartphone image we measure a star with a background-corrected B and G-band difference of -0.6, it probably has a temperature near 15,000 kelvins. Stars like our sun with temperatures near 6,000 kelvins would be expected to have a B-G magnitude difference of about -0.5.

By using your smartphone to photograph more constellations and measuring more stars, you can classify all of the stars in the sky brighter than about +3.0m according to their temperature and spectral type!

Experiment 6: Stellar Photometry with RAW images

Most digital cameras can export a variety of file formats including the popular jpeg format, which compresses the data so that it takes up less file space but still preserves the RGB color values with enough accuracy to provide over $256^3 = 16$ million different colors. For astronomical purposes, an intensity scale of 256 steps only covers a magnitude range of $2.5\text{Log}(256) = +6.0$, so if your brightest star is Sirius with $m = -1.4$, then your faintest star would be $6.0 - 1.4 = +4.6$ in a perfect world where the sky background and array do not contribute more than 1 brightness level. In fact, the dark image created in Experiment 3 had typical intensity values near 90 units so you only have $256 - 90 = 166$ levels or 5.6 magnitudes of range between a saturated star image like Sirius and the sky background. That works out to a faint star limit near $5.6 - 1.4 = +4.2$. When you add in the actual sky lighting from clouds, city lights and the moon, this limiting magnitude is closer to +3.0 as we have found. Fortunately, we are not restricted to using only the compressed jpeg files, either for astrophotography or photometry.

What we want to work with are the actual pixel values, not their compressed versions used in jpeg files. The uncompressed data from an array is in a format called RAW, and most DSLRs provide this format, but they are rarely used by amateur photographers because they are huge in size, typically many tens of megabytes.

Imaging arrays and DSLRs like the Nikon D3000 use arrays in which the data is represented in 2^{14} levels rather than $2^8 = 256$. Because the shading resolution in intensity is $2^6 = 64$ times finer than for jpeg images, much more precise intensity measurements are possible with RAW files. All of the techniques we used in Experiment 3 to work with jpeg files can be applied to working with RAW image files, but now we can create more accurate data. Because of their higher-quality data, we need to adjust our measurement technique accordingly.

Flat Field Image: If I photograph a uniform sky intensity, I should expect that all of the pixels in the array will record the same intensity of light within each of the RGB color bands. This generally is not the case because arrays have manufacturing defects, lenses have dust flecks, and the pixels in one part of the array may not have the same sensitivity to light as in other areas. Each pixel also has a slightly different response to light than its neighbor, called the pixel gain. The flat field image is divided into the science image to correct for the gain variations so that 1 unit of intensity on one pixel is the same as one unit of intensity on all other pixels in the image.

Dark Field Image: Pixels are like small wells that capture electrons generated by the photons of light that strike the well material. But electrons can flow in to these wells from simple thermal noise in the well material. To measure how much of this noise is present in each pixel, you cover the lens so the array is perfectly dark, and use short exposures to measure what is called the leakage rate. When you make a sky photograph of 2 seconds exposure, the dark field image obtained at the same 2-second exposure with a blanked-off lens, will indicate how much of the intensity in each well is contributed by leakage, and so this can be subtracted from each pixel value.

Bias Image: The pixels in an array are given a slight offset by the electronics that control each pixel. This has nothing to do with the light hitting it or the accumulation of dark current. This image is created by using the shortest exposure on the camera under ‘no exposure’ conditions. The resulting image is then subtracted from the dark, flat and science images.

Procedure:

- Step 1 – In your camera/app select the output format as RAW.
- Step 2 – Select ISO 800 and an exposure of 2 seconds for your ‘Science’ image.
- Step 3 – Cover the lens and with the same exposure settings take the dark image.
- Step 4 – Open the lens and with the same camera setting take the flat image.
- Step 5 – Select the shortest exposure time, say 1/10000 seconds, and with the lens covered take the bias image.
- The next steps require a program like *DeepSkyStacker* which is free, (<http://deepskystacker.free.fr/english/index.html>) to manipulate the array data.
- Step 6 – Subtract the bias image from each of the other images in Step 2, 3 and 4.
- Step 7 – Subtract the dark image from the Science image.
- Step 8 – Divide the resulting (Science – bias – dark) image by the flat image to get the calibrated image.
- You now have a properly-corrected image that has removed artifacts of the array from the final data, and has placed all of the pixels on the same intensity scale. You now need a program like *RawDigger* to display the RAW pixel data values and not the 8-bit compresses RBG values used in jpeg/tiff files.
- Step 9 – Open RawDigger
- Step 10 – In File, select ‘Open’ and find the calibrated science image.
- Step 11 – Select it and click to open in the work area. Information on the origin of the file is available in the top-left information bar area, eg. ‘Apple iPhone 6s, 0.,3sec, f/2.2, ISO 800.
- Step 12– Click on View and select ‘actual pixels’. The cursor should now display the actual R, G, B-channel values from 0 to 3567.
- Step 13 - For each star, place the cursor over the star and find the maximum-value pixel. Record these values under RGB. Do similar for nearby background pixels. You can average 4 background pixels in a square surrounding the star for better estimate.

I stacked 9 Nikon images of the Orion field, each 30 sec x 800 ISO saved and exported as RAW images to get this image



Figure 87. Nikon camera image of Orion.

Compare the stars in the photo with those in a star chart of the constellation to identify the stars and their magnitudes. Try to identify the faintest stars viewable in the photo to set a limit to the sky brightness and to the sensitivity of the camera/image settings.

Orion - Ori - The Hunter

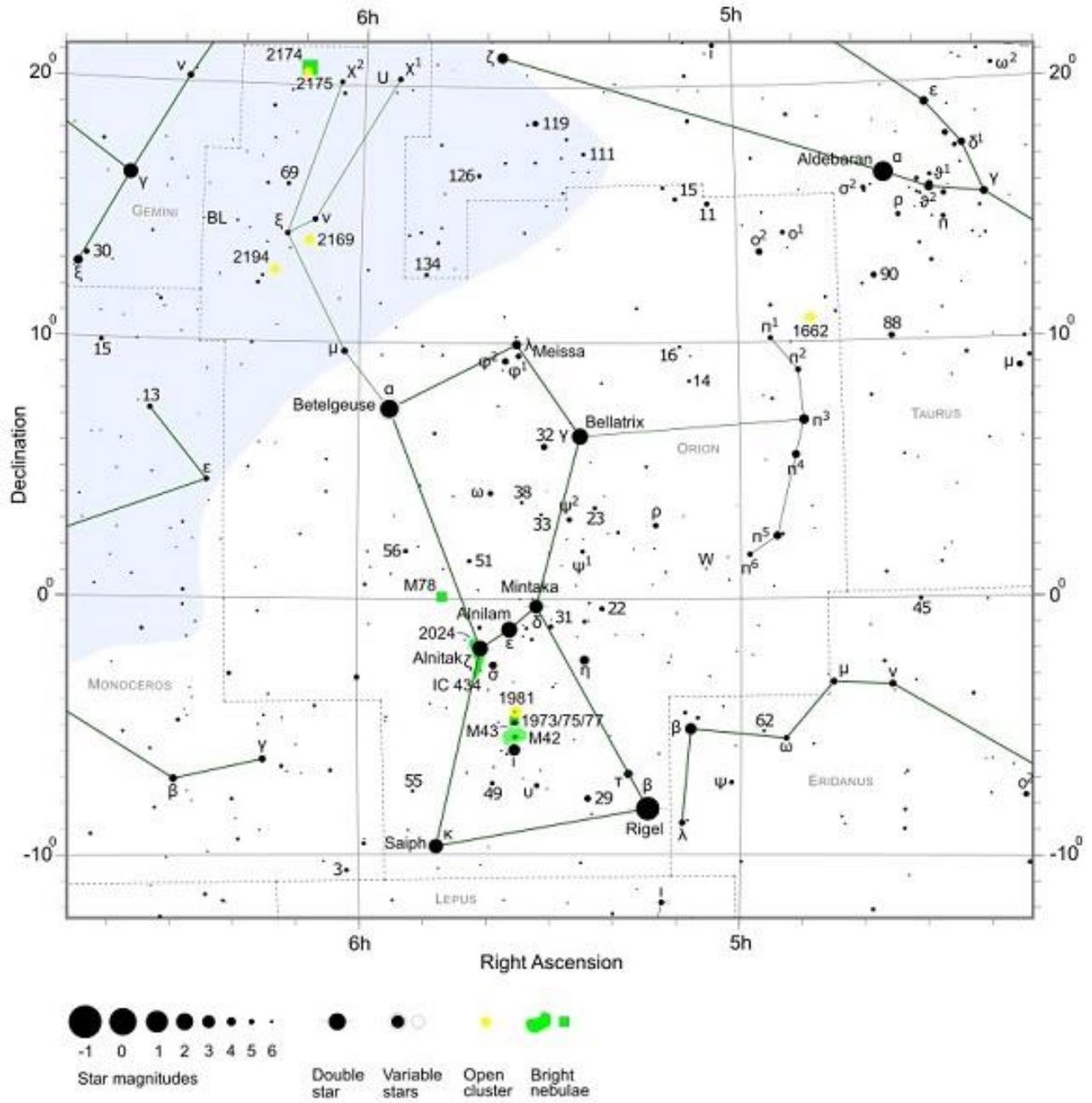


Figure 88. Map of Orion



Figure 89. Map of Orion.

In *RawDigger* operating on the RAW image (1/3sec ISO 2000), the star photometry looked like this, which includes stars in Orion and in Ursa Major (shaded).

Table 23 RGB values for stars in Nikon image of Orion region

Star	Magnitude	R	G	B
Rigel	+0.12	124-4=120	722-6 = 716	196-5=191
Saiph	+2.1	27-5=22	56-6=50	45-5=40
Betelgeuse	+0.42	175-4=171	249-6=243	54-5=49
Bellatrix	+1.64	60-4=56	71-6=65	218-4=214
Mintaka	+2.25	33-5=28	74-6=68	79-4=75
Sirius	-1.43	616-4=612	2261-5=2256	1613-4=1609
Dubhe	+1.79	30-4=26	63-4=59	50-4=46
Merak	+2.37	33-4=29	41-5=36	34-4=30
Phecda	+2.44	22-5=17	52-5=47	38-4=34
Alioth	+1.77	30-4=26	67-5=62	49-4=45
Mizar	+2.23	27-4=23	49-7=42	49-4=45
Alkaid	+1.86	39-4=35	57-6=51	60-4=56

Table 24. Temperatures and B-G color index for Orion stars

Star	m	Temp	mR	mG	mB	mB-mG
Rigel	+0.12	11,000	-5.2	-7.1	-5.7	-1.4
Saiph	+2.1	26,500	-3.4	-4.2	-4.0	-0.2
Betelgeuse	+0.42	3,500	-5.6	-6.0	-4.2	-1.7
Bellatrix	+1.64	22,000	-4.4	-4.5	-5.8	1.3
Mintaka	+2.25	60,000	-3.6	-4.6	-4.7	0.1
Sirius	-1.43	9,940	-7.0	-8.4	-8.0	-0.4
Dubhe	+1.79	4,282	-3.5	-4.4	-4.2	-0.3
Merak	+2.37	9,377	-3.7	-3.9	-3.7	-0.2
Phecda	+2.44	9,355	-3.1	-4.2	-3.8	-0.4
Alioth	+1.77	10,800	-3.5	-4.5	-4.1	-0.3
Mizar	+2.23	9,000	-3.4	-4.1	-4.1	0.1
Alkaid	+1.86	16,800	-3.9	-4.3	-4.4	0.1

Comparing mB-mG to the temperature we get the following graph. The coolest star is Betelgeuse with mB-mG=+1.7 at 3,500k. Generally, the more negative mB-mG is, the hotter the star. That is because hot stars have more B-band light than cooler stars, and so mG will be more positive compared to mB.

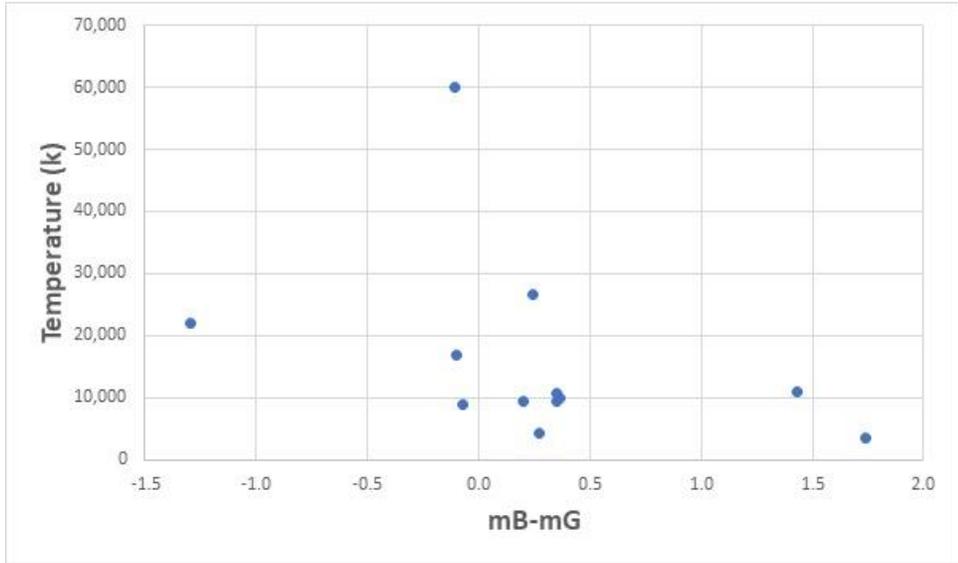


Figure 90. Plot of star temperature versus Blue-Green color.

Smartphone Sensor Math

By Dr. Sten Odenwald

NASA Space Science Education Consortium

Draft 1 November 2018



Working with real-world measurements

Significant Figures

For problems involving pure mathematics, one seldom takes the point of view that some digits in a stated number are more important than others, however in science where numbers represent measurements, not all digits are created equally! The examples below show the four basic rules of how to count 'significant figures'. The basic rule is that you never state more digits in a number than the precision of your measurement. For example, if you can only measure to a precision of one meter, you should never state a measurement as 102.56 meters! When multiplying or dividing numbers, the answer must have the same number of significant figures as that of the least known number. Pure numbers such as '5' can be written with as many decimal places as needed.

Example 1: 1.234 grams has 4 significant figures,

Example 2: 30.07 Liters has 4 significant figures.

Example 3: 0.012 grams has 2 significant figures.

0.20 grams has 2 significant figures

Example 4: 0.0230 Liters has 3 significant figures,

SpaceMath@NASA <https://spacemath.gsfc.nasa.gov/weekly/7Page25.pdf>

The Rules:

1. Non-zero digits are always significant
2. Any zeros between two significant digits are significant
3. A final zero or trailing zeros in the decimal portion **ONLY** are significant
Example: 0.500 or 0.632000 the zeros are significant
0.006 or 0.000968 the zeros are NOT significant
4. When in doubt, represent the number in scientific notation

Problem 1 – Significant Figures. For the numbers below, indicate how many SFs are involved:

A) 450.12 B) 450.120 C) 1.234x10⁻¹¹ D) 0.00234500 E) 0.002345

Answer: A) 5 SF B) 6 SF C) 4 SF D) 6 SF E) 4 SF

Problem 2 – Significant Figures in Formula Calculations . The Rydberg Constant is an important number in atomic physics and is given by the formula

$$R = \frac{2\pi^2 m e^4}{ch^3}$$

Evaluate the formula to the correct number of SF where:

Electron mass: $m = 9.10956 \times 10^{-28}$ grams

Speed of light: $c = 2.997925 \times 10^{10}$ cm/sec

Electron charge: $e = 4.80325 \times 10^{-10}$ ESU

Planck's Constant: $h = 6.62620 \times 10^{-27}$ erg sec

Answer: Answer: First evaluate the function using only the exponents:

$$(10^{-28})(10^{-10})^4 / (10^{10})(10^{-27})^3 = 10^3 .$$

Now evaluate all the measured constants:

$$\frac{2(3.141592654)^2(9.1095)(4.80325)^4}{(2.997925)(6.62620)^3}$$

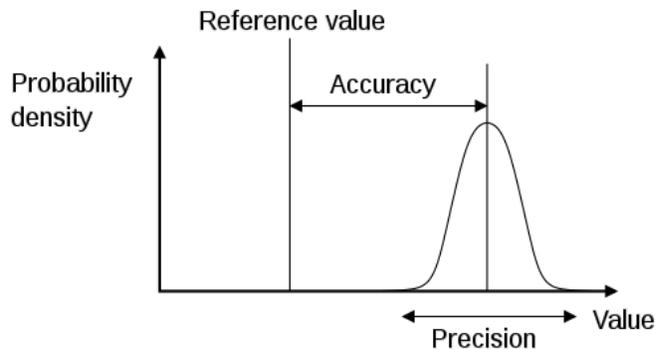
Using a calculator, one obtains the 'answer' $R = 109.7371112$ which has to be combined with the previous calculation for the exponents of 10^3 to get

$$R = 109.7371112 \times 10^3 \quad \text{or in scientific notation} \quad R = 1.097371112 \times 10^5$$

The smallest number of SFs occurs for the constants m , e and h which have 6 SF, so we round the answer to $R = 1.09737 \times 10^5$

Accuracy versus Precision

There are two situations that often confront scientists. In the first instance, the value of some measurable quantity is known, and the issue is how close to this value a series of measurements can get. The range of measurements of this known quantity represents what is called the accuracy of the measurement.



For example, your height is a fixed quantity so depending on how you measure it with a ruler, tape measure or some other device, the numbers they report will differ by some amount from your true height. The difference between the measured values and the true value is the accuracy of the measurement.

Another example is the ratio of the circumference of a circle to its radius, which is exactly 2π . Actual measurements of the ratio from repeated independent measurements of the radius and circumference will not achieve a perfect measure of 2π , but depending on the measuring device will achieve a range of numbers close to 2π . This range is a measure of accuracy.

However, in most cases when a scientist is measuring a new property or phenomenon for which the true value is not known beforehand, repeated measurements of the property will yield a consistent average, but individual measurements will sometimes be larger or smaller than this average value. The range or dispersion of these measurements around the average is called the precision of the measurement.

For example, if you use a tape measure to determine someone's height in millimeters, the average of these measurements might be 1854 millimeters (6' 1"), but the individual measurements might vary from this average by 3 millimeters more or less than this value. The measurement is then 'precise to within 3 millimeters'.

In the previous example, if the true height of the person is 1879 millimeters (6' 2") we would say that this measuring method is also accurate to within $1879 - 1854 = 25$ millimeters. A set of height measurements can then be very precise (within 3 millimeters), but not very accurate (within 25 millimeters).

Another example is the observational measurement of Hubble's Constant. One set of techniques use Type 1A supernovae as 'standard candles' and obtains a value of 73 km/sec/mpc to within $\pm 2\%$ so by these measurements it could be in the range from 72 to 74 km/sec/mpc. However

measurements made of the cosmic background radiation 380,000 years after the Big Bang by the Planck satellite derive a value of 67.8 km/sec/mpc over a range from 66.9 to 68.7 km/sec/mpc. We do not know what the 'correct' value should be but here we have an example of a quantity measured by two very precise methods (within $\pm 2\%$) but that disagree significantly on whether then true value has an average value of 67.8 or 73.0 km/s/mpc. So the accuracy assuming a single value for Hubble's Constant is $70.4 \pm 4\%$. This 4% accuracy is much worse than the 2% precision of either two methods.

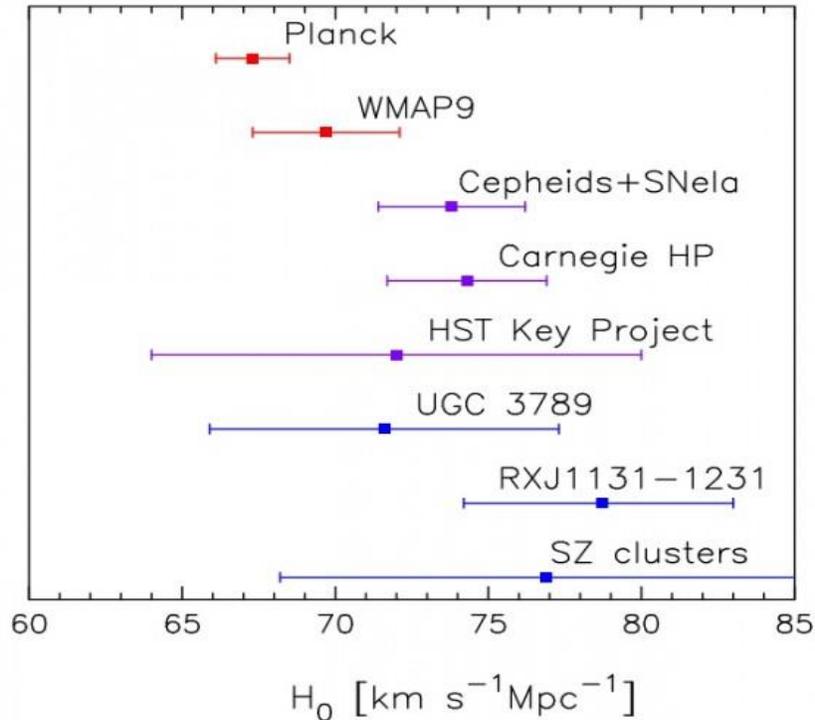


Figure 91 – Hubble constant data.

Figure 91 shows a variety of methods, their mean values and their range of precision. We do not know beforehand what the 'true' value for the Hubble Constant might be, but if it is close to the sample mean of 71 km/sec/mpc, the various methods have different levels of accuracy with the SZ Cluster and RXJ1131-1231 methods having the relatively worse accuracy.

How Smartphones Create Data

Your smartphone generates a huge amount of data to help figure out how to display the screen images as you move your phone. It also processes image data taken by your camera. By one estimate, a standard smartphone today has a million times the computing power of the computers on the Apollo 11 mission in 1969. Originally, the first generation of smartphones with viewing screens only needed a magnetometer, an accelerometer, and a gyroscope to figure out the basic functions for ca 2008 phones. App developers and camera designers soon came up with a variety of additional sensors to supply the needs of newer generations of smartphone users. Today, the list of sensors built-in to a typical smartphone is quite impressive.

- 1. Microphone** (Decibel, Frequency, Noise cancellation)
- 2. Camera/Image sensor** (Scanner, Barcode , Colour temperature-Kelvin)
- 3. Proximity sensor** (Obstruction detection/Like Laser Mouse Operation-Infrared)
- 4. Ambient light sensor** (Light Intensity-Luminance/lux)
- 5. Motion sensor** (Can be used by Accelerometer)
- 6. Gyroscope** (orientation, Constellation degree-Google sky map)
- 7. Accelerometer sensor** (Acceleration, Gravity, Speed)
- 8. Digital compass / Magnetometer**
- 9. Magnetic field sensor** (xyz wise-micro tesla)
- 10. Hall sensor**
- 11. Temperature**
- 12. Humidity**
- 13. Barometer** (Air pressure, Altimeter)
- 14. Battery temperature**
- 15. Fingerprint scanner** (Ultrasonic or Optical Scanner)
- 16. Iris Scanner** (Retina scanner-can be used by front camera)
- 17. Pedometer or Step counter** (mostly used by Accelerometer)
- 18. Heart rate Monitor** (Samsung Galaxy S5, Lenovo ZUK Z2 Pro)
- 19. Pulse oxymeter** (Samsung Galaxy S5, Lenovo ZUK Z2 Pro)

20. **Geiger Counter** (Harmful Radiation level detector-Sharp Pantone 5 in japan)
21. **NFC** (Type of radio frequency tag scanner, Connectivity)
22. **Infrared Blaster** (For using TV remote data connection etc)
23. **Laser** (Auto focus, Distance Measurement, Also one type of laser bar-code scanner available- Panasonic Touchpad FZ-F1 Mobile)
24. **Touch screen** (Conductivity or by pressure)
25. **Air Gesture** (by using front camera)
26. **3D Air gesture & 3D scanning** (use multiple camera 2 to 5- Takee 3D mobile)
27. **GNSS (Global Navigation Satellite System- GPS, GLONASS, BeiDuo-BDS, IRNSS-Indian Navigation Satellite system- Global position, Elevation or altitude, Speed, Time measurement, Distance measurement, Satellite scanning)**
28. **Other signal receiver spectrum and band width** (1G,2G,3G,4G,5G,WiFi,Bluetooth,FM radio, Television)
29. **LiFi** (Data connectivity by visible spectrum light)
30. **Clock** (Normal and Atomic clock-Distance measurement and Satellite positioning)

The outputs from these sensors can be read-out by the smartphone JavaScript software, which is constantly running to provide the overall smartphone experience, however, literally millions of application programs (apps) are now available that use selected data from these sensors for other purposes not considered by the smartphone developers. As of March 2017, Figure 92 shows there were 2.8 million available apps at the *Google Play Store* (Android phones) and 2.2 million apps available in the *Apple's App Store* (iOS phones) - the two leading app stores in the world. Most of these apps are nothing more than computer games, but a significant subset are tools that display specific sensor data in a variety of formats from digital dials to real-time graphs.

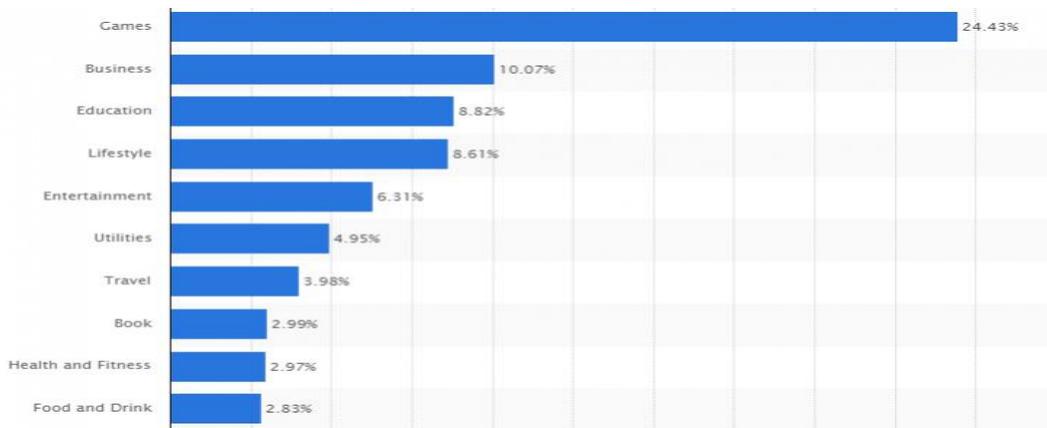


Figure 92 – Number of apps available for smartphones.

Analog and digital data

An analog signal can be represented as a continuous series of data values in time. That signal can be a voltage or current created by a sensor directly, or it can be produced indirectly as a transducer. Transducers are electronic devices that convert one kind of signal into another. For example, piezoelectric transducers convert pressure, which deforms crystal structure, into a voltage signal. Hall Effect transducers convert the strength of a magnetic field into a change in resistance that can be detected as a change in the electrical current flowing through the transducer. The preponderance of sensors are analog sensors.



Figure 93 – Analog signals from sound and electricity

Computers operate not with analog data but with digital information because they are based on a base-2 binary 1 or 0 language and memory architecture. The fundamental unit of information in a computer is the ‘word’ which is a number with 1 bit of information for simple 2^1 which can represent a simple True or False state for a gate in the vast collection of millions of logic elements.

Most modern computers use a 64-bit architecture.

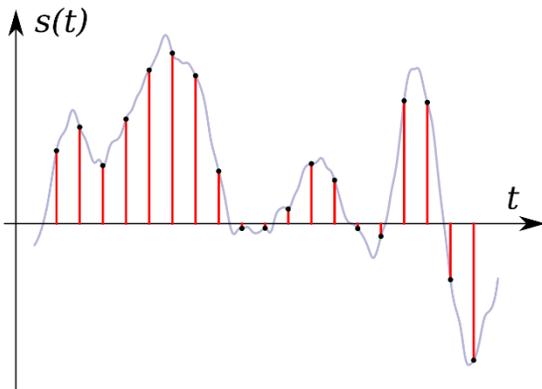


Figure 94 – An analog signals being sampled into a digital signal.

A typical 32-bit data word needs one bit to represent the sign of the data (+ = True, - = False) and allows 31-bits for the value of the data so that numbers as large as +/- 2,147,483,648 can be defined in a ‘signed’ word, or $2 \times (2,147,483,648) = 4,294,967,295$ for unsigned data. For 64-bit

architecture, unsigned numbers as large as 4.6×10^{18} can be defined. These large computer data words, however, do not represent actual data because the largest data word generated by sensors specifically smartphone cameras is only 16-bits, representing some 65,535 unsigned values. If we want to mathematically understand the data provided by smartphones, we need to understand how analog data is represented by digital data words. An example of an analog signal converted into a digital one is shown in Figure 4.

Imagine a temperature transducer, called a thermocouple, that is set up to register a range of values from 0° Celsius to 160° Celsius as a voltage change from 0 milliVolts to 80 milliVolts. The conversion of the temperature scale to the voltage scale leads to 2°C per millivolt. But both scales are analog so they are not as yet suitable to be manipulated by a computer or smartphone circuitry and programming. Suppose we define a new kind of digital scale based on a 3-bit data word, which has a total of 8 different levels. If we ‘stuff’ the analog voltage range of 80 milliVolts into the 8-level data word, we can get the following table:

Table 25: Data word digital values

Binary word	Analog Value	Temperature
000	00 to 09 mV	0 to 19° C
001	10 to 19 mV	20 to 39° C
010	20 to 29 mV	40 to 59° C
011	30 to 39 mV	60 to 79° C
100	40 to 49 mV	80 to 99° C
101	50 to 59 mV	100 to 119° C
110	60 to 69 mV	120 to 139° C
111	70 to 79 mV	140 to 159° C

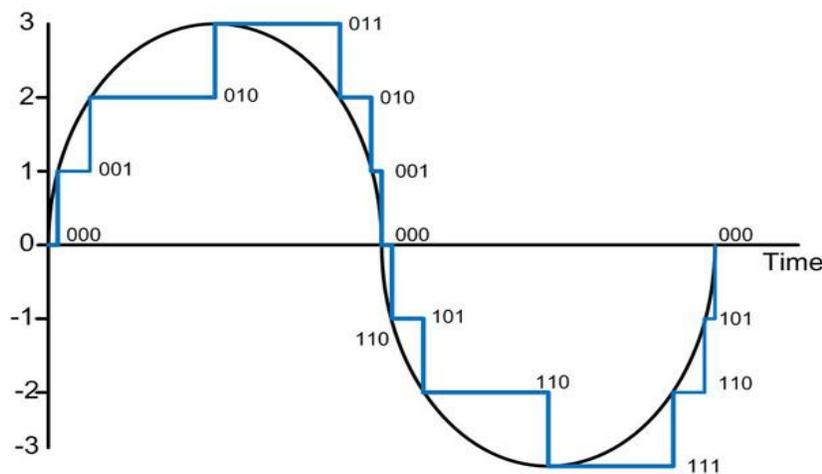


Figure 95 – Analog to digital conversion.

This is accomplished by an electrical device not surprisingly called the Analog-to-Digital Converter (ADC).

Problem 3 – Minimum ADC bits. A temperature in the range of -40°C to 55°C is to be measured with a resolution of 0.1°C . The minimum number of ADC bits required to get a matching dynamic range of the temperature sensor is

- (A) 8 (B) 10 (C) 12 (D) 14

Answer: The range is $55 - (-40) = 95^{\circ}$, and we want the 1-bit resolution to be 0.1°C , so we need at least $95/0.1 = 950$ steps. The answers are A) $2^8 = 256$ levels, B) $2^{10} = 1024$ levels, C) $2^{12} = 4096$ levels and D) $2^{14} = 16,384$, so the minimum number of ADC bits is B) 10-bits.

Problem 4- Data Word Size – A scientist needs to measure the acceleration of gravity at Earth’s surface to a resolution of 2 milliGals where one milliGal is 0.001 cm/sec^2 . On this scale, the average acceleration of gravity is 980000 milliGals. It is not expected that the variation of the surface gravity will be outside the range from 990000 to 970000 milligals. If the signed digital representation is such that 980000 corresponds to the zero value in the digital range, what size data word will be needed to achieve 2 milliGal resolution?

Answer: The range of the signed data word will be from 0 to 100000 milligals. To get a resolution of 2 milliGals as the LSB you will need $2^N = 100000/2$ or $N\log_2 = \log(50000)$ so $N = 15.7$ or a minimum of 16-bits for a total of 17 bits including the sign.

Problem 5 – Memory Fill Rate. An iPhone 6s has a memory capacity of 16 gigabytes (1 byte = 8 bits) of data. If the magnetometer read-out app generates one 14-bit data word 100 times a second, how long will it take for the app to swamp the smartphone’s storage capacity?

Answer: 16 billion bytes x 8 bits/byte = 128 billion bits.

$$\text{Time} = 128 \text{ billion} / (100 \times 14) = 91 \text{ million seconds or } 2.9 \text{ years!}$$

A/D Converters

The basic operation of an A/D converter is that it compares an input analog voltage with a series of reference voltages that form each of the levels in the digital data word.

When it identifies a match between the signal voltage and the reference voltage for a given bit in the data word, it returns a value of ‘1’. For example, in our previous table, if the input signal were between 60 to 69 mV, it would return a ‘1’ in the first and second bits of the data word to make ‘110’ in the 3-bit data word format.

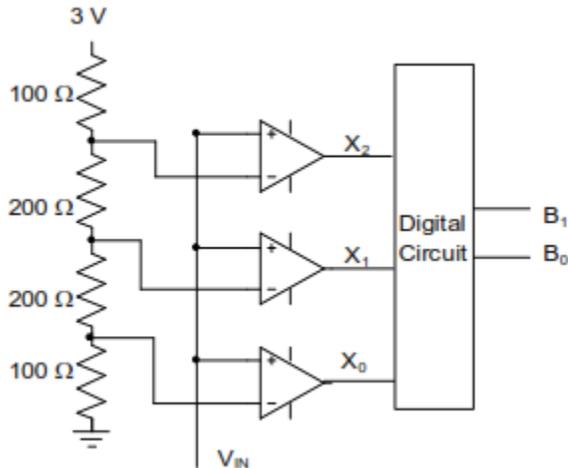


Figure 96 Circuit for an analog to digital converter.

Figure 96 is a schematic diagram for a simple 2-bit ADC. The reference voltage 3.0 volts, and the signal analog voltage is V_{in} . The resolution of this converter is 1-bit = $3.0 \text{ V}/2^2 = 0.75 \text{ V/bit}$. The resistors are selected in value to provide the following intermediate voltages for the three comparators:

$$V_2 = 3.0 \times 100 / (100 + 200 + 200 + 100) = 0.5 \text{ Volts}$$

$$V_1 = 3.0 \times (100 + 200) / (100 + 200 + 200 + 100) = 1.5 \text{ Volts}$$

$$V_0 = 3.0 \times (100 + 200 + 200) / (100 + 200 + 200 + 100) = 2.5 \text{ Volts.}$$

The triangle symbols are devices called operational amplifiers or OpAmps that can be configured to compare the two analog voltage inputs and pass the value of '1' if the voltage inputs are within the voltage range of the comparator, or '0' if they are not.

Table 25: Comparator assignments

Comparator	Voltage Range
X2	0v to 0.5 v
X1	0.51v to 1.5 v
X0	1.51v to 2.5 v

The 'Digital Circuit' block is a set of logic gates that takes the comparator outputs X0, X1 and X2 and actually builds the 2-bit digital word bits B0 and B1 where B1 is the bit representing the highest value (called the Most Significant Bit or MSB).

Table 26: Analog to digital bit assignments

B1	B0	Word	Voltage range
0	0	00	0 to 0.74
0	1	01	0.75 to 1.49
1	0	10	1.50 to 2.24
1	1	11	2.25 to 2.99

For example, in the above example, there are three voltage levels $X_2=0-0.5V$, $X_1=0.51-1.5V$, and $X_0=1.51-2.5V$. If the input voltage $V_{in} = 1.75V$, X_2 will be '0', X_1 will be '0' but X_0 will be '1'. This is passed to the logic gate block that composes the data word with $B_1='1'$ and $B_0 = '0'$ forming the data word '10' which is the code for $V_{data} = 1.50V$.

The bottom line is that Table 26 is the only one you need to consider unless you want to design an ADC device yourself! From this table we see that the smallest resolution of this ADC is $0.75V/bit = 3.0V/2^2$. When combined with the calibration of the transducer or sensor output, these digital voltage steps can be related to the specific physical parameter being measured such as decibels for sound intensity, teslas for magnetic field intensity, or m/sec^2 for accelerometers. The ADC doesn't care what these physical units are. It is only interested in digitizing an input voltage signal.

Problem 6 - Number of ADC Comparitors. If a 2-bit ADC requires 3 comparators and a 3-bit ADC requires 7 comparators, how many comparators are required for a 12-bit ADC?

Answer. $3 = (2^2-1)$ and $7 = (2^3-1)$ so we need $N = (2^{12}-1) = 4095$ comparators.

Problem 7 - Reference Voltage Intervals. In a comparator voltage divider string of resistors, the first and last resistors have half the value of the other resistors in the string. Also, for N comparators there are N+1 resistors. If the input reference voltage is 3.0 volts, what are the voltage drops in the voltage divider string for a popular 8-bit ADC?

Answer: The 8-bit ADC has $2^8-1 = 255$ comparators and so following the pattern for the 2-bit comparator in the figure above there have to be $N=255+1 = 256$ resistors. The voltage of 3.0 V will equal the sum of all the voltage drops in the divider.

$R_{total} = 254xR + 2(R/2) = 255xR$. Each resistor, R, creates a voltage drop of $3.0/255 = 0.01176$ or 11.76 milliVolts, but the first and last resistors with $1/2R$ create a voltage drop of 0.00588 or 5.88 milliVolts each.

Quantization (Digitization) Noise

No electronic device operates perfectly. In order for an ADC to yield reliable results when it converts an analog signal into a digital representation, not only must the reference voltage remain stable, but the warm devices themselves cannot contribute thermal agitation or noise to the conversion process. The stability of the input reference voltage and the thermal 'kTc' noise of the resistors in the voltage divider are the major causes for noise within an ADC.

For example, suppose we had a perfectly constant signal voltage of +1.7500000 volts. Suppose that the corresponding data word generated by the ADC was the 8-bit word +0010000 because the reference voltage was 2.56000 volts. The resolution of the ADC is $2.56 \text{ volts}/(2^8) = 10 \text{ millivolts}$. If the reference voltage dipped momentarily to 2.54 volts, the resulting data word would now become 1-bit less or $(+0010000) - 0000001 = 0001111$. In order for the ADC to produce a stable output in the face of a constant signal, the input voltage and other sources of noise in the electronics cannot exceed 10 millivolts.

Quantization noise is the term used to describe how well an ADC can represent an analog signal and is related to the step size (1 LSB) according to

$$\sigma = \frac{LSB}{\sqrt{12}}$$

For example, if you have a 16-bit ADC that is digitizing a signal representing the strength of a magnetic field from -40,000 to +40,000 μTeslas , it will use 15-bits to represent 40,000 μT for a resolution of $40,000 \mu\text{T}/2^{15} = 40,000/32768 = 1.22 \mu\text{T/bit}$. The quantization noise is then $1.22/(12)^{1/2} = \pm 0.35 \mu\text{T}$.

What this means is that the minimum magnetic field strength you can detect is a factor of $1.22/0.35 = 3.5$ times the quantization noise, which is the minimum precision you can expect to get with this 16-bit ADC.

Problem 8 - Calculating Quantization Noise. You are designing an app that will be measuring atmospheric pressure, based upon the pressure sensor that came with the iPhone 6s. The output of the sensor is a 14-bit unsigned word and the range of pressures encoded varies from zero to 100000 Pascals (1000 milliBars). What is the smallest pressure change you can detect with this system? How many significant figures should there be in your display?

Answer: Resolution = $100000 \text{ Pa}/2^{14} = 100000 \text{ Pa}/16384 = 6.0 \text{ Pa/bit}$. Then $\sigma = 6 \text{ Pa}/(12)^{1/2} = \pm 1.7 \text{ Pa}$. Because the minimum noise is close to 2 Pa, you only need 6 significant figures for which the least significant number will be varying randomly by $\pm 1.7 \text{ Pa}$. There is no need to include decimal values for the barometric pressure measured in Pascals, which are the smallest unit of pressure commonly used. Note: 1000 milliBars (mB) = 100,000 Pa. Typical meteorological variations are in the range from 900 mB to 1100 mB.

How Smartphones Corrupt Data

Smartphones are perhaps one of the most electrically noisy platforms for hosting sensors that one can imagine. Figure 97 shows the interior of a Samsung Galaxy 5s. There is a large battery (Item 8) with flowing currents and a powerful RF transceiver (Item 12 and 28) all crammed into proximity with a variety of sensors trying to make precise measurements of acceleration, spin, magnetism, pressure, temperature and other quantities. Analog-to-Digital Converters used in all of the sensor systems are also notoriously temperature-sensitive.

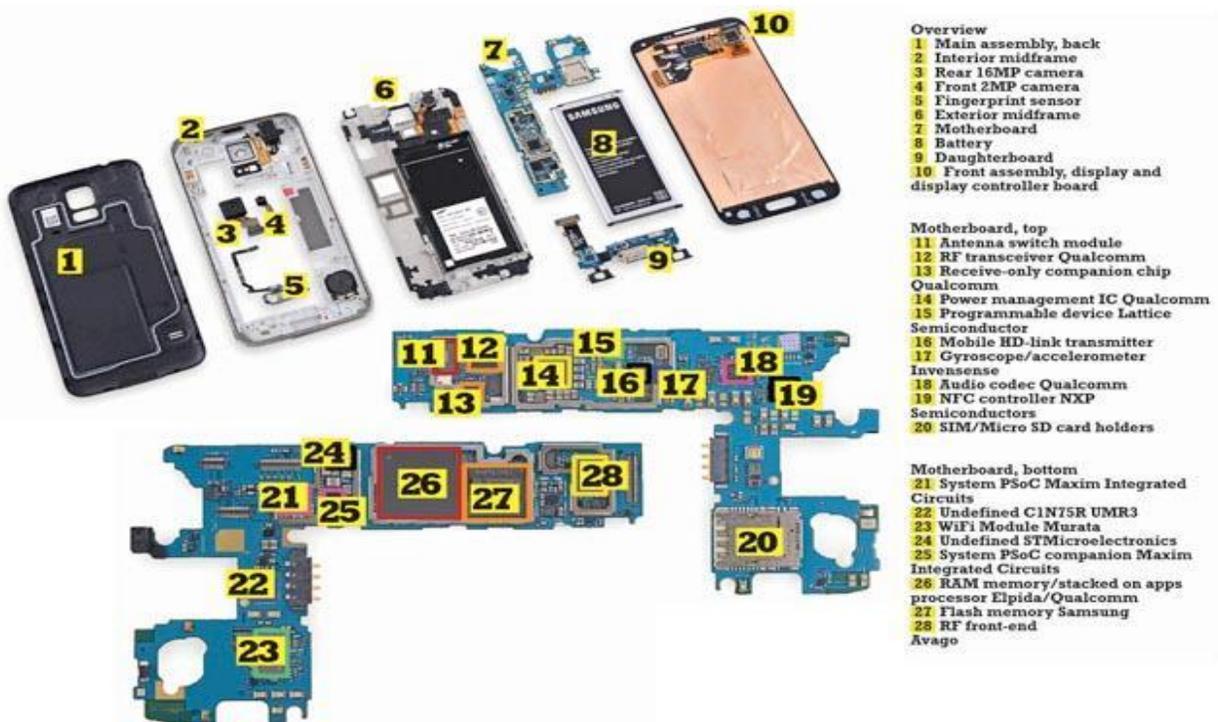


Figure 97 – Inside a typical smartphone. Image credit:

One of the worst sources of data corruption is in the complex thermal environment of the interior of a smartphone as shown in this thermal picture.

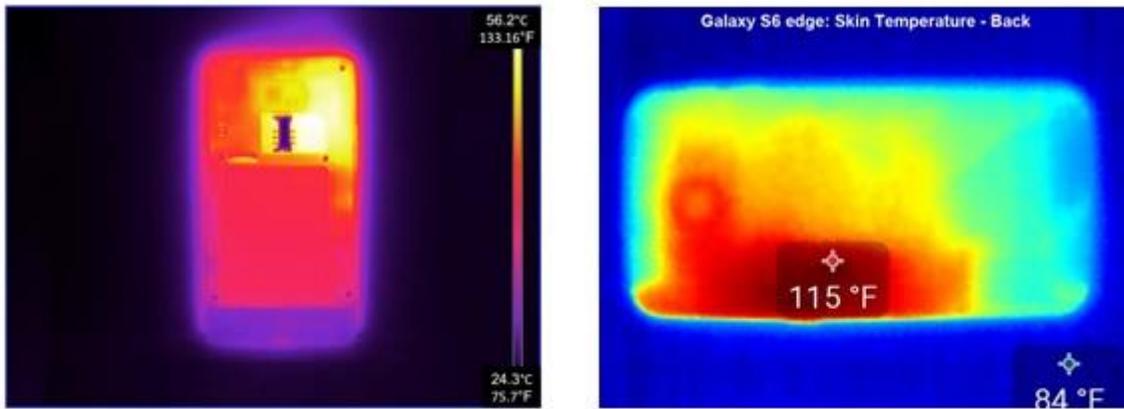
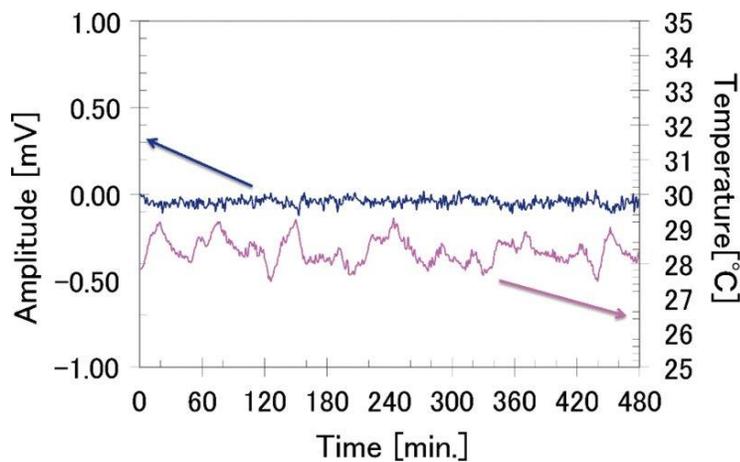


Figure 98 – Infrared images of smartphones



Because many of the sensors are electro-mechanical, minute changes in the thermal expansion and contraction of these MEMS devices can result in measurement error at the front-end of the measuring process. In addition, ADC devices are all subject to thermal drift which can cause voltage levels to change and thereby interfere with the reliable conversion of input signals into output digital words.

Figure 99 - Voltage conversion drift of the ADC and the environmental temperature variation, as a function of time. The thermal voltage drift of the ADC output is less than 0.2 millivolts/K.

Electrical Glitches

Electrical measuring devices are generally known to be subject to both internal and external conditions that can directly affect the quality of the data they produce. With uncorrected instrumental artifacts, the measurement error is limited by the white-noise floor set by the magnitude and frequency of the artifacts.

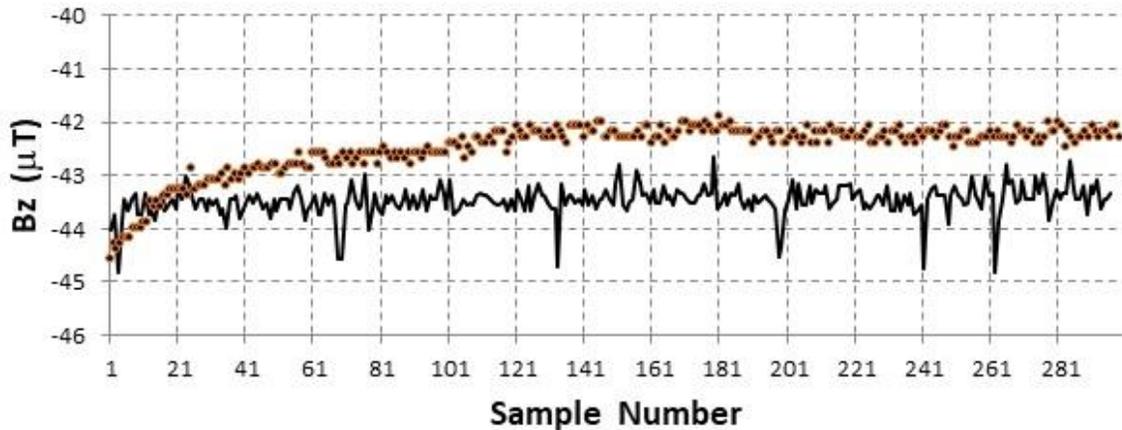


Figure 100 – Glitch artifacts seen in Samsung data (solid line) but not iPhone data (dots).

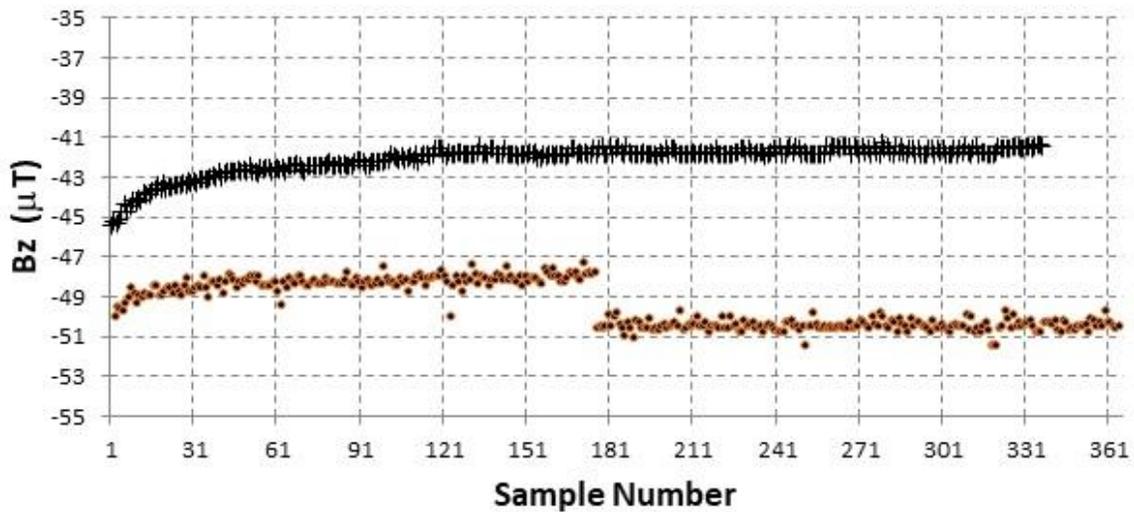


Figure 101 – Sudden DC shift artifact seen in side-by-side tests of the iPhone and Samsung platforms. Samsung data (dots), iPhone data (crosses).

Both smartphones were placed on an indoor, leveled tabletop at room-temperature conditions, with no metallic items within two meters of their positions. *Tesla Recorder* was allowed to take data uninterrupted for four hours. The two significant kinds of artifacts are shown in Figure 101 and include sudden ‘glitches’ that last for a single sample period (30 seconds), as well as longer-term DC shifts that can last for hours at a time. Glitches typically have small amplitudes of order $\pm 1 \mu\text{T}$ and are sufficiently few in number that they will rapidly ‘average out’. The DC shifts are far more troubling with amplitudes of 2 to 5 μT . The transition, as for the glitches, is very abrupt, occurring at the 30-second sample timescale. DC shifts are a significant contributor to elevating the data noise well above the nominal ADC level of $\pm 200 \text{ nT}$.

The glitches are not random in time and do not correspond to fixed time (ADC clocking) intervals or represent fixed amounts of data (buffering). On occasion, the largest of these glitches leads to an actual DC jump that remains constant until the next severe glitch or until the app is turned off. Similar jumps and glitches are known to be a problem with some ADCs in which the input analog data is not properly filtered (Engineer Zone, 2012, 2018). It is possible that the Samsung platform is less well-regulated than the iPhone platform, but the details are not in the public domain.

Problem 9 – Glitches and Drifts Oh My! Based on the data shown in Figure 10, what is the most common glitch interval in the Samsung data if the samples were obtained at intervals of 30 seconds?

Answer: The plot shows glitches at Samples 69, 133, 198, 241 and 262. Between Glitch 1-2, 2-3 and 3-5 the intervals are 64, 65 and 64 for an average of 64 samples or $64 \times 0.5 = 32$ seconds.

Problem 10 – Investigating glitch causes – Compare the simultaneous data in Figure 11 for the iPhone 6s and the Samsung Note 5. What can you deduce about the cause of the sudden change at sample 176-177?

Answer: These data series were made at the same time and so to find the glitch in one of the phone data sequences and not the other implies that the cause of the glitch was not from an external event to the smartphone. The glitch must be the result of some internal electrical change in the Samsung Note 5 during the course of the data-taking operation.

Other Kinds of Variations

Two smartphones, a Samsung Note 5 and an iPhone 6s, were placed on an indoor, leveled tabletop with no metallic items within two meters of their positions. *Tesla Recorder* was allowed to take magnetic data uninterrupted for two simultaneous sessions of one hour each under room temperature (73°F) and geomagnetically calm ($K < 3$) conditions. The batteries were fully charged at the start of the data-logging, and the results are shown in Figure 102.

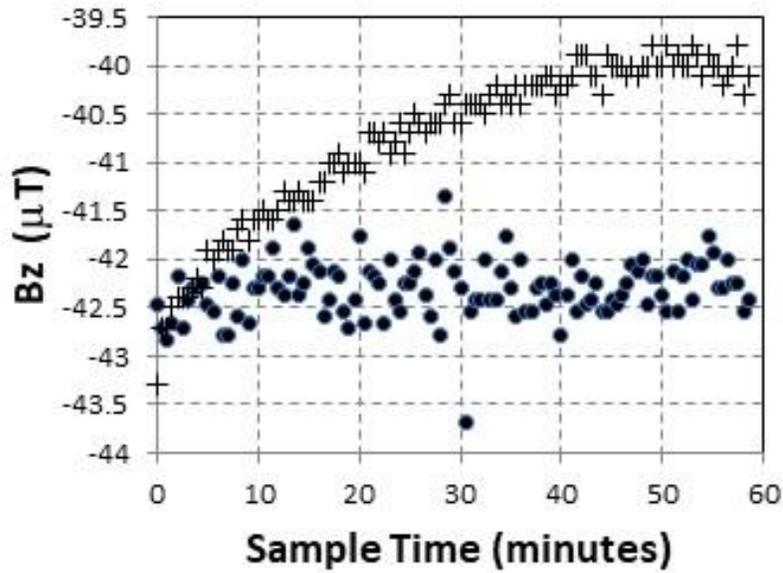


Figure 102 Comparison of Bz geomagnetic field data: Samsung data (dots) and iPhone data (crosses).

For the iPhone platform, following power-up and once data-taking commences, the magnetometer readout does not instantaneously record the value of each field component but requires a prolonged period of time before the values stabilize. For the Bx and By components, this happens comparatively quickly within 30 samples (15 minutes) but for the Bz component, this relaxation process takes up to one hour before the reported value is within ± 200 nT of its final, mean value. This long time constant in the component measurement process is problematical because the difference between a 'quick look' measurement after a few samples can be as much as 5 to 10 μ T different than the value obtained after the process reaches its final, stable, value. This asymptotic process for the iPhone is repeatable, and a feature of all measurements made with this platform. The Samsung phone, in rather stark contrast, shows no such effect and reaches its asymptotic, and stable, value within a few minutes.

Problem 11 – When is data not valid? – According to the data in Figure 102, when would be the most reliable time to log the measurement of the magnetic field intensity?

Answer: The only time would be within the first few minutes of operating the app because at other times the iPhone values drift significantly away from the Samsung values and lead to inconsistent answers between the two platforms.

Accelerometer

The data from a series of measurements show that the Samsung phone displayed in Figure 103 suffers from complex changes. The Samsung platform metrology appears to include a considerable number of short-term ‘glitches’ with amplitudes of up to 0.03 m/sec^2 , a frequency of about two samples per minute, and also appear to be semi-periodic.

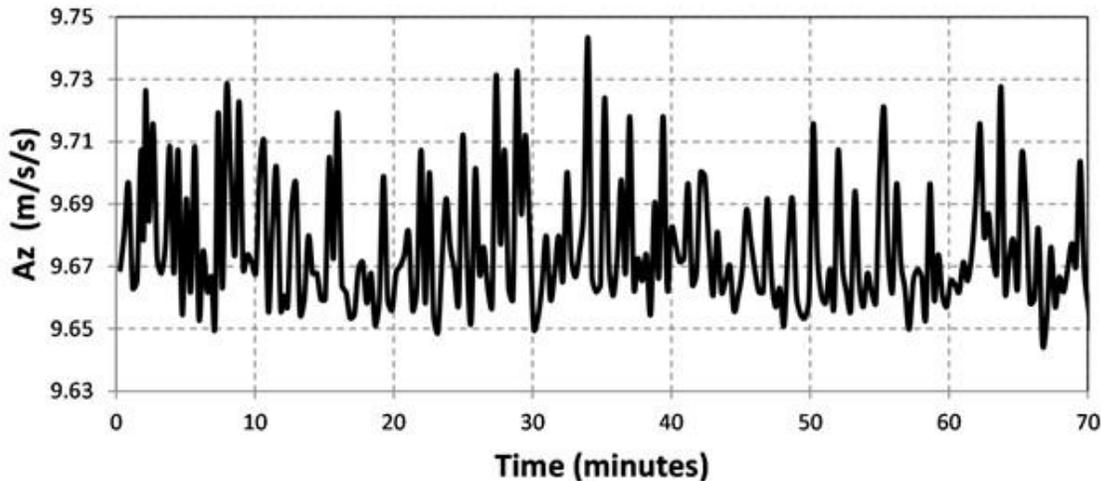


Figure 103 – Variations in smartphone accelerometer data output.

The changing internal temperature of a smartphone can also be a factor influencing measurement. An investigation of bias changes due to temperature by Kos, Tomazik and Umek (2016) demonstrates that bias changes by up to 0.7 mGs (e.g. 0.007 m/s^2) in 20 minutes are not uncommon. The effect is variously caused by power dissipation from the CPU, GPS units, screen and other devices, which increase with time as the smartphone operates.

The data-logging *Sensor Kinetics* app was tested on an iPhone and Samsung platform to gauge the stability of the local measurement. The smartphones were placed side-by-side on a table under near-room-temperature conditions measured with the *OmniTemp* app and an external sensor. The data was sampled for 10 minutes at 19 Hz. During this time, the battery charge declines were only 3%. We see in Figure 104 that, on average, the smartphones both display a statistically constant reading throughout the temperature range, however, even over a 10-minute measuring window, the bias can vary by as much as 0.05 m/s^2 so data-taking over short timescales is favored over running the app continuously for minutes at a time.

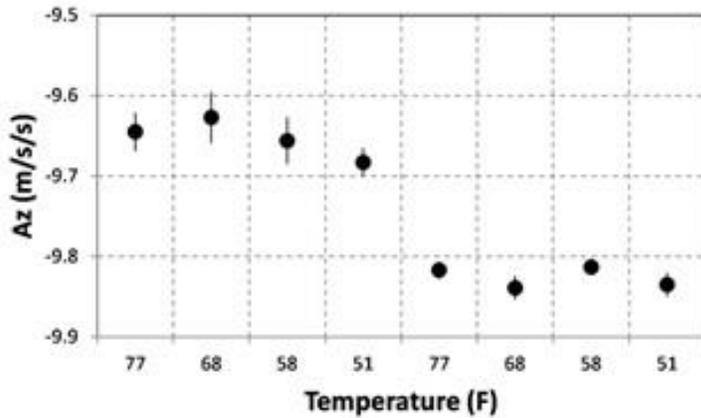


Figure 104 Acceleration measurement versus temperature.

Another discovery was made with iPads. When the measurements are displayed according to time, a clear correlation is shown in Figure 105. This is presumably a temperature effect.

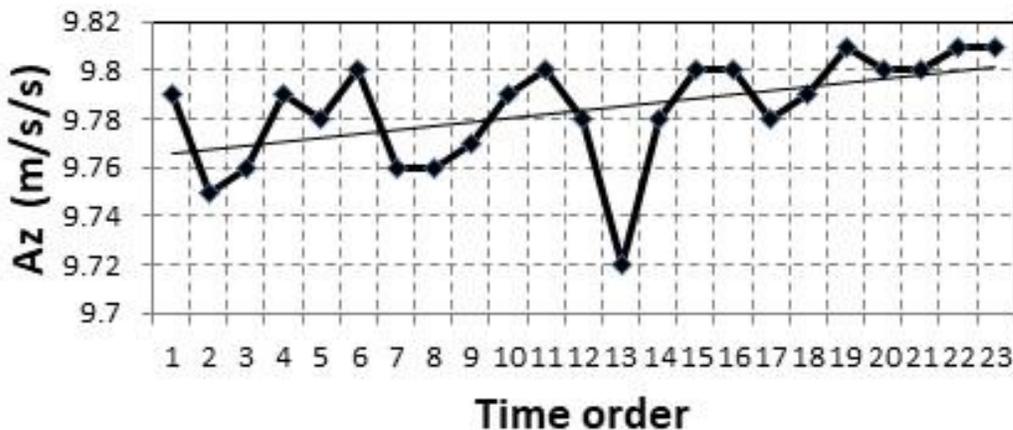


Figure 105: Time ordering of iPad acceleration measurements displaying a systematic drift of 0.02 m/s² in 90 minutes.

Problem 12 – Peak to Peak Variations – In Figure 103, what is the maximum variation between the highest and lowest measurement change between samples?

Answer: This occurs at a time of about 35 seconds from the start of the measuring process where the maximum value is about 9.75 and the minimum value is about 9.66 for a difference of 0.09 m/sec².

How the Environment Corrupts Data

Local magnetic changes

Below in Figure 106 and 107 are a typical series of data points measured during November 2016 from the same location with no obvious sources of environmental interference. Although the individual magnetic measurements are precise to $\pm 0.2 \mu\text{Teslas}$, it is clear that there are a variety of unknown sources of variation that make the analysis of this data challenging.

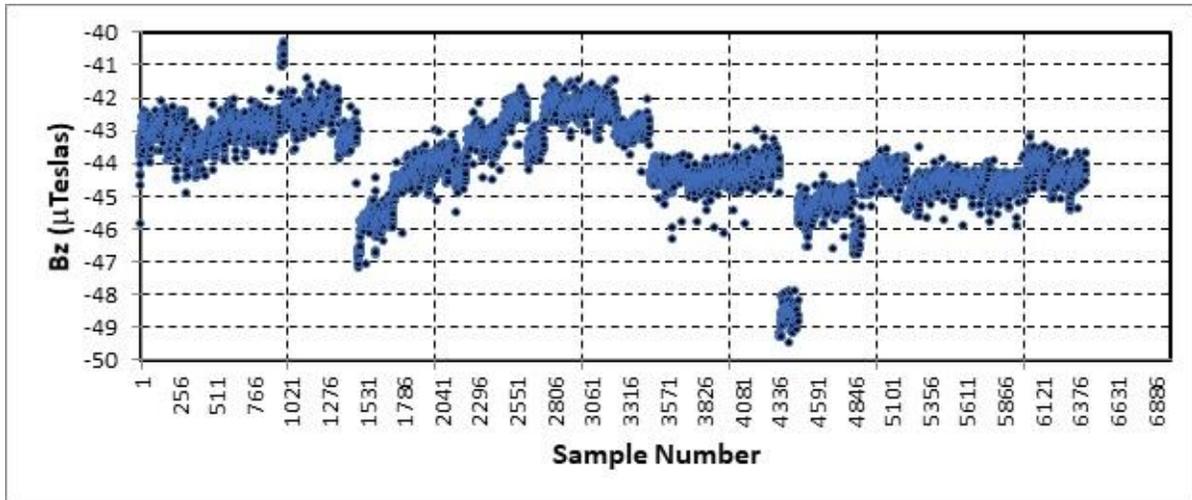


Figure 106. A series of magnetometer measurements spanning six hours

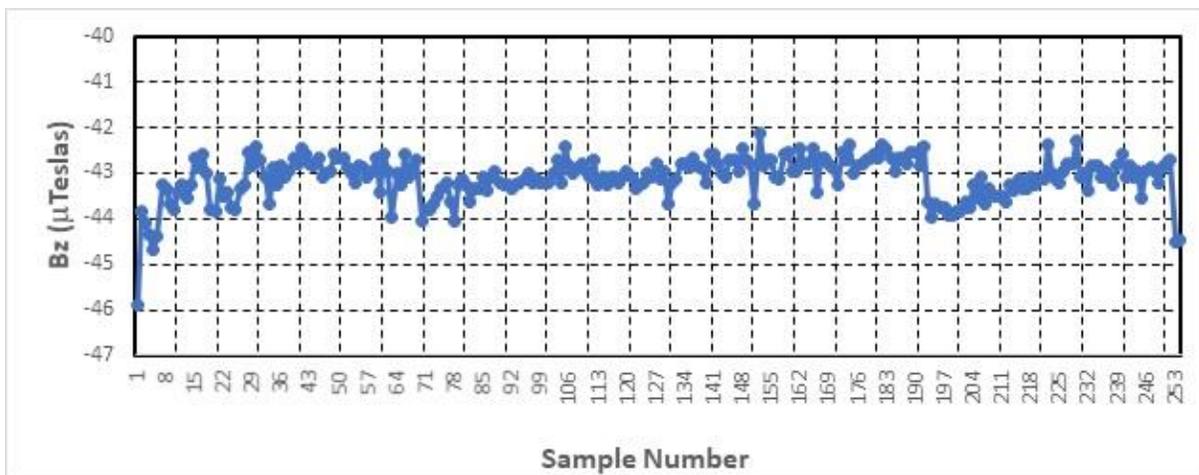


Figure 107. A shorter series of data spanning three hours shows a different series of changes including sudden 'glitches' at Samples 73 and 194.

Problem 13 – When not to ‘go long’ – Comparing Figures 106 and 107, which were taken at a rate of one sample every second, what would be the longest period of time over which the measurement values show the least change?

Answer: Figure 106 appears to show a variety of changes in Bz for measurements more than about 100 samples (1.4 minutes) long, however Figure 107 shows that very constant measurement values can be obtained over 20 to 30 samples (up to 1 minute).

Barometric changes due to weather

Barometric data from smartphones can display what appear to be oscillations on both long and short timescales. Although the average pressure reading in Figure 108 might indicate 999.94 milliBars, the measurement must contend with large swings by as much as ± 0.07 milliBars over a span of an hour.

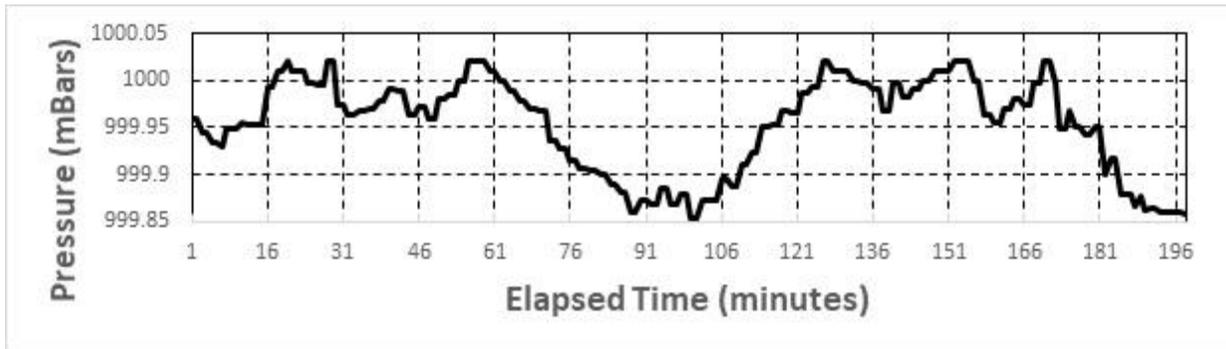


Figure 108. Barometer data showing short-term changes

On longer timescales, slow variations appear in response to the inevitably changing atmospheric conditions. (Note 100 pA = 1 milliBars so the graph in Figure 109 covers a range from 1007 to 1014 mB)

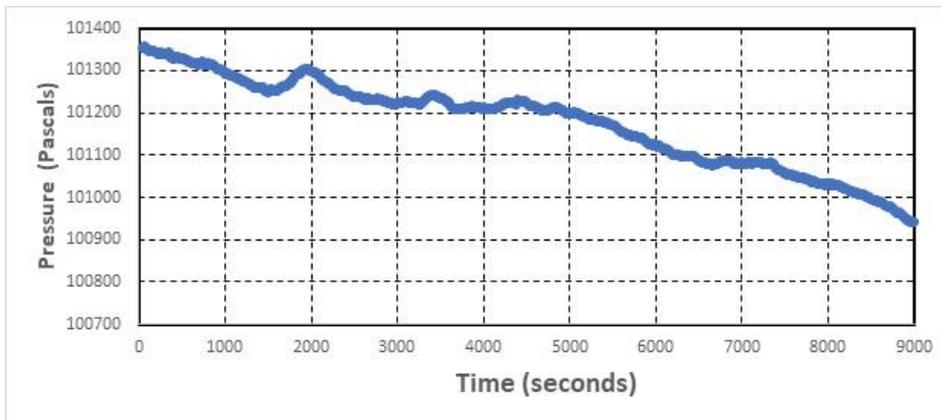


Figure 109. Barometer measurements showing long-term changes.

A slow and constant increase is a sign for an improvement of the weather conditions as a high-pressure system approaches. A constant decline in pressure indicates an approaching low-pressure center and bad weather. A rapid increase (more than 100 Pa/hour) can be a sign for strong winds ("high pressure storm") lasting perhaps several days. Air pressure also changes according to a semicircadian (12 hr) rhythm and a superimposed circadian (24 hr) rhythm with changes up to 500 Pa.

NOAA Weather Service Pressure Terminology

- Falling (or rising) slowly: 10 - 150 Pa in 3 hours
- Falling (or rising): 160 - 350 Pa in 3 hours
- Falling (or rising) quickly: 360 - 600 Pa in 3 hours
- Falling (or rising) very rapidly: > 600 Pa in 3 hours

This data is from the *Weather Handbook* by Alan Watts (2005).

Pressure Change and Wind Speed

- 100 Pa/hour: likely Force 6
- 200 Pa/hour: likely Force 7 - 8
- 300 Pa/hour: likely gale exceeding Force 8

Table 27 – Pressure changes and weather conditions

Pressure change	Character	Conditions
1016 to 1023	Steady	Fair and slight temperature changes for a few days
1016 to 1023	Rising rapidly	Fair followed in 2 days by warmer temps and rain
1016 to 1023	Falling rapidly	Warmer and rain within 24 hours
1023 or above	Falling rapidly	Warmer and rain within 36 hours
1023 or above	Variable	No early changes
1016 or below	Falling slowly	Rain in 18 hours continues for a few days
1016 or below	Falling rapidly	Rain and high winds followed by cold/clear
1016 or below	Rising	Clearing and cold within 12 hours
1009 or below	Falling rapidly	Severe storm of wind and rain in 24 hours
1009 or below	Rising rapidly	Clearing and colder

The rate at which barometric pressure changes is a good indicator of whether your measurements are sensible or not. In Figure 108 near the measurement at 16 minutes, the pressure

changed from 999.96 to 999.99 millibars in just 2 minutes. That would be a rate of 0.9 millibars/hour or 900 Pa/hour suggesting gale-force winds, which is not sensible. By comparison, Hurricane Wilma dropped from 982 millibars to 882 millibars in 24 hours, or a rate of 4.2 millibars an hour. Hurricane Gilbert dropped at 3 millibars an hour over 24 hours. Hurricane Beulah's drop of 6.3 millibars an hour in six hours in 1967.

Problem 14 - Pressure and Weather - In Figure 110 the pressure measured by a smartphone sensor is declining. From the rate of decline over the sampling period and the pressure and wind speed weather tables, what is your forecast?

Answer: The measured pressure starts out at 101350 Pa and declines to 100950 Pa, so this means that bad weather is approaching as the current high-pressure, fair-weather conditions are replaced by low-pressure, stormy conditions. The rate of decline is $(101350-100950)/9000 \text{ seconds} = 400 \text{ Pa}/2.5 \text{ hrs}$ or 160 Pa/hr . Table 4 is in 3-hour increments so the smartphone seems to indicate a rate of $dP/dt = 53 \text{ Pa}/3\text{-hours}$. This would be classified as a 'slowly-falling' barometric pressure by the NOAA Weather Service. The resulting winds would be between Force 6 and Force 7.

In Figure 110 we see a continuous record of barometric measurements made between 8:00 am November 15 during a snow storm, through 10:00 am, November 16 during clear sky conditions. The data was taken with a Samsung Galaxy 8s phone running the Physics Toolbox app on the 'barometer' feature. The slowest data rate of 5 Hz was selected and files were downloaded every 8 hours and processed with MS Excel. The battery drain was 5% per hour. The snowfall ended at Noon on November 15 (running time 240 minutes) followed by steady rainfall until 3:00 PM (running time 420 minutes) followed by cloudy conditions until sunset (running time 600 minutes). Sunrise occurred at 6:50 am (running time ca 1380) with clear blue skies thereafter.

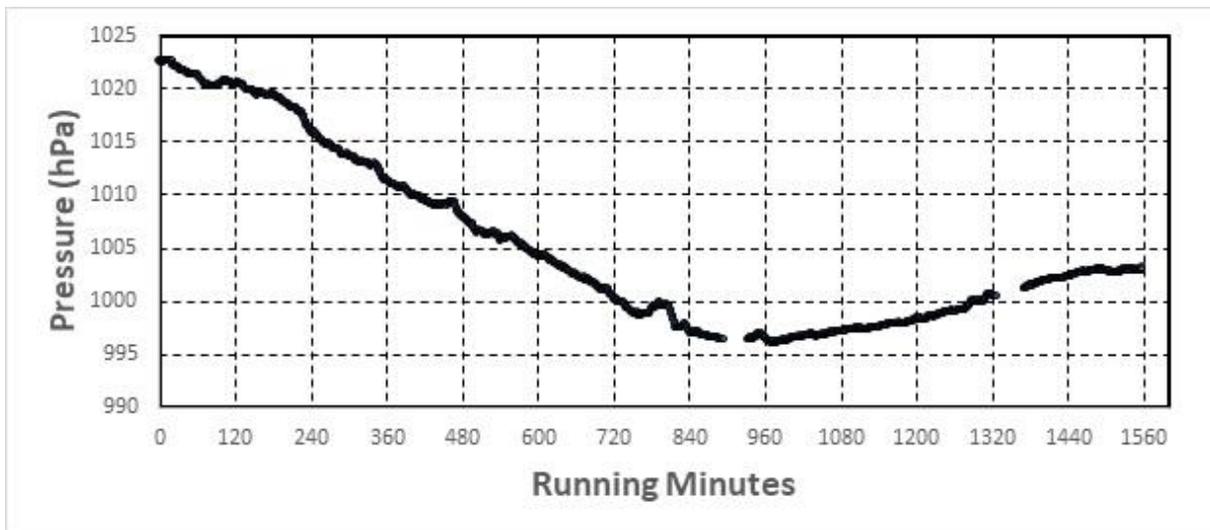


Figure 110. Continuous record of barometric readings November 15-16, 2018 during transition from snow storm to clear skies using Samsung Galaxy 8s and the *Physics Toolbox* app.

The Zen of Data Analysis

Smartphone data can come in many forms but we now have to think critically about what the numbers mean. This is a process called ‘data analysis’ in which we work with numbers in the data to extract useful information such as trends and patterns. This information, in turn, will help us understand something about what we are measuring, perhaps even leading to a new discovery!

Scaling and Rounding

Numbers can be used for counting the things you have in a set or group. They can also be used to represent physical dimensions such as length, temperature, mass, speed. To understand how to work with data, we can begin with the very intuitive case of studying the nearest exoplanets to our sun as of 2018, out to 11.7 light years in Table 28:

Table 28 – Exoplanets among near-by Stars

	Star	Exoplanets		Star	Exoplanets
1	Proxima Centauri	1	18	Lacaille 9352	0
2	Alpha Centauri A	0	19	EZ Aquarii A	0
3	Alpha Centauri B	0	20	EZ Aquarii B	0
4	Barnard’s Star	1	21	EZ Aquarii C	0
5	Luhman 16A	0	22	61 Cygni A	0
6	Luhman 16B	0	23	61 Cygni B	0
7	WISE 0855	0	24	Procyon A	0
8	Wolf 359	0	25	Procyon B	0
9	Lalande 21185	0	26	Struve 2398 A	0
10	Sirius A	0	27	Struve 2398 B	0
11	Sirius B	0	28	Groombridge 34 A	0
12	Luyten 726-8A	0	29	Groombridge 34 B	0
13	Luyten 726-8B	0	30	DX Cancri	0
14	Epsilon Eridani	1	31	Tau Ceti	4
15	Ross 154	0			
16	Ross 248	0			
17	Ross 128	1			

Problem 15 – Scaling from one set to another. From Table 28, how many exoplanets would you predict that could exist in a small star cluster containing 200 stars?

Answer: The table suggests that for 31 stars one might expect to find 8 exoplanets, so if this sample is typical of the stars in the star cluster, we might expect to find $N = 200 \text{ stars} \times (8 \text{ exoplanets}/31 \text{ stars}) = 51.61 \text{ exoplanets}$.

The answer we got in Problem 15 was 51.61 exoplanets, but of course there is no such thing as 0.61 exoplanets. That means we have a choice in making this answer sensible. This requires that we either round-down our answer to 51 exoplanets, or round it up to 52 exoplanets. The established convention for numbers in the range from 0.00 to 0.49 is to round them down to 0, while numbers between 0.50 and 0.99 are rounded-up to 1.0, so in our problem, we are entirely justified in rounding the answer to 52 exoplanets!

Problem 16 – Scaling and Rounding – Suppose that for Problem 15 we had a star cluster with 1000 members. How many exoplanets would you expect to find?

Answer: $N = 1000 \text{ stars} \times (8 \text{ exoplanets}/31 \text{ exoplanets}) = 258.06 \text{ exoplanets}$ or after rounding we get 258 exoplanets.

Do we have a problem that involves significant figures? No. The specific star cluster was counted to have exactly 1000 stars. The number of stars in Table 28 is 31 and the number of exoplanets is 8 which are also known to the same number of SF as the star cluster, so the answer of 258 exoplanets stated to only 3 SF is an accurate representation.

Statistical Analysis

Computing Mean, Median, Mode

Simply put, if you make a series of measurements of the same thing and get 7 values, $m_1=A$, $m_2=D$, $m_3=C$, $m_4=B$, $m_5=D$, $m_6=F$, $m_7=G$, then:

Mean: The ‘average’ value $\langle M \rangle = (A+B+C+D+D+F+G)/7$. Mathematically, this is written as

$$\langle M \rangle = \frac{\sum_1^N m_i}{N}$$

Median: is the value, you get if you rank the seven values from largest to smallest and select the fourth value, $m_4=B$, where half of the data values are greater than this value and half are smaller.

Mode: is the most frequent value in the series, in this case ‘D’ because it appears twice.

Example, the smartphone magnetometer gives this series of $N=11$ measurements in units of μ Teslas in Table 29

Table 29 – Ranked data

Bz	Ranked
-42.96	-42.42
-43.02	-42.54
-42.42	-42.66
-42.84	-42.66
-42.66	-42.66
-42.66	-42.78
-43.32	-42.78
-42.54	-42.84
-42.78	-42.96
-42.66	-43.02
-42.78	-43.32

Mean: From the first column in Table 29, the average $\langle M \rangle = -470.64/11 = -42.7855 \mu\text{T}$ but since the numbers only have 4 significant figures (SF) the answer is $\langle M \rangle = -42.79 \mu\text{Teslas}$.

Median: Then second column above shows these numbers ranked from largest to smallest. There are 11 numbers so the median is in row 7 or $m_7 = -42.78 \mu\text{Teslas}$.

Mode: The most frequent number in the list is $m_5 = m_6 = -42.66 \mu\text{Teslas}$.

Which one of these do you use if you want to describe this set of measurements? This is a judgement call that depends on what you want to emphasize. For example, an economist wants to know the ‘typical’ income of a room-full of people. They all have exactly the same income of \$60,000 per year so the mean=median=mode. Then a millionaire enters the room and the economist recalculates the numbers to find that the median is \$60,000 and the mode is \$60,000 but the mean has now jumped to \$90,000. She decides that the median/mode is the best representative of the typical income because it more closely reflect the salaries that the majority of the people will receive. The average person in the room has not just received a \$30,000 raise, after all! In the case of the smartphone magnetometer measurements, the mean and median are nearly identical so $-42.78 \mu\text{T}$ is just as good a descriptor of the data as $-42.79 \mu\text{T}$.

For smartphone data, which can undergo rapid changes due to the environment or the smartphone electronics, the issue of mean, median and mode is equally important when you are trying to make an accurate measurement of some process. This is especially true when the data suffers from random glitches and slowly varies because of environmental weather or temperature changes. Figure 111 is a graph of a smartphone magnetometer output and the corresponding data table.

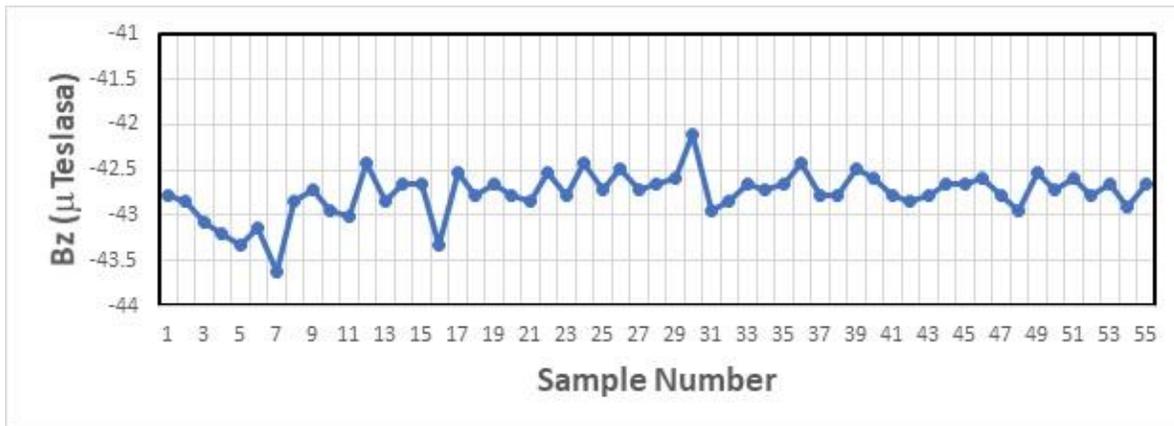


Figure 111. Example of 54 consecutive magnetometer measurements

Table 30 – Tabulated data for Figure 111

C1	C2	C3	C4	C5	C6
-42.78	-42.96	-42.66	-42.66	-42.78	-42.6
-42.84	-43.02	-42.78	-42.6	-42.78	-42.78
-43.08	-42.42	-42.84	-42.12	-42.48	-42.96
-43.2	-42.84	-42.54	-42.96	-42.6	-42.54
-43.32	-42.66	-42.78	-42.84	-42.78	-42.72
-43.14	-42.66	-42.42	-42.66	-42.84	-42.6
-43.62	-43.32	-42.72	-42.72	-42.78	-42.78
-42.84	-42.54	-42.48	-42.66	-42.66	-42.66
-42.72	-42.78	-42.72	-42.42	-42.66	-42.9

The 54 data points are grouped into 6 columns and 9 rows in Table 30, because when computing the mean $\langle M \rangle$ you can use the associative law to group and add the 54 points in any convenient manner. Example: $(A+B+C+D) + (E+F+G+H) = K$ but so does $(A+B) + (C+D) + (E+F) + (G+H) = K$

Problem 17 – Calculating Means with the Associative Law – It is a common practice to bin a collection of data points and calculate the mean/median/mode values for each bin. This allows errant data to be identified and removed to improve the accuracy of the derived statistical values. Calculate the median values for the points in columns 1-6 in Table 30, and the median for all 54 points. Make proper allowance for significant figures!

$$\begin{aligned} \text{Answer: } \langle C1 \rangle &= -387.54/9 = -43.06 \mu\text{Teslas} & \langle C4 \rangle &= -383.64/9 = -42.62 \mu\text{Teslas} \\ \langle C2 \rangle &= -385.20/9 = -42.80 \mu\text{Teslas} & \langle C5 \rangle &= -384.36/9 = -42.71 \mu\text{Teslas} \\ \langle C3 \rangle &= -383.94/9 = -42.66 \mu\text{Teslas} & \langle C6 \rangle &= -384.54/9 = -42.73 \mu\text{Teslas} \end{aligned}$$

$$\text{Mean for all points } \langle M_{54} \rangle = -42.76 \mu\text{Teslas}$$

Median for all 54 points would be the value in sample 27 which is $-42.72 \mu\text{T}$.

So, the initial analysis of the magnetometer data shown in Figure111 indicates that the most reliable mean for the series is $\langle M \rangle = -42.72 \mu\text{Teslas}$. We have not considered the initial set of nine

data points in column C1 in Table 30, which appeared to be rather low in value compared to later samples.

Data Histogramming and Median Filtering

Another way to represent the data is to histogram the values. Take the individual data numbers and sort them from smallest to largest. For digital data, count the number of times the same value occurs in the sequence as shown in Table 8. Graph in Figure 112 the result as a bar graph in terms of the rank and number of values at each rank.

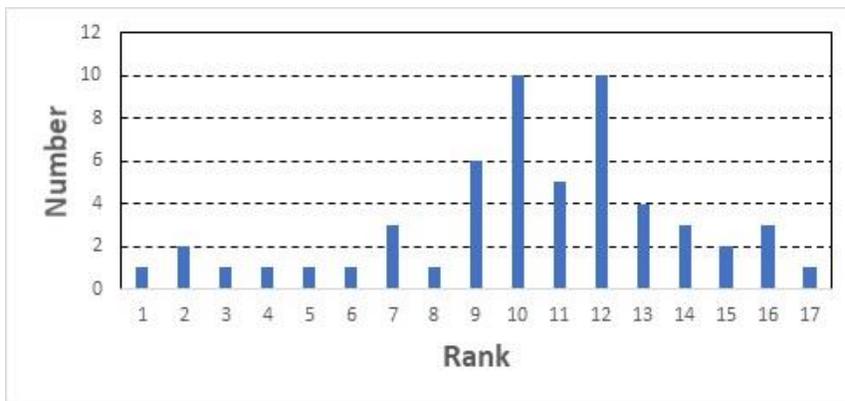


Figure 112. Bar graph of tabulated data

Table 31 – Ranked data

Rank	Value	N	Rank	Value	N
1	-43.62	1	10	-42.78	10
2	-43.32	2	11	-42.72	5
3	-43.2	1	12	-42.66	10
4	-43.14	1	13	-42.6	4
5	-43.08	1	14	-42.54	3
6	-43.02	1	15	-42.48	2
7	-42.96	3	16	-42.42	3
8	-42.9	1	17	-42.12	1
9	-42.84	6			

When we look at all N=54 data values in rank-order, the 27th value is the median -42.78 μ Teslas (Table 31, rank 10). The average is -42.76 μ Teslas (table rank '10.5'). The histogram distribution has a main portion between about Rank 7 and 17, and a long 'tail' from Rank 1 to Rank 6. The process of 'median filtering' removes data outliers by clipping off the tail and considering only the core where the majority of the data values are located.

Problem 18 – Median Filtering – A set of data has been median filtered so that the new distribution of data values does not include magnetic values smaller than -43.00 μ Teslas. From the data in Table 31, calculate the new mean and median for the sample.

Answer: The data dropped is shaded in gray. The new median will occur at Rank 12 with a value of -42.66 μ Teslas. The new average will be involve N= 3+1+6+10+5+10+4+3+2+3+1 = 48 data samples $\langle M \rangle = (3(-42.96)+1(-42.9)+6(-42.84)+10(-42.78)+5(-42.72)+10(-42.66)+4(-42.6)+3(-42.54)+2(-42.48)+3(-42.42)+1(-42.12))/48 = -42.69 \mu$ Teslas.

From our answer to Problem 18, and looking at the graph in Figure 111, this median filtering technique has eliminated all of the points with values smaller than -43.00 μ Teslas measured during the first 17 samples of the 54-sample measurement process, which appeared to be systematically lower than the other points. Removing this tail by median filtering results in an improved median of $\langle M \rangle = -42.69 \mu$ T.

Another method of analyzing data is to bin the values to reduce the noise in a sample. If the measurements are dominated by random measurement fluctuations, then the binning process smooths the data and reduces the variation by $1/\sqrt{N}$. For instance, if you have 45 samples and bin them into 5 bins each with 9 samples, the computed average value in each bin will have its variation reduced by $1/\sqrt{9}$ or a factor of 3. Binning the data in this way may let you see a trend in the average values of the five bins that is much easier to discern with a sample noise reduced by a factor of 3.

Problem 19 – Binning data to remove outliers

The data in Figure 111 can be grouped into six consecutive nine-sample bins corresponding to the columns in Table 30. A) Based on the column/bin averages $\langle C1 \rangle$ to $\langle C6 \rangle$, what is the mean and median of these bins? B) A common statistical method is to remove pairs of outliers that may have significantly different values than the mean value. Calculating the mean of the resulting reduced set of points usually produces a more accurate value for the mean of the sample. Identify two outliers on either side of the initial mean value and re-calculate the mean and median for the sample.

Answer: A) Mean: $\langle C \rangle = (-43.06 - 42.80 - 42.66 - 42.62 - 42.71 - 42.73)/6 = -42.76 \mu$ Teslas

Median: = $(-42.71 - 42.73)/2 = -42.72 \mu$ Teslas

B) The outliers (largest and smallest values) are $\langle C1 \rangle = -43.06$ and $\langle C4 \rangle = -42.62$ so that

$$\langle C \rangle = (-42.80 - 42.66 - 42.71 - 42.73)/4 = -42.72 \text{ mTeslas}$$

$$\text{Median} = (-42.71 - 42.73)/2 = -42.72 \text{ } \mu\text{Teslas}$$

So what should we record as the ‘final’ value for the magnetic field measured by the smartphone during this period? Our best answer from Problem 18 is $-42.69 \text{ } \mu\text{Teslas}$, and from Problem 19 we get $-42.72 \text{ } \mu\text{T}$. The average of these two methods yields $\langle M \rangle = -42.71 \text{ mT}$, which differs from the original values by only $\pm 0.01 \text{ } \mu\text{T}$. In each case we have controlled the data for what appear to be outliers caused by any number of environmental or measurement factors possibly not even within our control. Most measurement answers can be decided by performing one of the above methods. If the data has many known artifacts and glitches, median filtering can be very effective in removing ‘bad data’ and offering a more reliable and stable estimate.

The effect of digitizing the data

Smartphone measurements are derived from the sensor outputs after they have been processed through devices known as analog-to-digital converters (ADCs), which will affect the quality of the data. In this example, we look at the data from a smartphone pressure sensor in Figure 113.

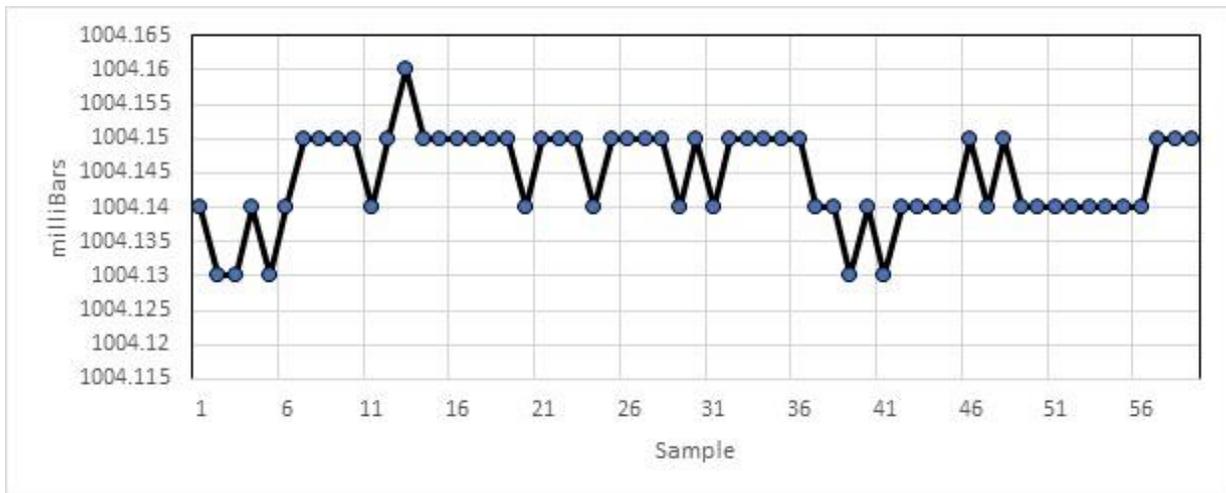


Figure 113. Pressure data showing digitization steps.

Table 32 – Data for figure 113

C1	C2	C3	C4	C5	C6
1004.14	1004.15	1004.14	1004.15	1004.14	1004.14
1004.13	1004.14	1004.15	1004.14	1004.13	1004.14
1004.13	1004.15	1004.15	1004.15	1004.14	1004.14
1004.14	1004.16	1004.15	1004.15	1004.14	1004.14
1004.13	1004.15	1004.14	1004.15	1004.14	1004.14
1004.14	1004.15	1004.15	1004.15	1004.14	1004.14
1004.15	1004.15	1004.15	1004.15	1004.15	1004.14
1004.15	1004.15	1004.15	1004.14	1004.14	1004.15
1004.15	1004.15	1004.15	1004.14	1004.15	1004.15
1004.15	1004.15	1004.14	1004.13	1004.14	1004.15

Notice in Table 32 that there are only four unique values: 1004.13, 1004.14, 1004.15, and 1004.16.

Problem 20: Exploring Digital Data - From Table 32, calculate A) the mean, median and mode. B) The minimum difference between the measurements. Remember to incorporate significant figures in your answer.

Answer: A) Mean = $60248.67/60 = 1004.145$ milliBars which becomes 1004.15 milliBars.

Median = 1004.15 milliBars. Mode = 1004.15 milliBars.

B) The minimum difference is 0.01 milliBars.

The step-like pattern you see in the data is an indication that we are seeing the individual bit values change in the data word for this pressure level. In other words, each level change corresponds to a value of one bit. From the minimum difference in the levels of 0.01 milliBars, we can deduce that the analog data for the pressure is being resolved at a level of one bit corresponding to 0.01 milliBars.

Standard Deviations and Measurement Uncertainty

Histograms and the basic properties of the mean, median and mode are concepts well known to most middle-school and high-school students, and are analysis tools frequently encountered among the ranks of science fair projects developed every year. The mean of a series of N measurements is simply

$$M = \frac{\sum_1^N m_i}{N}$$

A second statistical quantity called the standard deviation tells us about how the measurements are spread out around the mean value. This quantity is represented by the Greek letter sigma, σ , or simply as s.d. There are actually two different standard deviations: The population s.d and the sample s.d. The population s.d is given by

$$\sigma = \sqrt{\frac{\sum_1^N (m_i - M)^2}{N}}$$

and is used when you have a fixed population of N objects that are unique to your study. For instance, there were a fixed number of smartphones of a specific model ever produced and sold, or your garden has a fixed number of rose bushes. If the sample is a fixed population you use the above formula for the s.d. Usually, however, you are measuring some process, m_i , for which an unlimited number of measurements can be made. In that case you use the sample s.d given by

$$\sigma = \sqrt{\frac{\sum_1^N (m_i - M)^2}{N - 1}}$$

Note that when you are making many measurements, such as $N=100$ or $N=1000$, the difference between N and $N-1$ becomes very small so the population and sample s.d. become nearly identical. For most laboratory situations, N is usually small so it is more accurate to use the sample s.d. which we will call sigma and represent by the Greek letter, σ .

Another way to look at the sample s.d or σ , is in terms of degrees-of-freedom (DOF) of the data. When you have N data points, the DOF of the sample is equal to N when you compute the average M, but in the calculation of σ , you have subtracted the mean value, M from each data point so now you have reduced the DOF in the calculation by one, so that σ is based on N-1 DOF. Once again, if you are dealing with hundreds or thousands of sample points, the difference between N and N-1 becomes progressively less important, but when you are only dealing with a few dozen measurements, the difference can be numerically significant, and the correction has to be made. Generally, you do not know beforehand if N will be large or small for a given experiment so it is always best to use (N-1) in the calculation for σ .

Problem 21 Degrees of Freedom

A set of data is modeled with a polynomial function of degree 5 defined by

$$Y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

A) How many DOF are represented by this equation?

Answer: [a,b,c,d,e,f] = 5 DOF

B) If you have a set of data with 21 values that are fit by this equation. You calculate the σ by subtracting the model from each data point. How many DOF will be used in defining σ ? What is the equation for σ ?

Answer $21 - 5 = 16$. So the equation for σ is as follows:

$$\sigma = \sqrt{\frac{\sum_1^{21}(m_i - M)^2}{16}}$$

How do we visualize σ ? For a set of measurements, m_i , when you subtract the average value of the set, M, from each point you are removing a trend in the data leaving behind a series of numbers that represent how far away from the average a particular data value is ($m_i - M$). What the s.d does is to take each of these residuals and square them, (called the variance) and divide by the number of DOF. Then it takes the square-root of this average variance to determine the s.d of the sample per point. The reason this is an important gauge of a data set is that if the various measurements were influenced by a random measurement ‘error’ the histogram of the s.d should look like a ‘bell curve’ or Gaussian curve shown in Figure 114.

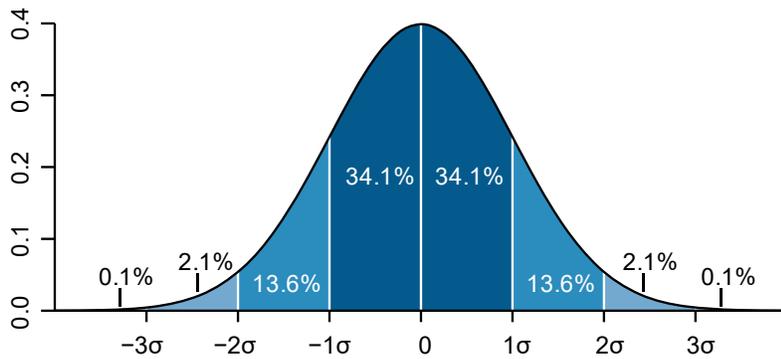


Figure 114. Definition of statistical histogram and standard deviation frequencies.

This is the histogram curve you would get when you take all of the individual measurements and subtract from them the mean value M . The horizontal axis is

the calculated residual value ($m_i - M$), and the vertical value is the fraction of the measurements with the computed value of the residual. If your measurement is only determined by random error, it should yield a value that is within $\pm 1\sigma$ of the mean value M about 64.3% of the time. It should yield a measurement value that is within $\pm 2\sigma$ about 95.4% of the time, and within $\pm 3\sigma$ about 99.7% of the time. If your measurement falls outside these limits, this is a very good indicator that your measurements are not dominated by random error but still have some additional systematic effect hiding in the measurements that have to be controlled. For example, if you subtracted only an average value from each measurement but did not subtract some correlated trend (linear, quadratic, etc) then the resulting sample will still have a residual systematic error that will contribute to σ .

Problem 22 Sigma and Significance.

Consider a series of sound measurements in the figure 115 and Table 33 below. The samples seem to follow an average level near 48.2 decibels, but between samples 17-19 the sound volume seems to take a dip to 39.2 decibels. A calculation of the σ for the samples from 1-15 yields $\sigma = 1.0$ dB.

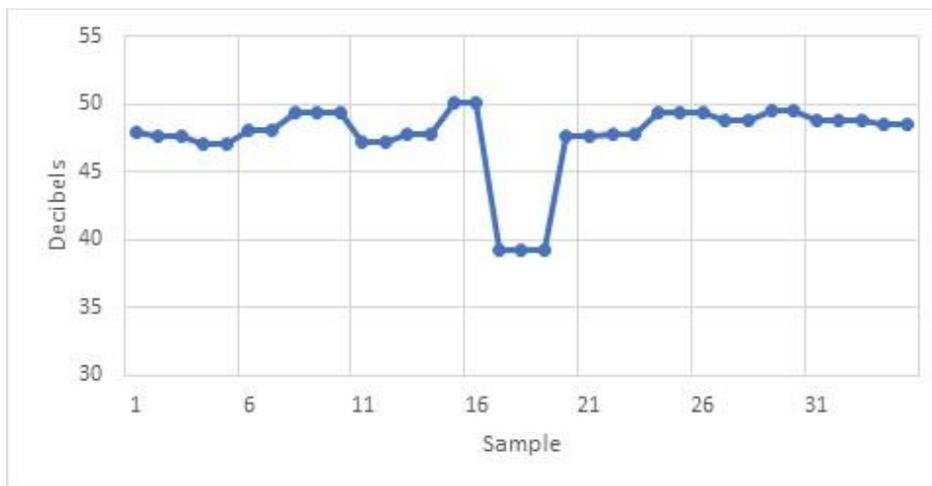


Figure 115. Sound measurement series.

Table 33 Sound data tabulated for Figure 115

47.94	47.65	47.65	47.05	47.05	48.08	48.08	49.43	49.43	49.43
47.16	47.16	47.73	47.73	50.12	50.12	39.2	39.2	39.2	47.7
47.7	47.74	47.74	49.41	49.41	49.41	48.75	48.75	49.45	49.45

- A) What is the difference between the sound level between samples 1-16 and the sound level of the event between samples 17-19? Answer: $(48.2-39.2) = 9.0$ dB.
- B) In multiples of the calculated sigma between samples 1-16, how large is the event between samples 17-19? Answer $(9.0 \text{ dB}/1.0 \text{ dB})= 9$ sigma units.
- C) A frequency table for random ‘Gaussian’ deviations from the mean shows the following
- $2-\sigma = 1$ event in 22
 - $3-\sigma = 1$ event in 370
 - $4-\sigma = 1$ event in 15,800
 - $5-\sigma = 1$ event in 1,700,000 (https://en.wikipedia.org/wiki/Standard_deviation)
- What is the frequency table given we have made 30 measurements?
 Answer: $2-\sigma = 1$ event and $3-\sigma = 0$ events,
- D) How significant do you think the measured 9-sigma event is? Answer: It is very significant and not a random event because for such a small number of samples, we expect to see only 1 event as large as $3-\sigma$. The $9-\sigma$ event is even rarer than that.

Problem 23 – Calculating M and σ .

A smartphone app was used to measure the elevation angle of an object and obtained the following measurements shown graphically in Figure 116. Then first ten measurements are also provided in the table. A) Compute the average M and the value for σ for the first ten samples. B) Draw a horizontal line on the graph showing M and the range $+2\sigma$ and -2σ . (Be careful of the significant figures!)

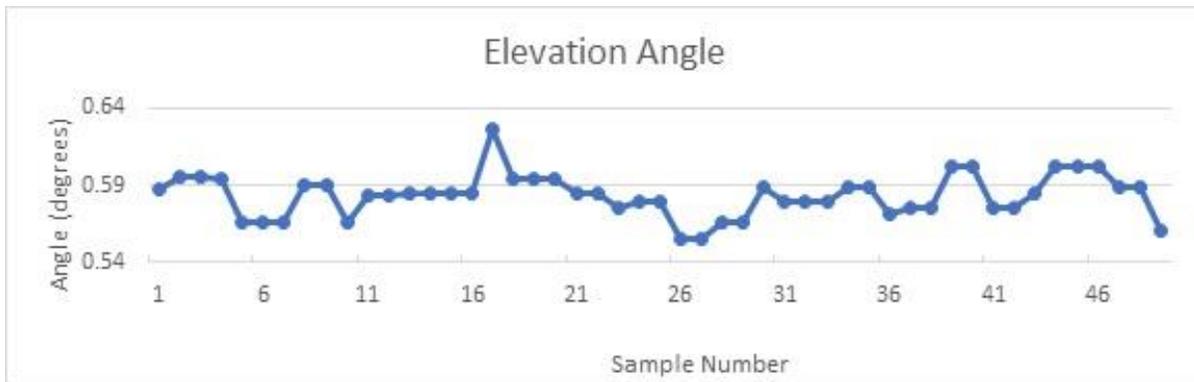


Figure 116. Elevation angle measurements

Table 34 Data for elevation angles

0.588	0.595	0.595	0.594	0.566	0.566	0.566	0.59	0.59	0.566
-------	-------	-------	-------	-------	-------	-------	------	------	-------

A) $M = (0.588+0.595+0.595+0.594+0.566+0.566+0.566+0.590+0.590+0.566)/10$
 $= 0.582 \text{ degrees}$ shown with horizontal red line in Figure 117

B) $\sigma^2 (N-1) = (0.588-0.582)^2 + 2(0.595-0.582)^2 + (0.594-0.582)^2 + 4(0.566-0.582)^2 + 2(0.59-0.582)^2$

$\sigma^2 (N-1) = 0.000036 + 0.000338 + 0.000144 + 0.00102 + 0.000128$

$\sigma^2 (N-1) = 0.00167$

$\sigma^2 = 0.000185$

$\sigma = \pm 0.0136 \text{ degrees}$ shown with horizontal blue lines in Figure 117

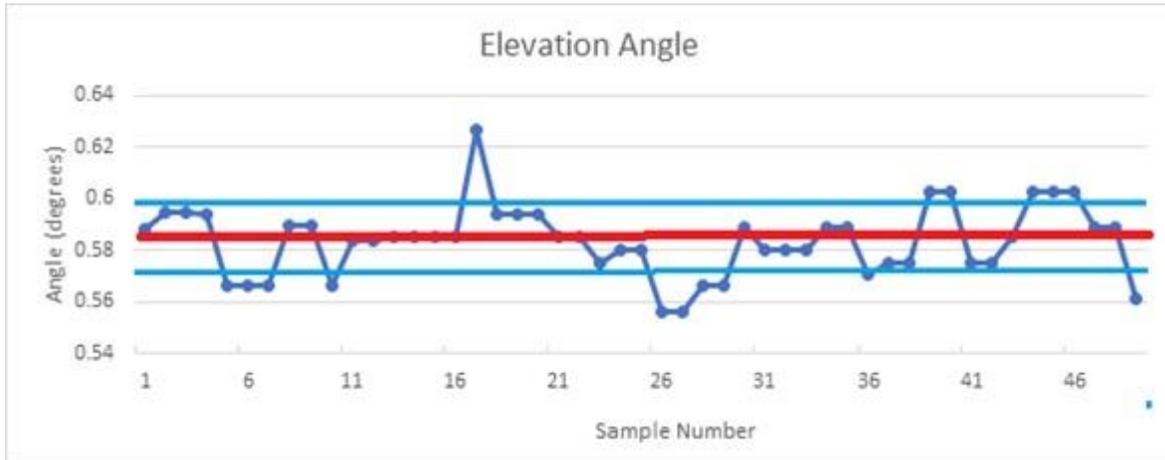


Figure 117. High resolution elevation data showing digitization

The previous example was for the case where a single average value was subtracted from the data, but what happens when the data shows a trend? Consider the following measurements of the local air pressure as the weather is changing over the course of 2 minutes in Figure 118 and Table 35.

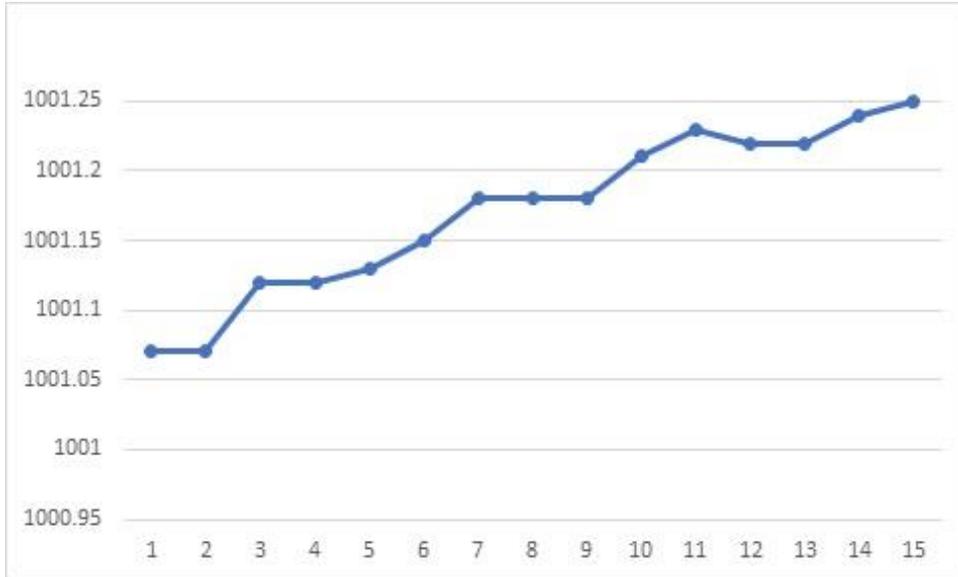


Figure 118. Pressure data graph

Table 35 Pressure data

1001.07, 1001.07, 1001.12, 1001.12, 1001.13, 1001.15, 1001.18,
1001.18, 1001.18, 1001.21, 1001.23, 1001.22, 1001.22, 1001.24

Problem 24 Pressure data analysis

- A) Compute the average of the 14 data points. Answer: $M = 1001.17$ milliBars.
 B) Compute the σ for the 14 data points. Answer: $\sigma = \pm 0.057$ milliBars.

Although we cannot determine whether the value for M is sensible or not, we might have a concern about the derived value for σ because we know that our barometer instrument is fully capable of making accurate measurements to within $\sigma = \pm 0.01$ milliBars. Why is the computed value for σ over 5-times larger? This could suggest that there was something going on with the data that might be interesting to study, or that for some reason the barometer was malfunctioning. Either of these issues would be of interest to study further. In fact if we plotted the data as in Figure 118 we would see an increasing trend, which is why the total σ is so large.

For some purposes the derived M and σ may suffice to answer some questions about the current pressure change in the atmosphere, but if we were interested in studying a long-term problem about the changes in the atmospheric pressure, we will probably want to study how the pressure is changing over time or ‘trending’. Mathematically we can investigate this by a process

of fitting the data to various mathematical functions. Perhaps, for instance, we have a theory about weather that predicts via a formula how the pressure change should vary in time.

A glance at the plotted data in Figure 119 suggests that one very good model that could fit the data is a linear equation shown by the dotted line. Lets find the line of the form $y = mx + b$ that would define such a line. Sample 1 has a value of 1001.07 and sample 14 has a value of 1001.24 so the slope is just $m = (1001.24 - 1001.07)/(14-1) = 0.0131$ and the intercept $b = 1001.07$, so the linear equation is $P = 0.0131 x + 1001.07$. The dashed line in the plot below shows the predicted values for the pressure at each sample. What we can now do is to subtract this model from the data, just like we subtracted the average value, M , from each data point. The result is this plot of the residuals in Figure 120.

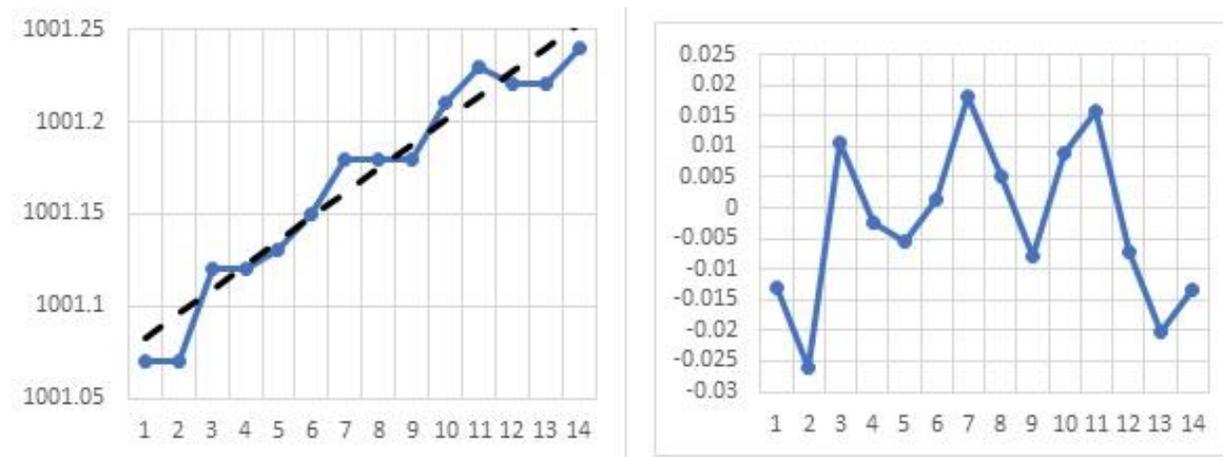


Figure 119 and 120. Pressure trend (dotted line) and residual after subtracting trend.

In our earlier definition of σ , we subtracted M from each measurement, but now for each measurement we subtract the predicted value P_i for each measurement where $P_i = 0.0131 i + 1001.07$. This gives us the formula:

$$\sigma = \sqrt{\frac{\sum_1^N (m_i - P_i)^2}{N - 1}}$$

The following Table 36 shows this calculation: Column 2 is the measured data. Column 3 is the individual variance $(m_i - M)^2$. Column 4 is the predicted value for the pressure P_i . Column 5 is the difference between the actual and predicted values $(m_i - P_i)$, and column 6 is the variance $(m_i - P_i)^2$ computed by squaring the entries in column 5.

Table 36 – Pressure data

1	2	3	4	5	6
Point	Data	Var	Pi	Diff	Var
1	1001.07	0.009216	1001.083	-0.0131	0.000172
2	1001.07	0.009216	1001.096	-0.0262	0.000686
3	1001.12	0.002116	1001.109	0.0107	0.000114
4	1001.12	0.002116	1001.122	-0.0024	5.76E-06
5	1001.13	0.001296	1001.136	-0.0055	3.03E-05
6	1001.15	0.000256	1001.149	0.0014	1.96E-06
7	1001.18	0.000196	1001.162	0.0183	0.000335
8	1001.18	0.000196	1001.175	0.0052	2.7E-05
9	1001.18	0.000196	1001.188	-0.0079	6.24E-05
10	1001.21	0.001936	1001.201	0.009	8.1E-05
11	1001.23	0.004096	1001.214	0.0159	0.000253
12	1001.22	0.002916	1001.227	-0.0072	5.18E-05
13	1001.22	0.002916	1001.24	-0.0203	0.000412
14	1001.24	0.005476	1001.253	-0.0134	0.00018
M / σ	1001.166	0.057			0.014

We see that by subtracting the linear model, P_i , from each point, the final σ has decreased from ± 0.057 to ± 0.014 milliBars, which is very close to the ± 0.010 milliBar accuracy of the barometer. By eliminating the linear trend, we have extracted all of the available systematic information available in the data and now have simply the expected random measurement ‘noise’ left. Another way to state this is that the data is consistent with the prediction that there should be a linear relationship with the pressure measurements over time.

One thing that we see in the residual plot is that the data seem to follow a curve. This suggests that there could be some additional information hidden in the data that is not completely covered by an exact linear correlation. We can use *Excel* to study this issue in more detail. If we import the 14 points into a spreadsheet and then plot them we get our initial figure, but now let’s use the ‘trendline’ function to plot a polynomial curve of degree-2 (a quadratic curve) through the data. We get the following plot in Figure 121.

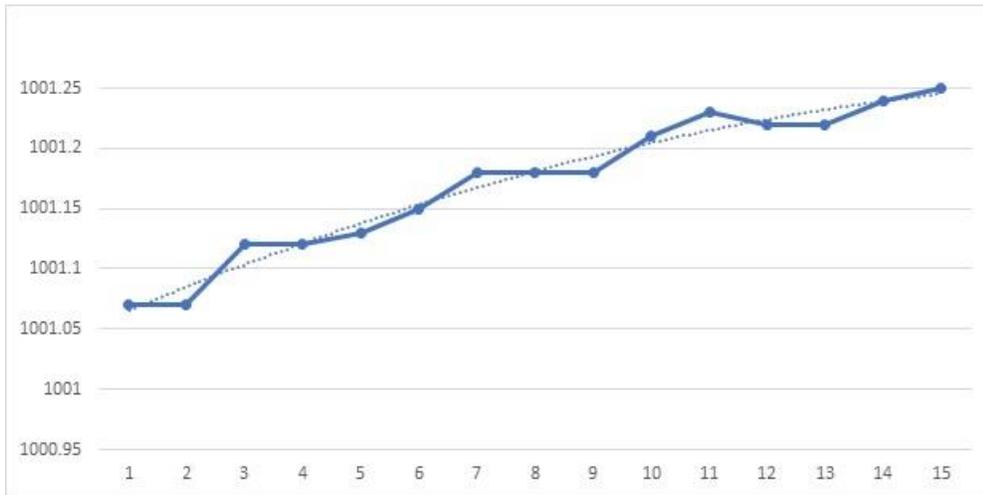


Figure 121. Pressure data with quadratic trend (dotted line)

The software tells us that the best-fit quadratic D=2 equation has the form $P = -0.00052x^2 + 0.0212x + 1.01.045$. If we now create Table 37 with this model we get Figure 201.

Table 37 – Analysis of pressure data using two regressions.

Point	Data	var	mx+b	diff	Var	Poly D=2	Diff	Var
1	1001.07	0.009216	1001.083	-0.0131	0.000172	1001.066	0.00432	1.86624E-05
2	1001.07	0.009216	1001.096	-0.0262	0.000686	1001.085	-0.01532	0.000234702
3	1001.12	0.002116	1001.109	0.0107	0.000114	1001.104	0.01608	0.000258566
4	1001.12	0.002116	1001.122	-0.0024	5.76E-06	1001.121	-0.00148	2.1904E-06
5	1001.13	0.001296	1001.136	-0.0055	3.03E-05	1001.138	-0.008	6.4E-05
6	1001.15	0.000256	1001.149	0.0014	1.96E-06	1001.153	-0.00348	1.21104E-05
7	1001.18	0.000196	1001.162	0.0183	0.000335	1001.168	0.01208	0.000145926
8	1001.18	0.000196	1001.175	0.0052	2.7E-05	1001.181	-0.00132	1.7424E-06
9	1001.18	0.000196	1001.188	-0.0079	6.24E-05	1001.194	-0.01368	0.000187142
10	1001.21	0.001936	1001.201	0.009	8.1E-05	1001.205	0.005	2.5E-05
11	1001.23	0.004096	1001.214	0.0159	0.000253	1001.215	0.01472	0.000216678
12	1001.22	0.002916	1001.227	-0.0072	5.18E-05	1001.225	-0.00452	2.04304E-05
13	1001.22	0.002916	1001.24	-0.0203	0.000412	1001.233	-0.01272	0.000161798
14	1001.24	0.005476	1001.253	-0.0134	0.00018	1001.24	0.00012	1.44E-08
	1001.166	0.056937			0.013622			0.010186585

The new value for $s = \pm 0.010$ milliBar, which is exactly the expected instrumental noise due to random error. The residual plot now looks like this in Figure 122. There is no longer any obvious trend left in the residuals after a quadratic model has been subtracted.

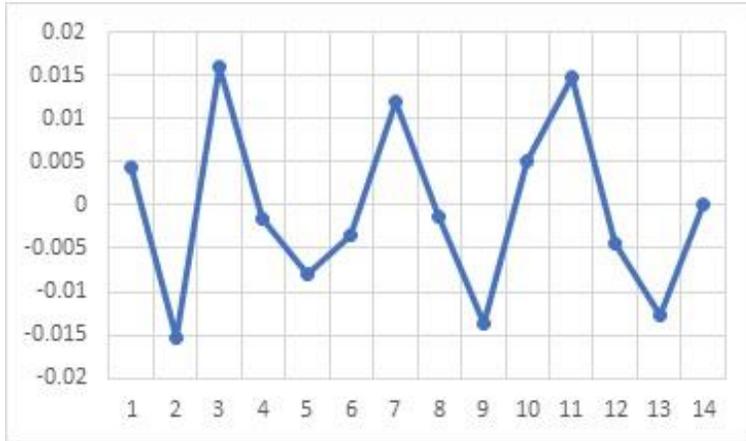


Figure 122. Residual after subtracting quadratic trend.

Now the question is, which of the two models for the data, linear or quadratic, is the more significant? Our theory says it should be a linear correlation but we have also found a quadratic correlation that fits the data even better. If only a few end points had been dropped, there would no longer be enough information to model a quadratic function. Is there some way to test which of these two models is the most significant?

Data Modeling and Significance Testing

The basic idea is that you have a set of measurements and you want to test whether these measured values are consistent with a particular mathematical model. This sounds like a straight-forward process, but what complicates the problem is that every measurement is imprecise and inaccurate. We call this inaccuracy by various names such as ‘measurement error’ or ‘noise’, and it is found in all instruments and methods of measurement. It is especially severe when, for instance, you have a millimeter ruler but are trying to measure something whose length falls in-between the millimeter marks. Even the width of the millimeter mark itself can lead to measurement error. In Figure 123 below, is the length of the red bar equal to 23.3, 23.4 or 23.5 millimeters? If the right edge was placed at the left edge of the ‘0’ mark or at the right edge of the ‘0’ mark, what would be the uncertainty in the measurement?

Note, because the thing we are measuring has a knowable fixed length, our measurement error is a measure of the precision of the measurement. If we didn't know its length beforehand, the error would be a measure of accuracy.

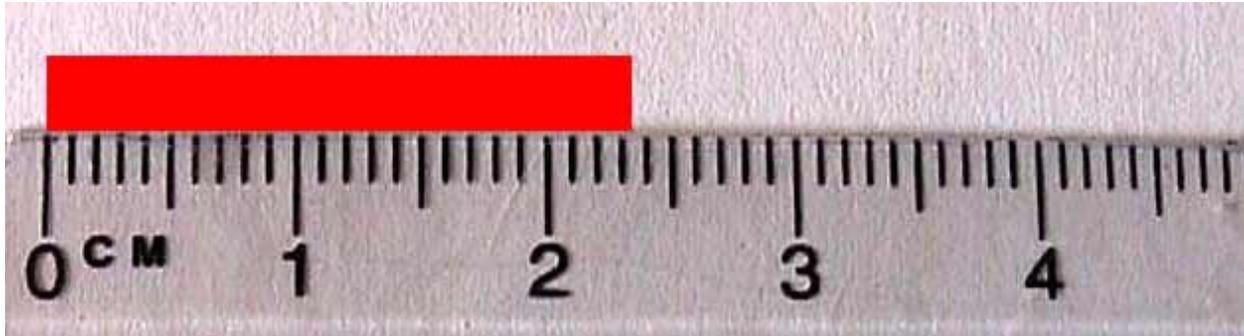


Figure 123. Example of measuring a known object.

So when we are testing a mathematical model to see whether it is reliable (accurate) or not, how do we avoid dismissing a perfectly good model just because the measurements are themselves prone to error and uncertainty? Alternatively, if we have two mathematical models how do we decide which one is more accurate? In our previous example, we found that a linear equation and a quadratic equation seemed to match the data with almost equal accuracy. This issue of comparing inaccurate data against a model and assessing whether the model is a good one in view of the measurement uncertainties is called ‘significance testing’. It is a key tool used by scientists to decide whether their theoretical models truly provide an accurate representation of the data.

Method of least-squares:

This approach takes a series of measurements, m_i , and compares them to their corresponding predictions, P_i , based upon a mathematical model. The square of the difference is computed for each measurement and summed for all the measurements to get the total variance

$$Var = \sum_1^N (m_i - P_i)^2$$

If we have two different models, P_1 and P_2 , we calculate Var_1 and Var_2 . The model that makes the variance the smallest is selected as the best model that ‘fits’ the data. In our previous example of the pressure measurements we can form Table 38.

Table 38 – Data for regressions of pressure trends.

Point	Data	var	mx+b	diff	Var	Poly D=2	Diff	Var
1	1001.07	0.009216	1001.083	-0.0131	0.000172	1001.066	0.00432	1.86624E-05
2	1001.07	0.009216	1001.096	-0.0262	0.000686	1001.085	-0.01532	0.000234702
3	1001.12	0.002116	1001.109	0.0107	0.000114	1001.104	0.01608	0.000258566
4	1001.12	0.002116	1001.122	-0.0024	5.76E-06	1001.121	-0.00148	2.1904E-06
5	1001.13	0.001296	1001.136	-0.0055	3.03E-05	1001.138	-0.008	6.4E-05
6	1001.15	0.000256	1001.149	0.0014	1.96E-06	1001.153	-0.00348	1.21104E-05
7	1001.18	0.000196	1001.162	0.0183	0.000335	1001.168	0.01208	0.000145926
8	1001.18	0.000196	1001.175	0.0052	2.7E-05	1001.181	-0.00132	1.7424E-06
9	1001.18	0.000196	1001.188	-0.0079	6.24E-05	1001.194	-0.01368	0.000187142
10	1001.21	0.001936	1001.201	0.009	8.1E-05	1001.205	0.005	2.5E-05
11	1001.23	0.004096	1001.214	0.0159	0.000253	1001.215	0.01472	0.000216678
12	1001.22	0.002916	1001.227	-0.0072	5.18E-05	1001.225	-0.00452	2.04304E-05
13	1001.22	0.002916	1001.24	-0.0203	0.000412	1001.233	-0.01272	0.000161798
14	1001.24	0.005476	1001.253	-0.0134	0.00018	1001.24	0.00012	1.44E-08
	1001.17	0.057			0.0024			0.0013

We see that for the linear model, $\text{Var1} = 0.0024$ and for the quadratic model we have $\text{Var2} = 0.0013$. We conclude that the quadratic model fits the data better than the linear model. There is, however, one statistical difference between the two results. For the linear fit we only needed to define two coefficients to make the fit while for the quadratic model we needed three coefficients. These had to be determined by using information extracted from the data set itself. In essence, we are comparing apples to oranges and we need to control for the number of degrees-of-freedom (DOF) being used.

Linear model: $P = mx + b$, parameters: m and b DOF = 2

Quadratic model: $P = ax^2 + bx + c$ parameters: a, b and c DOF = 3

The full data set represents 14 DOF. The linear model requires 2 DOF and the quadratic model requires 3 DOF, so the final number of DOFs are for the linear model, $14 - 2 = 12$, and for the quadratic model we have $14 - 3 = 11$. If we ‘normalize’ each variance by the number of DOF, we get $\text{ML} = 0.0024/12 = 0.0002$ and $\text{MQ} = 0.0013/11 = 0.00012$. So once again we see that the variance for the quadratic model is just a little bit smaller than for the linear model. But the question is whether the difference between the variances is significant or not given the amount of noise in the data. Sometimes we can remove one or two data points and discover that in calculating the new variances, their values become almost identical.

The Fischer F-Test

This test compares a set of data to a mathematical model called the regression and determines whether the model fits the data. The test is expressed as a ‘Null Hypothesis’ that is to be rejected or accepted.

The Null Hypothesis in the case of the previous pressure data is that *the quadratic model does not provide a better fit than the linear model*. This hypothesis will be rejected if a test statistic called F is greater than the critical F from the Fisher F-distribution in Figure 124 for the stated DOFs. In other words, suppose F is calculated to be 10.0 and the table gives a critical Fc of 8.0 for 0.90 confidence. Since $F > F_c$, we have to accept the Null Hypothesis at the 90% confidence level. This means that in 9 times out of 10 for the same number of measurements, the quadratic model does not provide a better (more confident) fit than a simple linear equation even though the variance for the quadratic model is less than for the linear model.

In our previous problem, to decide between the linear and quadratic models we are going to test the significance of the quadratic model by defining the test statistic as:

$$F = \frac{\Delta}{MQ} = \frac{Abs(Var1 - Var2)}{MQ}$$

which for $Var1 = 0.0024$, $Var2 = 0.0013$ and $MQ = 0.00012$ in our previous problem gives us a value of $F=9.2$. To decide whether this value of F lets us accept or reject the Null Hypothesis, we consult a table that relates the value of F to a probability. What this table gives is the area under a probability curve as a function of the number of DOF of MQ versus the DOF of $\Delta=abs(Var2-Var1)$. Figure 124 below shows what this table calculation looks like this. The curve is the F

distribution function for the number of DOF in the problem.

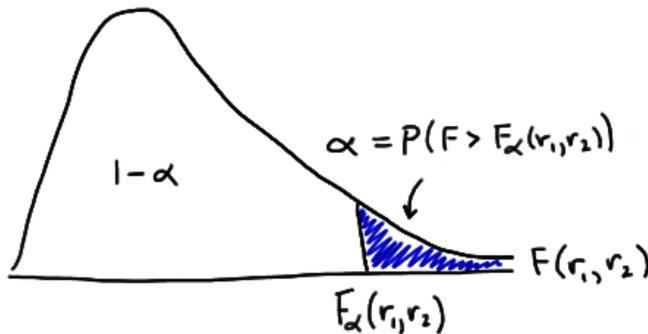


Figure 124. F Test distribution curve.

The area under the complete curve is exactly 1.000 (a probability of 100%). The shaded area is the probability corresponding to a value of F greater than the calculated value. The smaller this area becomes as F increases, the less likely will be the calculated difference between the linear and quadratic models.

The quadratic model, MQ has 3 DOF and Δ has $3-2 = 1$ DOF. The results from this table can be found at <https://stattrek.com/online-calculator/f-distribution.aspx>

If you input DOF $\nu_1 = 1$, DOF $\nu_2 = 3$, and enter $F=9.2$ you get a cumulative probability (1-alpha) of 0.94 from the table. What this means is that 94% of the time the quadratic model does not provide a better fit to the data than the linear model, and only 6% of the time does it give a better fit compared to the linear model.

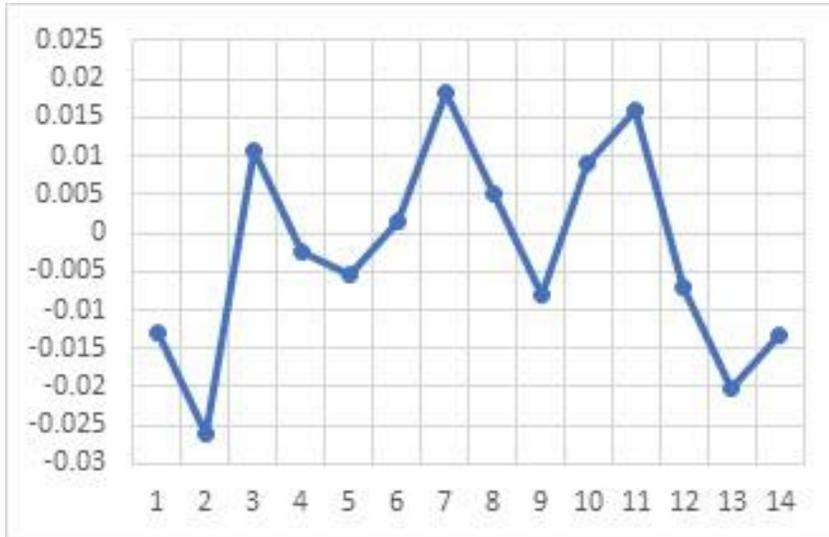


Figure 125. Residuals after removal of quadratic trend

This reflects the fact that if we had removed samples 2 and 13 in Figure 205, it would have dramatically reduced the significance of the quadratic model compared to the linear one given the noise in the data measured by its variance. With these points dropped, $\text{Var1} = 0.0013$ and $\text{Var2} = 0.0010$. This is why for the small number of points that enter into the computation of F, the differences in the variances are not considered to be highly significant.

The R^2 coefficient of determination statistic.

Many curve-fitting or trend line-fitting routines such as those found in *Microsoft Excel* use a parameter called R^2 to gauge how significant a fit is to a set of data. It is a measure of what fraction of the variation in the data is accounted for by the selected model. If the model perfectly accounts for the data values, $R^2 = 1.0$ but if it accounts for, as an example, only 40% of the variation its value will be $R^2=0.4$. This statistic is a means for deciding how well several different possible models fit the data. It does this by comparing the variance in the measured values after the model is removed, to the total variance in the measured values. It is calculated as follows:

$$R^2 = \frac{\sum_1^N (m_i - P_i)^2}{\sum_1^N (y_i - Y)^2} = \frac{\text{Measured Variance}}{\text{Total } y \text{ variance}}$$

Table 16 below shows the values from the 14 pressure data points in the previous problem for the linear and quadratic models.

Table 39 – Analysis of data for different regressions

Point	Data	var	Linear			Quadratic		
			mx+b	diff	Var	Poly D=2	Diff	Var
1	1001.07	0.009216	1001.083	-0.0849	0.007208	1001.066	-0.10032	0.010064102
2	1001.07	0.009216	1001.096	-0.0718	0.005227	1001.085	-0.08068	0.006509262
3	1001.12	0.002116	1001.109	-0.0587	0.003446	1001.104	-0.06208	0.003853926
4	1001.12	0.002116	1001.122	-0.0456	0.002079	1001.121	-0.04452	0.00198203
5	1001.13	0.001296	1001.136	-0.0325	0.001056	1001.138	-0.028	0.000784
6	1001.15	0.000256	1001.149	-0.0194	0.000376	1001.153	-0.01252	0.00015675
7	1001.18	0.000196	1001.162	-0.0063	3.97E-05	1001.168	0.00192	3.6864E-06
8	1001.18	0.000196	1001.175	0.0068	4.62E-05	1001.181	0.01532	0.000234702
9	1001.18	0.000196	1001.188	0.0199	0.000396	1001.194	0.02768	0.000766182
10	1001.21	0.001936	1001.201	0.033	0.001089	1001.205	0.039	0.001521
11	1001.23	0.004096	1001.214	0.0461	0.002125	1001.215	0.04928	0.002428518
12	1001.22	0.002916	1001.227	0.0592	0.003505	1001.225	0.05852	0.00342459
13	1001.22	0.002916	1001.24	0.0723	0.005155	1001.233	0.06672	0.004451558
14	1001.24	0.005476	1001.253	0.0854	0.007293	1001.24	0.07388	0.005458254
	1001.17	0.042	1001.168		0.0390	1001.166		0.0416

The total Y data variance is 0.042. The measured variance for the linear model is 0.0390 and for the quadratic model it is 0.0416. The resulting R²s are:

$$\text{Linear: } R^2 = 0.039/0.042 = \mathbf{0.929} \quad \text{Quadratic: } R^2 = 0.0416/0.042 = \mathbf{0.990}$$

So both models account for over 93% of the information contained in the data, with the quadratic model doing a better job, mainly because it has one more parameter (DOF) with which to represent the data than the linear model. Generally, polynomial models always yield higher values for R² because the individual bumps and wiggles in the data can be more completely modeled leaving smaller residuals behind. Whether the quadratic model is actually ‘better’ than the linear model can often be checked by clipping out ‘outliers’ in the sample. In the above case, we discovered that by removing only two data points (sample 2 and 13), the difference between the linear and quadratic model vanished suggesting that the quadratic model being ‘better’; was probably a spurious result. This was supported by the F-test which showed that the difference between the linear and quadratic model residuals (i.e their respective variances) was only significant at about the 6% level, and that 94% of the time you could get as-good fits by random variation.

Noise Analysis

In none of the above cases do we compare the variation we see against the known measurement variation in our instrument. Instead, we let the variation in the measured values offer a clue about the total measurement variation, and against which the various statistical tests are applied. Generally, the measurement error or ‘data noise’ is the result of several other statistical processes, each with its own variance. If these sources of noise (error) are caused by a random process that follows a Gaussian distribution with a well-defined mean and s.d, the total data s.d or σ is the quadrature sum of these individual sources is given by:

$$\sigma^2 = \sigma_i^2 + \sigma_t^2 + \sigma_m^2 + \sigma_c^2$$

where σ_i might be the noise due to the set up of the instrument, σ_t might be the noise caused by changes in the instrument temperature, σ_m might be the noise introduced by using one model of the instrument versus another model, and σ_c might be the noise contributed by variations between the copies of the same instrument model.

To make the total measurement noise (error) the smallest, you need to minimize each of the different contributions as much as possible by, for instance, using the same copy for all the measurements so that $\sigma_c = 0$. You also need to make the measurement at the same temperature if you have concluded that the data has a temperature dependence so that $\sigma_t = 0$. That leaves σ_i and σ_m as the main contributors to the total measurement error. For smartphones, σ_m generally has to do with the electronics of the sensor, which may vary from model to model (i.e a Samsung Galaxy 8s may have a slightly different sensor set up than an iPhone 6s). The instrument set up noise, σ_i , could involve inadvertent vibrations in the table being used for an accelerometer measurement, or random noises in your basement when you are making a sound-level measurement.

Problem 25 – Exploring copy variance: σ_c

The same app and sound measurement were made on four copies of the Samsung Galaxy 8s and four of the Samsung Note 5.

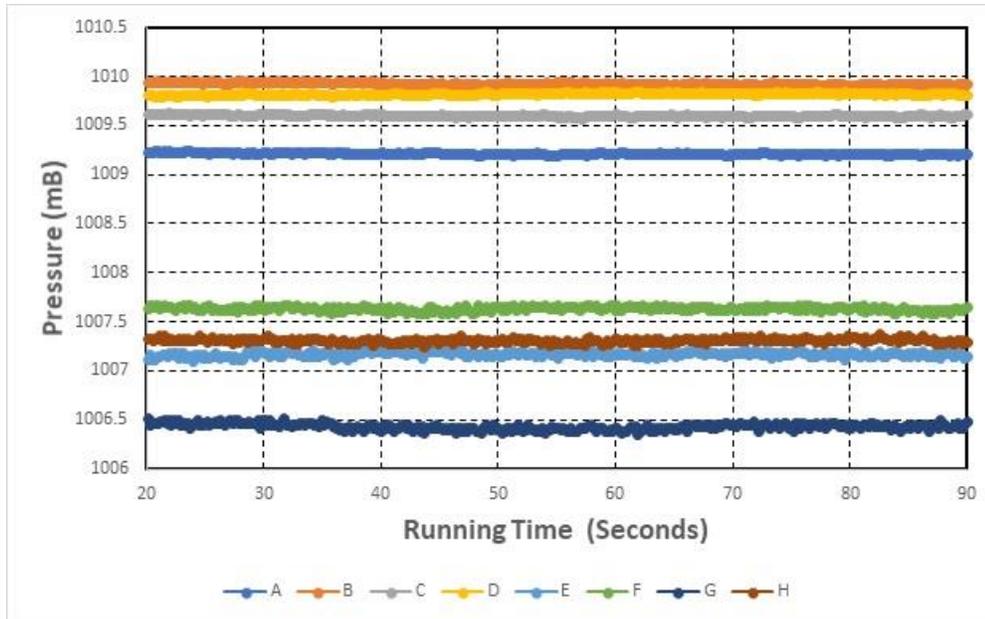


Figure 126. Pressure data for different copies of smartphones

The measurement variation within each model and copy in Figure 126 are consistently of the order of $\sigma = \pm 0.01$ mB. The measured mean values for the Galaxy 8s are A=1009.21 mB, B=1009.92 mB, C=1009.60 mB, D=1009.81 mB, and for the Note 5s: E=1007.16 mB, F=1007.63 mB, G=1006.44 mB and H=1007.31 mB.

A) Calculate the mean and σ for the Galaxy 8s phone models.

Answer: **Mean = 1009.64 mB. $\sigma = 0.32$ mB**

B) Calculate the mean and σ for the Note 5 phone models.

Answer: **Mean = 1007.14 mB, $\sigma = 0.51$ mB**

C) The values for σ calculated above represent the total sigma, σ_t , of the measurement where $\sigma_t^2 = \sigma_i^2 + \sigma_c^2$. If the instrument noise in each case is $\sigma_i = 0.15$ mB, what is the copy-to-copy noise, σ_c ?

Answer: Galaxy 8s: $\sigma_c^2 = \sigma_t^2 - \sigma_i^2$ so $\sigma_c^2 = 0.32^2 - 0.15^2 = 0.080$ so $\sigma_c = \mathbf{0.28}$ mB

Note 5: $\sigma_c^2 = 0.51^2 - 0.15^2 = 0.237$ so $\sigma_c = \mathbf{0.49}$ mB

We see from this analysis that the two smartphone models give significantly different readings that differ by $1009.64 - 1007.14 = 2.5$ mB. Also, the Note 5 has a larger copy variation than the Galaxy 8s. Which smartphone model is better to use? The Galaxy 8s has a smaller copy variance so measurements with this model would give more consistent readings. As for the significance of the systematic offset of 2.5 mB between the two phone models, it is difficult to assess this aspect of the data without knowing what the true atmospheric pressure was at the time of the measurements using a properly calibrated, professional-grade barometer.

A Variety of Smartphone Data Types

There are several apps available that record data for later analysis such as *Physics Toolbox*. This app reads-out data from 10 different sensors and saves them as .csv files for importing to programs such as Microsoft Excel. Here is a sample of these data taken over the course of 1 minute to explore their short-term and one hour to explore their long-term behavior under conditions in which they should read a fixed, but not necessarily a zero, signal level. The app was operated on a Samsung Galaxy 8s phone.

Problem 26 – Data Rates and File Size. If an app generates accelerometer sensor measurements at a rate of 15 samples per second, how many data points will be generated in one hour of operation?

Answer: 1 hour x (3600 seconds/hour) x 15 samples/sec = **54,000**. If each row in an *Excel* spreadsheet .csv file represents one data sample, the spreadsheet will have 54,000 rows of data for you to analyze.

Linear Accelerometer

These data were obtained with a smartphone placed on a level table top and left motionless during the measurement cycle. Under these conditions, the A_x and A_y accelerations should be zero, but the A_z acceleration should show only the effect of gravity at 9.8 m/s^2 . This gravity component was subtracted from the A_z data to get the net vertical acceleration of the smartphone which on a fixed table top should be exactly 0.

The measurements were made over the course of one minute at a frequency of 5 samples per second. The mean values in Figure 127 are $\langle A_x \rangle = 0.00 \text{ m/s}^2$ $\langle A_y \rangle = 0.00 \text{ m/s}^2$ and $\langle A_z \rangle = 0.00 \text{ m/s}^2$ and the values for σ are all $\pm 0.001 \text{ m/s}^2$. Calculating σ in this way assumes that the underlying noise is a random Gaussian and that the frequency of the noise data follows a ‘Bell Curve’.

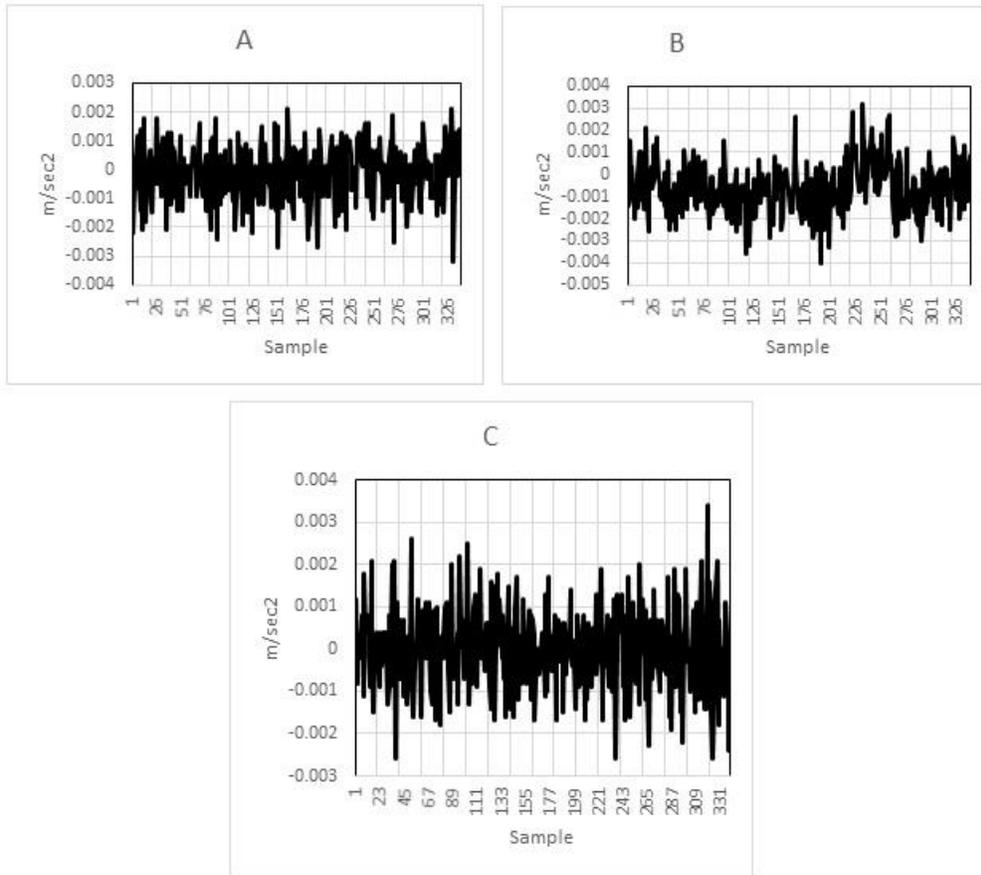


Figure 127. One minute data for a) Ax, b) Ay and c) Az.

We can check this by actually calculating the frequency distribution of the noise for each component and combining the frequency distributions into one plot in Figure 128. The dashed line shows the Gaussian distribution for a mean of zero and $\sigma = 0.001 \text{ m/s}^2$. The positive-side of the data pretty nearly follows a random distribution but the negative-side shows a different distribution. A search through the data shows that this is contributed by the data for the Az component shown in Figure 129.

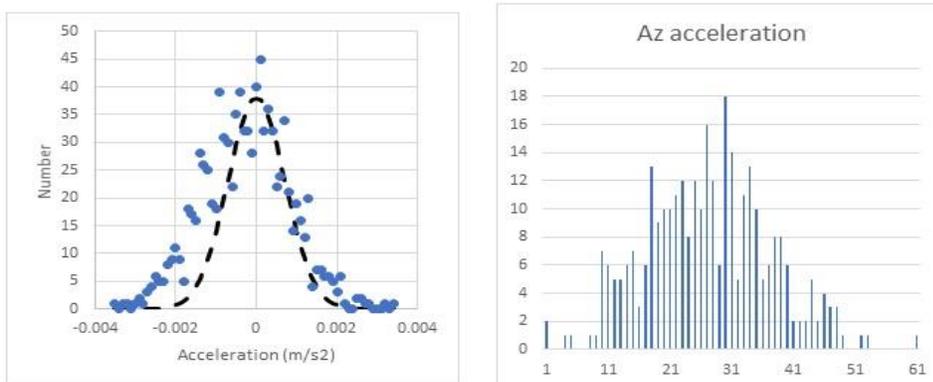


Figure 128 (left) and Figure 129 (right) showing distributions of the measurements and a Gaussian profile (dashed line).

The physical reason for the Az measurements to favor more values that are smaller than the mean value is difficult to assess without knowing more about the acceleration sensor used to measure the vertical acceleration. Nevertheless, the data seems to show a bias in this sensor that is different than for the other two sensors in the horizontal plane.

G-force meter on stationary tabletop

The g-force meter is used to record one minute-worth of data with the smartphone motionless on a leveled table. Unlike the linear acceleration meter, which subtracts the acceleration of gravity from each value of A, Ay and Az, the g-force meter records the acceleration in all three directions without subtracting earths acceleration, which should be near 9.8 m/s². The g-force meter gives the acceleration along axis measured in units of G's, where 1-G = 9.8 m/s². When we calculate the total acceleration from the three components using $|A| = (Ax^2+Ay^2+Az^2)^{1/2}$ the only acceleration on the motionless smartphone will be the acceleration due to gravity so that $|A| = g = 9.80665 \text{ m/s}^2$.

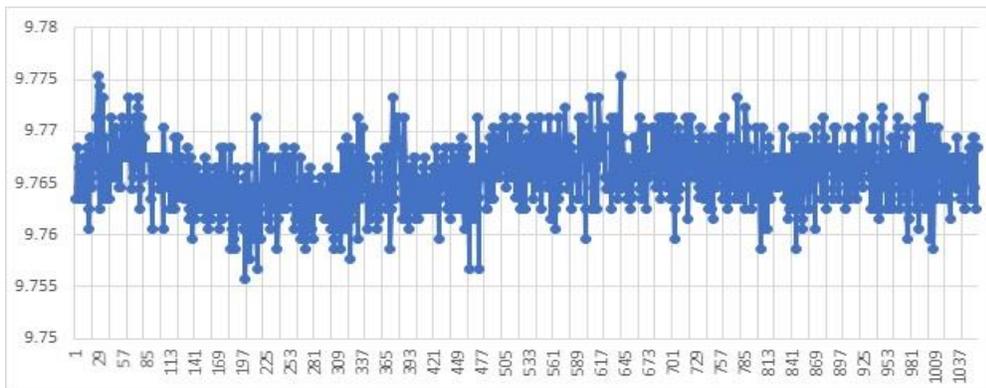


Figure 130. Vertical acceleration for one minute

The acceleration of gravity at Earth's surface is constant, so the first thing one notices in Figure 130 is that this smartphone sensor needed almost 30 seconds in order to stabilize at a relatively fixed average reading after Sample-505. We can replot this valid data in the Figure 211.

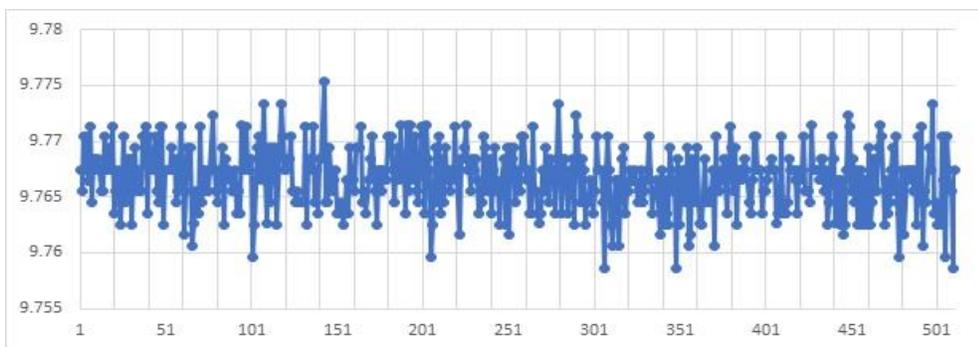


Figure 131. Vertical acceleration data.

The computed average for this edited data is $\langle g \rangle = 9.767 \text{ m/s}^2$ and $\sigma = \pm 0.003 \text{ m/s}^2$. Although the measurement appears to be accurate to ± 0.003 or 3%, the precision of this measurement is worse than this since the true acceleration of gravity is 9.80 m/s^2 which differs from $\langle g \rangle$ by 0.12 m/s^2 . So this particular smartphone model and sensor gives a systematically lower value for G by 0.12 m/s^2 . Assuming that this value is constant in time, it is called a systematic bias, which has to be added to all of the smartphone g -measurements to bring them into alignment with the actual acceleration of gravity at Earth's surface. This process of determining a measurement bias and correcting for it is called 'calibration', and will allow an accurate measurement to also become a precise measurement.

Barometer – indoors

The behavior of smartphone pressure sensors is hard to understand on minute to minute timescales as the graph in Figure 132 shows. Some of these effects could be internal to the smartphone, or caused by air conditioning systems and even low-frequency 'infrasound' events from thunderstorms and other sources. Variations of ± 0.05 milliBars are common.

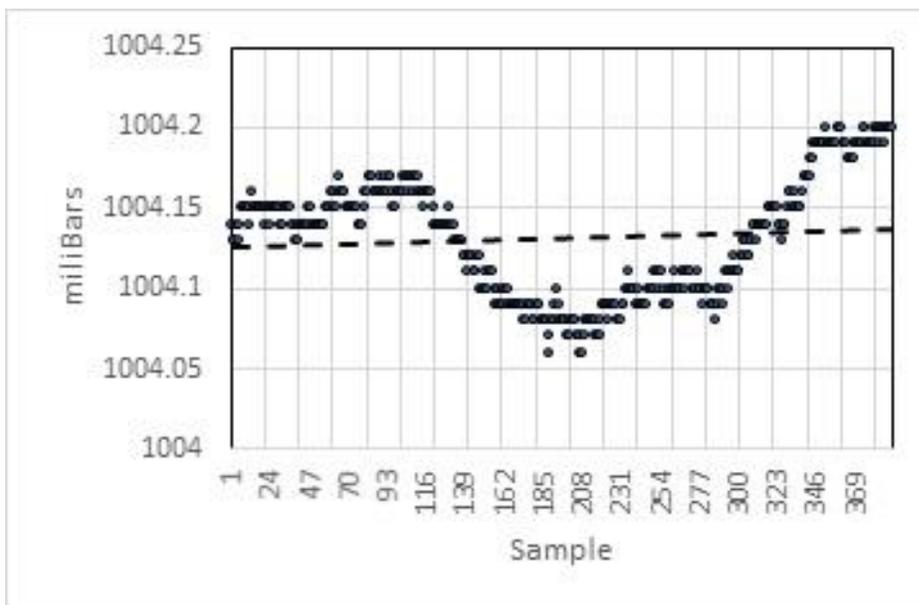


Figure 132. Smartphone pressure measurements.

When the baseline is extended to an hour as the graph in Figure 133 shows, long-term trends related to weather conditions start to become apparent, and can be tracked backwards to timescales of a few minutes during which similar trends can be seen in the very short-term data.

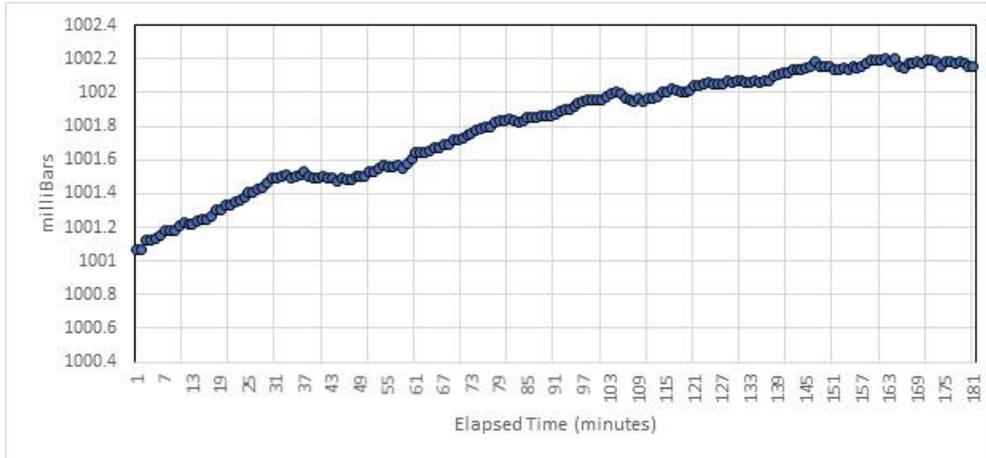


Figure 133. Long term pressure data.

Magnetometer – on tabletop

At any geographic location, Earth’s magnetic field should show a constant level that is unique to that location, but environmental influences such as local metal deposits, electrical systems and motors can be a source of noise for detecting the weak geomagnetic signal. The graphs in Figure 134 show a one-minute data series sampled at 15 times per second. The smartphone was placed on a leveled table top and remained motionless. It is obvious that there are several forms of ‘noise’ in what should be an otherwise constant signal measurement.

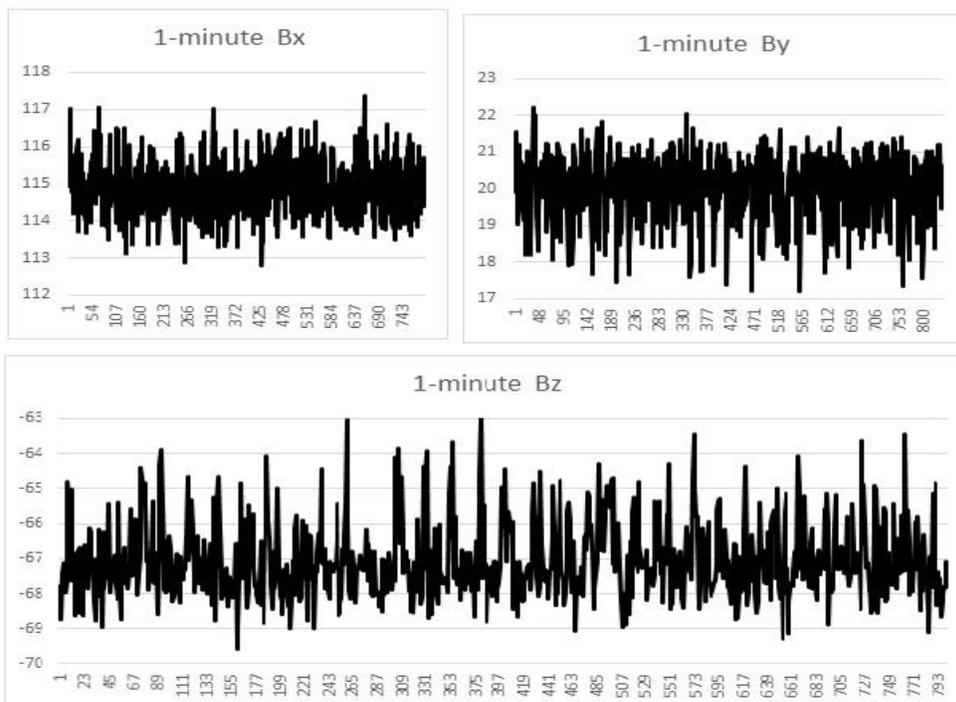


Figure 134. Magnetometer measurements.

The sample means and σ for each component are $B_x = 115.05 \pm 0.71 \mu\text{T}$, $B_y = 20.04 \pm 0.85 \mu\text{T}$, and $B_z = -67.14 \pm 1.1 \mu\text{T}$. The minimum noise for the magnetometer sensor is about $\sigma = \pm 0.2 \mu\text{T}$, so this data is considerably noisier than what would normally be expected. We can sample the data for a considerably longer period of time and get the result shown in Figure 135 for an iPhone 6s using the *Teslameter 11th* app to record the data every 30 seconds between October 30 and November 1, 2017.

The graphs in Figure 135 and 136 are scaled to the same 4.0 mT range in the respective values for B_x , B_y and B_z with samples made every 0.5 minutes. The data between 0 and 150 minutes is continuous, however at 151 minutes an 8-hour gap in data taking was introduced before data-taking was resumed at 152 minutes. From 152 minutes to 184 minutes, continuous data taking was resumed before another pause in data-taking was introduced at 184.5 minutes for a period of 8 hours, then again resumed until the end of the last session at 292 minutes. The two pauses at 151 and 184 minutes were introduced to test the long-term behavior of the data-taking after stopping and starting the magnetometer app.

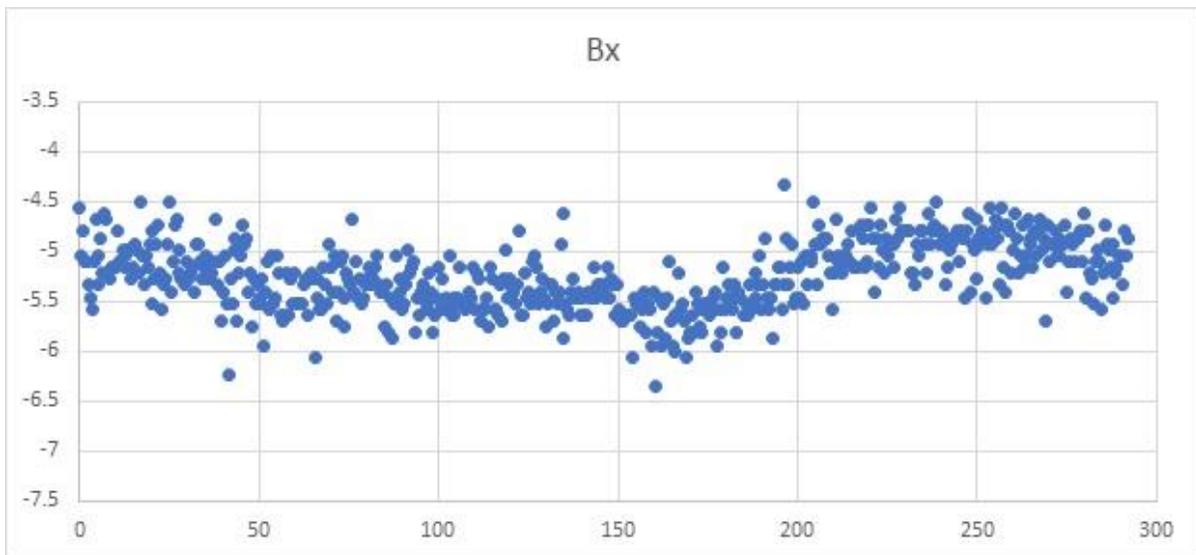


Figure 135. Data from iPhone 6s.

What we see in the B_x plot is that following the power-up of the app near $t=0$ minutes, the measurements decline steadily until they reach a constant value after about 50 minutes. The value for $\langle B_x \rangle$ and σ from 50 minutes to 150 minutes is calculated to be $\langle B_x \rangle = -5.43 \pm 0.22 \mu\text{T}$. The interruption at 151 minutes is followed by 32 minutes of data-taking but the B_x values never reached a constant level during this time. The data taking was resumed after an 8 hour pause at 185 minutes but the values never stabilized at a fixed level until 220 minutes. The value for $\langle B_x \rangle$ and σ from 220 minutes to 292 minutes is $\langle B_x \rangle = -4.96 \pm 0.24 \mu\text{T}$. The increase between -5.43 and $-5.00 \mu\text{T}$ is $0.43 \mu\text{T}$. The difference is only about twice the noise level of $\pm 0.23 \text{ mT}$, but the

graph shows that even such a statistically small difference is discernable in the data trends. We can conclude that over the roughly 19-hour data-taking session, the value of Bx increased slowly from -5.4 to -4.97 μT . Whether this is an instrumental effect in the smartphone or an environmental (geophysical) change remains to be determined.

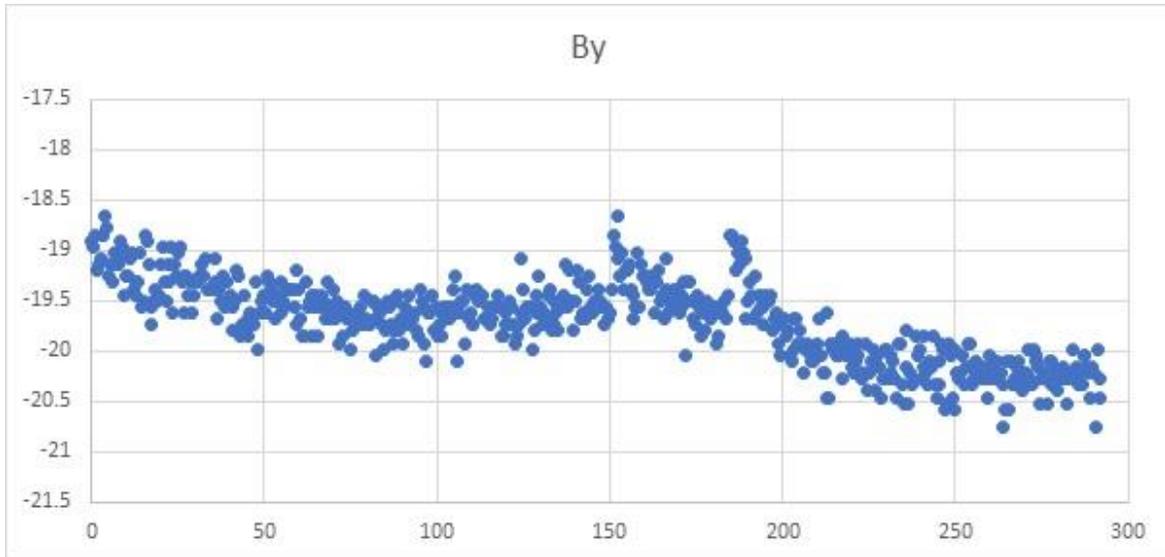


Figure 136. Magnetometer data.

The B_y data in Figure 136 in many ways resembled the B_x data in Figure 135. During the first 50 minutes, the measurements are declining to a roughly fixed value until 150 minutes for which $\langle B_y \rangle = -19.70 \pm 0.21 \mu\text{T}$. Between 150 minutes and 220 minutes the data cannot be used, but after 220 minutes it stabilizes at a mean level of $\langle B_y \rangle = -20.21 \pm 0.19 \mu\text{T}$. This magnetic component over the 19-hour period has declined an additional $\Delta B_y = + 0.50 \mu\text{T}$.

The geomagnetic components in the local horizontal plane are used by compasses to determine a magnetic bearing. The relationship between the bearing angle and the magnetic components is given by

$$\theta = \text{arcTan} \left(\frac{B_y}{B_x} \right)$$

Problem 27 – Magnetic Bearing Changes

From the above values of $\langle B_x \rangle$ and $\langle B_y \rangle$, what is the bearing change during this 19-hour period?

Answer: From 0-150 minutes, $\theta = \arctan (-5.43 / -19.7) = 15.4^\circ$.

From 220 to 292, $\theta = \arctan (5.0 / 20.21) = 13.9^\circ$, so your compass needle would have drifted about 1.5° from its true direction.

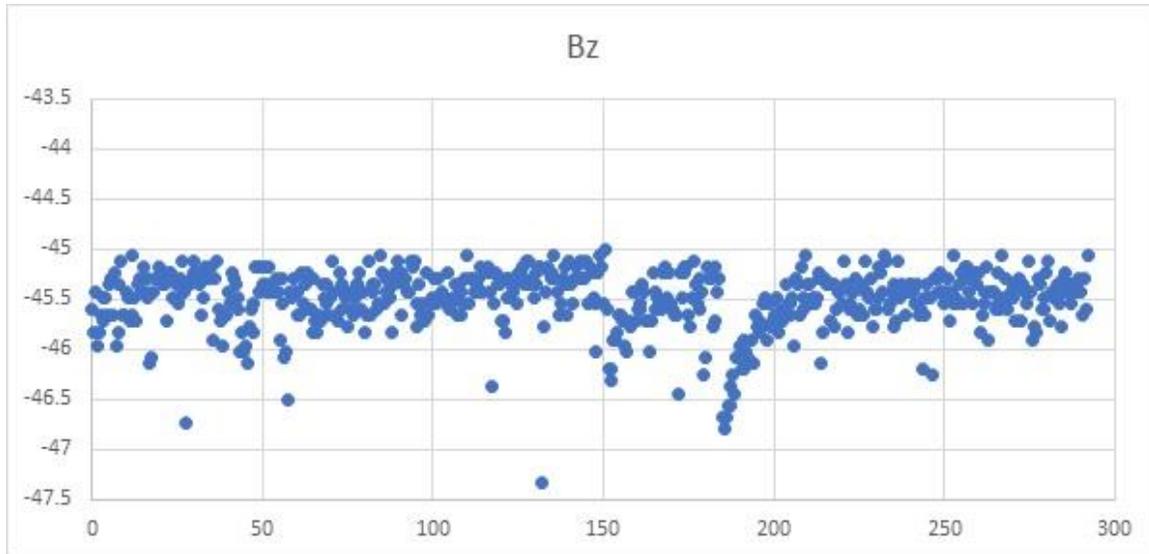


Figure 137. Magnetometer data

The magnetic ‘z’ component, when the smartphone is laying flat on a leveled table top, is along the vertical direction. This data in Figure 137 shows no sign of the initial 50-minute ‘power-up’ seen in the Bx and By components. The mean value for Bz from 0 to 150 minutes is $\langle B_z \rangle = -45.44 \pm 0.35 \mu\text{T}$. From 220 to 292 minutes $\langle B_z \rangle = -45.45 \pm 0.020 \mu\text{T}$. This shows that during this 19-hour period, the value for $\langle B_z \rangle$ did not change in a measurable way. The total magnetic field data in Figure 138 shows even less variation.

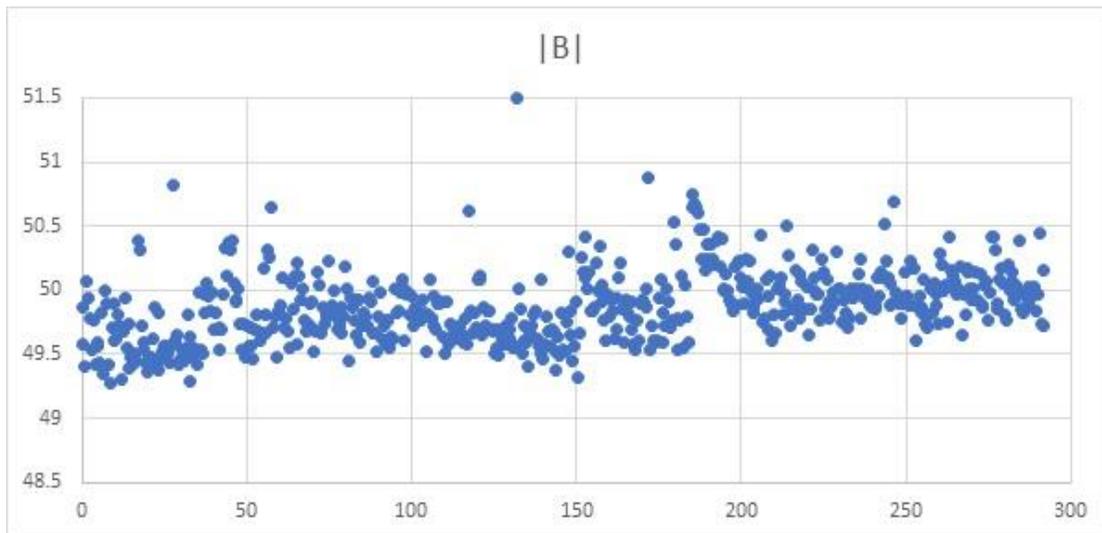


Figure 138. Total magnetic field value |B|.

Inclinometer – on tabletop

This app measures the orientation of the smartphone in space as rotations around the x, y and z axis of the smartphone. To find these A, P and R angles in a null position, place the phone on a leveled table top and start the *Physics Toolbox* app recording for a 30 seconds, which results in the data shown in Figure 139.

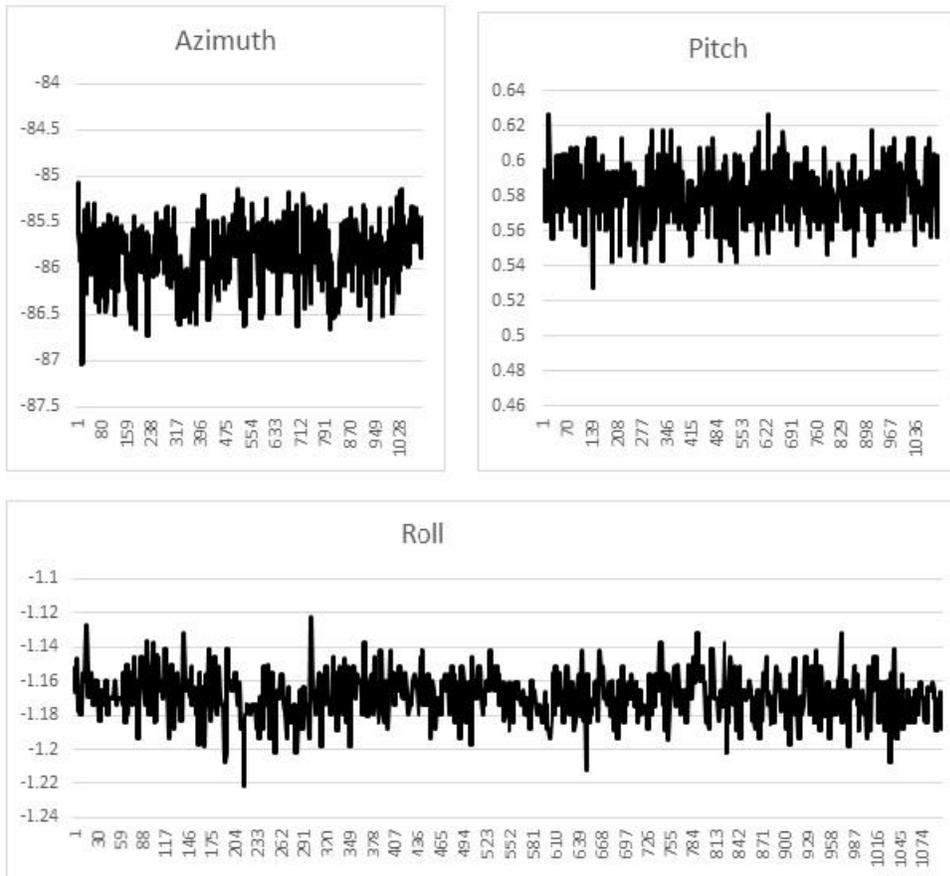


Figure 139. Orientation data for Pitch, Azimuth and Roll.

The azimuth, pitch and roll angles are $\langle \text{Azimuth} \rangle = -85.91 \pm 0.35$ degrees $\langle \text{Pitch} \rangle = +0.58 \pm 0.016$ degrees, and $\langle \text{Roll} \rangle = -1.17 \pm 0.015$ degrees. The smartphone is motionless so any features that appear within the above plots is due to the internal electronics of the smartphone and not externally generated. These plots give some sense of what to expect in terms of the noise in each direction for other measurements. Clearly the pitch and roll angles provide quite-high accuracy, although they seem to suffer from systematic offsets of about 0.5 to 1 degrees.

Light meter

The smartphone was placed outdoors at noon with the back-camera pointed skyward in shadow. The *Physics Toolbox* app took data every 0.2 seconds. The first plot in Figure 140 shows a sample of points spanning 10 seconds and shows the step-like changes in the lux level, which is caused by the digitization noise of approximately 2 lux.

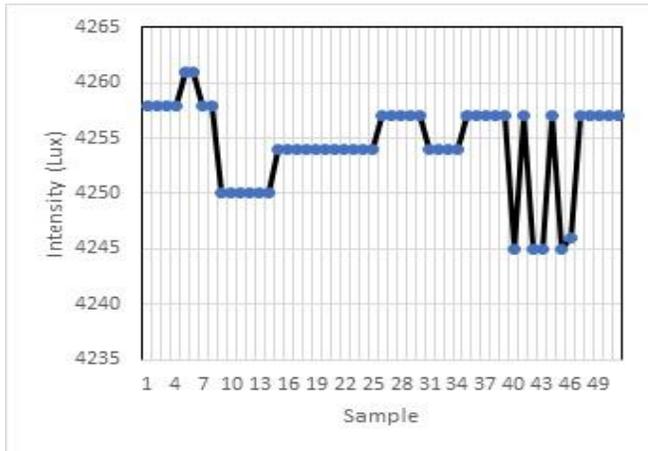


Figure 140. Illumination data

Intensity changes smaller than this cannot be resolved by this camera/app system. Although this is not a problem under typical daylight illumination, it is a problem for indoor and faint light applications near twilight.

A more extended logging of sky illumination levels was made over the course of one hour as shown in the following plot in Figure 141. The sky illumination was measured every two seconds. The general trend is to lower illumination, caused by the setting of the sun. During the measurement session, the sky illumination decreased by 1000 lux units, although the progress was not smooth, possibly due to atmospheric haze.

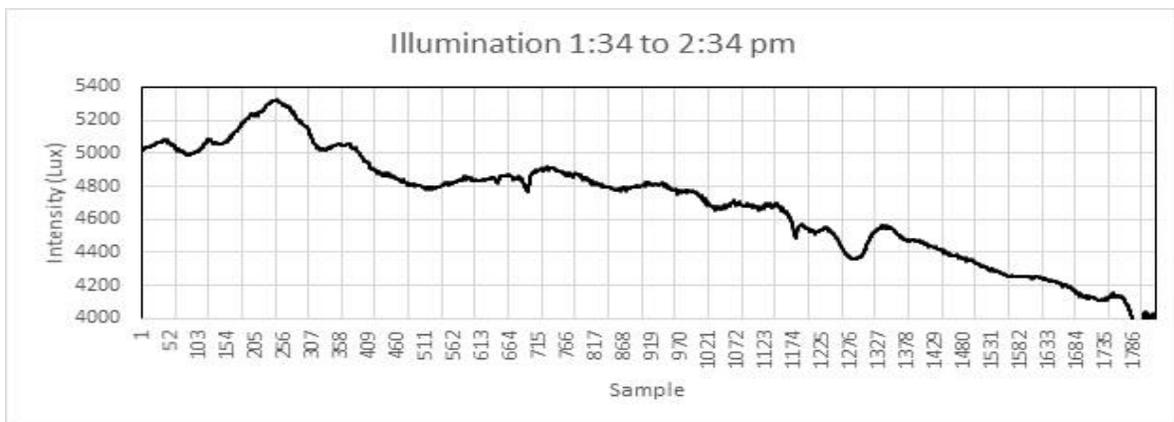


Figure 141. Long term illumination data.

Sound meter

Sound measurements were made in a quiet basement with no A/C or other obvious sources of noise. There was no traffic on the street outside. As the one-minute graph in Figure 142 shows, the sensor picks up a number of ‘spikes’ in the sound level that do not correspond to any known external phenomena and are probably artifacts of the electronics. If we delete these nine spikes, we can calculate an average and σ for the data as $\langle S \rangle = 48.2 \pm 1.0$ decibels.

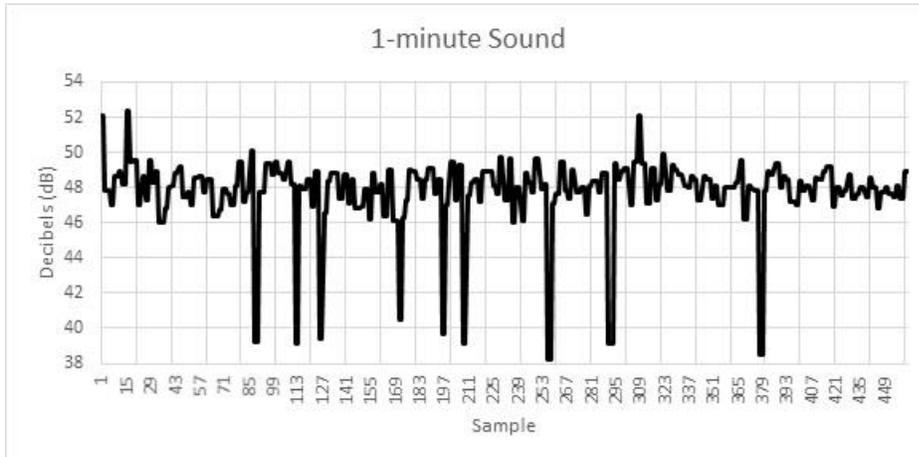


Figure 142. Sound intensity plot for quiet conditions.

We can do a longer-term study spanning a full hour and produce the graph in Figure 143. We continue to see many of the large spikes between 38 to 43 decibels, which we can ignore in the data. With these removed we can rescale the data to reveal in the second plot in Figure 144 a series of horizontal lines appearing near levels of 45.11, 46.16, and 47.2 decibels. These are the indications of the digitization of the sensor output at intervals of 1.0 decibels. The likely full range of the data spans 0 to 150 decibels, so with a single-bit level of 1-dB, the digitization is at a level of 8-bits to get 256-levels, or 7-bits to get 128-levels of sound intensity.

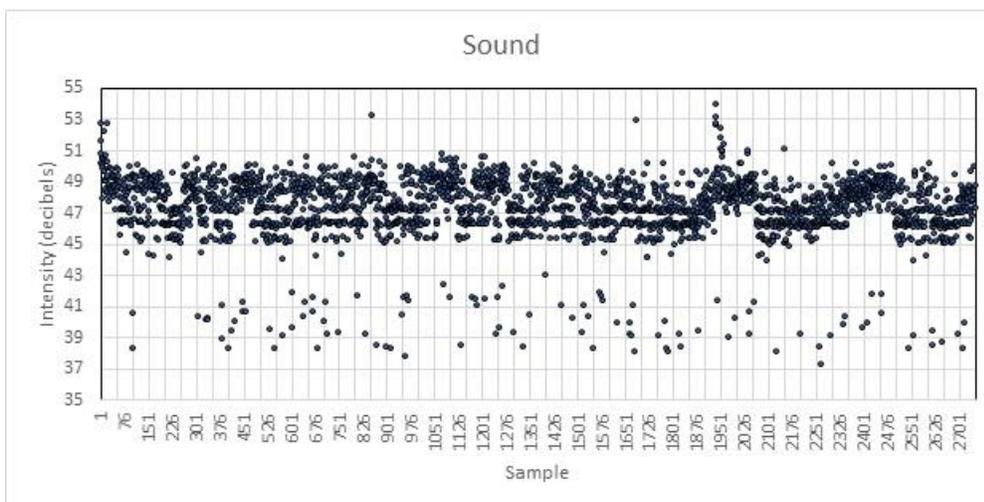


Figure 143. Sound intensity plot for one hour

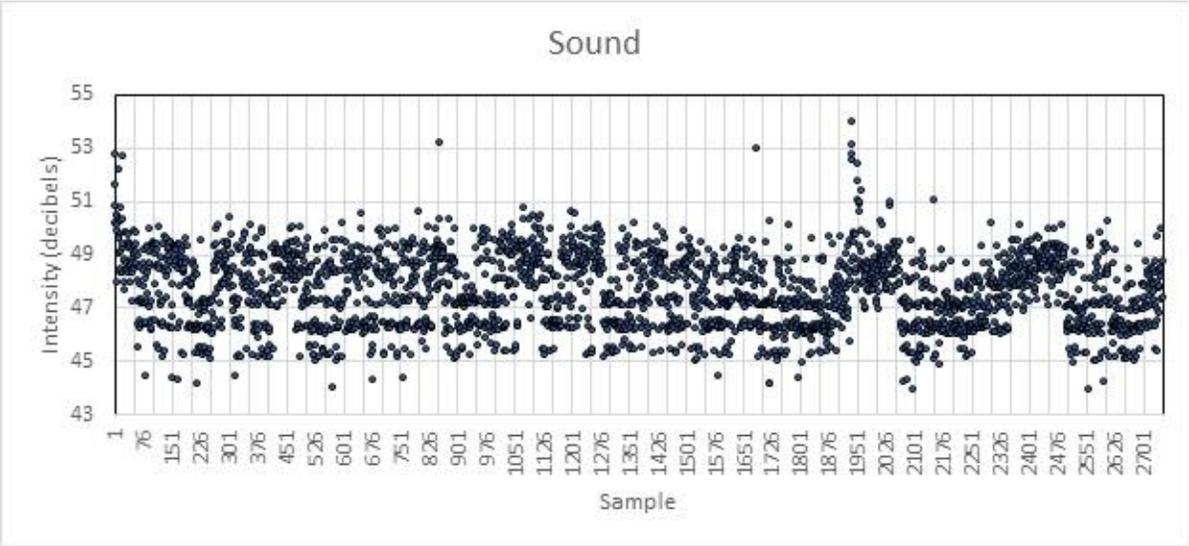


Figure 144. Sound intensity plot showing digitization levels.

Calibrating Sensors

The previous sections described how to use smartphone sensors and apps to make measurements of your environment and analyze the data statistically. In this section, we use these techniques to take the next step in analyzing sensor data: Calibration.

Calibration is a mathematical procedure for re-scaling the units provided by your sensor (voltage values, etc) into physically useful units such as temperature (Celsius, Fahrenheit, kelvin) or pressure (Pascals, MilliBars, PSI). What your sensor actually provides is an output (analog or digital) in units of voltage (Volts), so you need to compare these output voltage levels against some set of standard physical units, either by performing a calculation or by measuring a known physical system with your sensor. This step, of defining a set of standards, is actually very complicated and has its own issues of consistency. For instance, when your sensor is measuring a calibration standard for temperature at 30° Celsius and generates 3.05 Volts, how do you know that the temperature of the standard was actually 30° Celsius? Even the value of the standard you use can have its own built-in uncertainty so the calibration temperature might actually be $30.0^\circ \pm 0.03^\circ$ Celsius, which is an uncertainty of $100\% \times (0.03/30) = \pm 0.1\%$. What this means is that if your sensor is able to provide output voltages accurate to $\pm 1\%$, this calibration standard is a good choice, but if the sensor provides output voltages accurate to 0.05%, this standard may not be good enough to provide sensor calibration at the full level of sensitivity of the sensor.

In what follows, we will look at several smartphone sensor systems and perform a calibration of them by using the units already provided by the sensors and comparing them against more-expensive, professional-grade instruments which are in turn calibrated against high-quality standards: Gravitational acceleration and light intensity.

Sound Calibration

Sound levels are measured in units of decibels (dB) which like the magnitude units used by astronomers to describe star brightness, is a logarithmic unit of sound power per square meter through a surface, specifically, $\text{dB} = 10\text{Log}_{10}(S/P_0)$ where P_0 is a reference power level of 1 picoWatt (10^{-12} watts). An increase of 3 dB equals a power increase by a factor of 2.0. In terms of actual watts, S , a 60 dB sound leads to a sound power of 10^{-6} watts and 120 dB yields 1 watt.

All smartphones have one or more microphones to enable talking, and for rigorous testing this element is generally the only one whose properties are known in detail outside the company-proprietary umbrella of the smartphone designers. These apps take advantage of the microphone to measure ambient background sound levels. Because microphones are originally engineered to

accommodate the normal ranges of speech, they are limited in the range of loudest and weakest sounds they can detect and measure spanning a range from about 40 dB to 120 dB. Detailed acoustic studies (Brown and Evans, 2011) of an iPhone 3GS with a frequency range between 350 Hz and 48,000 Hz shows a minimum detectable signal at about 32 dB, a reduced response above 85 dB, and an accuracy of about ± 4.0 dB using acoustic sources of known intensity and frequency.

For this analysis we used the app *Decibel 10th* (or *Decibel X*) by SkyPaw Co. Ltd, which has a superior display including real-time plot, an analog meter dial, and digital display of the average, current and peak sound levels. It also allows for variable sampling rates from 4 to 20 Hz. It is also one of very few noise/sound meter apps on the market that the developer says has been pre-calibrated. The data can be recorded and exported as a .csv file.

Relative Performance Tests

An iPhone 6s was placed on flat surface face up with microphone exposed. The uncertainties in the measurements are estimated from the ranges of the maximum and minimum values displayed by each app. The results for various apps and circumstances are shown in Table 40.

Table 40 Acoustic levels at a variety of locations

Source	<i>DecibelMeter</i>	<i>Decibel Level</i>	<i>Decibel Sound Meter</i>	<i>DeciBel-10th</i>
Summit of Mt Greylock	63	65	38	39
Quiet Room	58	59	30	43
Toyota Corolla engine	75	76	55	65
Jet plane interior	92	88	70	86
Electric Lawn Mower	89	86	70	86
Jack hammer @ 10 meters	89	67	73	86
<i>Queen</i> Rock Concert	95	85	75	109

Given that the dB scale is logarithmic, the variation in the measured values from app to app under the same environmental conditions is significant when compared to the ± 5 dB for the individual app measurements. The range of measurements for a ‘quiet room’ spanned nearly 30 dB, which is a factor of 1000 in acoustic power. For the other end of the acoustic range, a rock concert measurement at the same location in the arena varied by 20 dB corresponding to a factor of 100 in power.

iPhone and Samsung comparison

Smartphone platforms use a variety of microphone technologies and so we need to compare sound sensitivity in each phone and also determine their zero-points. A Samsung Note 5 and iPhone 6s were placed side-by-side in a quiet room with the same *Decibel 10th* app in operation. Initially the Samsung phone registered the quiet conditions as being near 65 dB. The calibration adjustment feature in the app was used to shift the Samsung data by -13 dB so it matched the iPhone values, which were believed to be more typical of the ‘quiet room’ sound level near 40 dB.

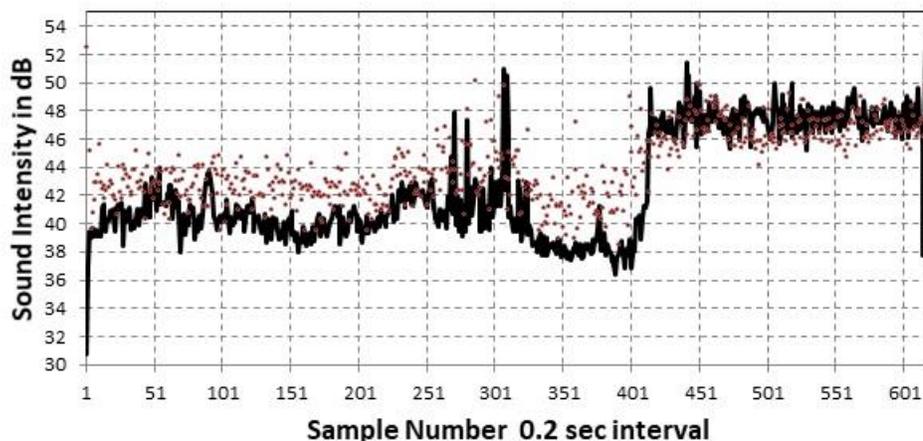


Figure 145. Quiet environment data for *Decibel 10th*. Solid curve is the Samsung Note 5 and dots are the iPhone 6s.

Figure 145 shows three minutes of data sampled at the app’s fast rate of 0.2 seconds. The sudden jump at Sample 400 was caused by the home air conditioner turning on, causing a DC level jump for subsequent samples. The iPhone detected this jump at about a difference of 4.5 dB, while the Samsung registered a larger 7 dB jump. Also, a study of the rms values of the data defined by $\sigma = \sqrt{(\langle x^2 \rangle - \langle x \rangle^2)/(N - 1)}$ shows that for the iPhone system, its measurement noise is ± 1.5 dB while the Samsung phone is much quieter at ± 0.5 dB. The Samsung smartphone with an offset added, seems to be a more sensitive system (lower σ) than the iPhone 6S. The question is, what is the proper offset to use?

Absolute Performance Tests

The relative sound test only tells us what each system indicates is the correct sound level, but not which one is actually the more accurate. The B&K Precision Model 732A Sound Level Meter (\$310) provides 30 to 130 dB measurement capability with an accuracy of $\sigma = \pm 1.5$ dB. The meter also meets the IEC 60651 Type II sound level meter standard that requires a measurement accuracy of $\sigma = \pm 1.0$ dB, which is the general-purpose grade for field work. Type I at $\sigma = \pm 0.7$ dB, by comparison, is considered precision-grade for lab work. The relative performance tests “...were

repeated and compared with an independent and professionally-calibrated intensity scale”, according to the manufacturer.

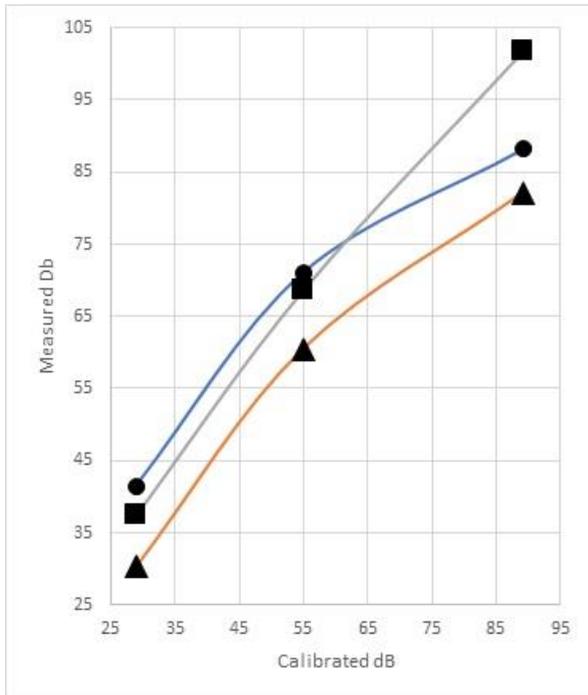


Figure 146. iPhone 6s (dot), Galaxy Note 5 (square) and Galaxy 8s (triangle).

What Figure 146 shows is that, when compared to a professional-grade sound meter capable of making high precision measurements, the smartphones show very different performances. For the best agreements, the smartphone measurements should fall on a diagonal line with a slope of 1.0 so that the Calibrated dB = Smartphone dB values.

To bring the Smartphone values into the best agreement with the diagonal 1:1 correlation, we can first apply offsets to the smartphone values to bring their curves into concordance with the same reference point at 29 dB. This is seen in Figure 147. We have had to shift the curve for the iPhone 6s by subtracting 12 dB, the Note 5 by subtracting 8 dB and not changing the measurements for the Galaxy 8s.

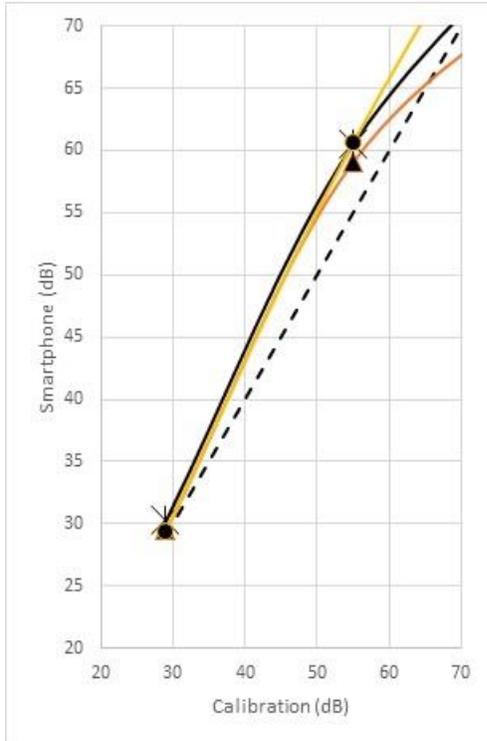


Figure 147. Data for smartphones compared to calibrated meter. iPhone 63 (triangle), Note 5 (dot), Galaxy 8S (asterisk).

The next correction is for the slopes of the measured curves, which are slightly steeper than the 1:1 slope of the calibration curve shown with the dashed line. For the Galaxy 8s, the equation that gives the best match is $C = 29 + 0.8(M - 29)$. For the iPhone 6s $C = 29 + 0.85(M - 41.5)$. For the Note 5 $C = 29 + 0.85(M - 37.4)$. These curves work over the range from 29 to 60 dB, and lead to Figure 148, which shows how the smartphone measurements now compare reasonably well with the dotted calibration curve.

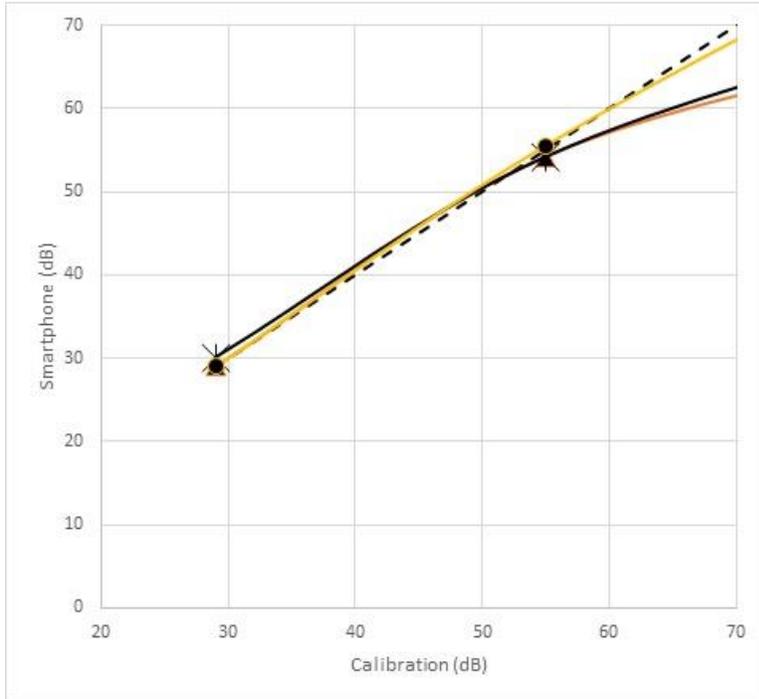


Figure 148. Calibrated sound measurements.

Once a measurement is made with the iPhone 6s, or the Samsung Galaxy 8s or Note 5, these measurements can be converted to a calibrated sound scale over a range from 29 to about 55 dB, where the difference between the 1:1 calibration curve (dashed line) is coincident with the individual calibration curves for the smartphones. Note that for sound levels greater than about 55 dB, the Samsung Note 5 continues to be nearly identical to the calibration curve to levels as high as 100 dB, while the other two smartphones rapidly diverge from the ideal (dashed) correlation, and show a systematically lower response. Nevertheless, even this problem can be corrected if we use a different and non-linear calibration process. Figure 149 shows an exponential fit to the iPhone 6s and Samsung Galaxy 8s data. The Galaxy 8s data can be fit by the curve $C = 14.983e^{0.0217M}$ and the iPhone 6s is fit by $C = 10.62e^{0.0238M}$. An example of one of these calibration curves is shown in Figure 59 for the Galaxy 8s phone.

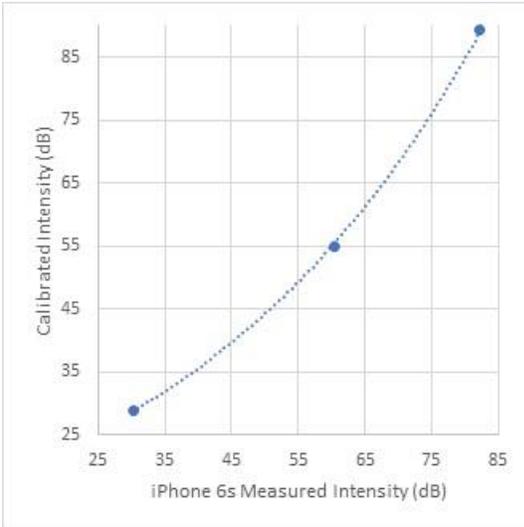


Figure 149. Calibrated sound measurements for an iPhone 6s

Comparison with multiple Samsung phones

All eight smartphones were placed in three different, constant-intensity sound environments and allowed to record data with the *Decibel 10th* app by SkyPaw. Table 41 shows that there is considerable copy-to-copy variation within the same model type by as much as +12 dB for Galaxy 8s and +20 dB for the Note 5. The Galaxy 8s seemed to perform better over-all, especially for the quieter sound levels. Within each model line, the sound measurements had an internal precision of about ± 3 dB, although one of the Note 5 phones gave dramatically different measurements than its other three copies.

The internal precision of the phone copies suggests that it might be possible to calibrate the sound measurements against a standard level, and that such a calibration might be constant across multiple copies of the same phone model, although the current sample of copies is admittedly limited. The total variation of the measurement would consist of the contribution from the calibration residual and the copy-to-copy variance. When added in quadrature we get a total accuracy of $\sigma^2 = \sigma_c^2 + \sigma_n^2$, where $\sigma_c = \pm 6$ dB and $\sigma_n = \pm 3$ dB, for a total accuracy of $\sigma = \pm 7$ dB. Such a large residual uncertainty after calibration would not meet the requirements of an IEC 60651 Type II metering system ($\sigma = \pm 1.0$ dB), which is considered a general-purpose sound metering system adequate for field work.

Table 41 Sound comparisons with Samsung phones

Sound Source	Calibration	Galaxy 8s				Note 5			
		A	B	C	D	E	F	G	H
Quiet room	29.8	30.3	29.9	32.9	29.9	50.9	50.2	50.6	37.0
Inside idling car	55.0	60.5	61.2	62.3	61.5	82.0	82.2	82.4	68.6
Electric lawn mower	89.2	82.1	77.0	78.5	76.8	104.5	105.0	105.5	91.3

This concludes a more-or-less thorough investigation of smartphone sensors as they can be applied to making high-quality measurements of a variety of physical parameters.

Figure Credits:

Figure 1 JustScience <http://www.justscience.in/articles/magnetic-field-and-magnetic-field-lines/2017/06/30>

Figure 2 onlinemathlearning <https://www.onlinemathlearning.com/vector-magnitude.html>

Figure 4 <http://www.rci.rutgers.edu/~schlisch/103web/Pangeabreakup/paleomag.html>

Figure 46 Casey Reed <https://www.skyandtelescope.com/astronomy-resources/transparency-and-atmospheric-extinction/>

Figure 53 <http://ogrisel.github.io/scikit-learn.org/sklearn-tutorial/tutorial/astronomy/classification.html>

Figure 56 <http://housecraft.ca/eco-friendly-lighting-colour-rendering-index-and-colour-temperature/>

Figure 88 https://freestarcharts.com/images/Articles/Constellations/Orion/Ori_Orion_Star_Chart.pdf

Figure 91 Planck Collaboration: P. A. R. Ade et al., 2013, A&A Preprint.

Figure 92 <https://thinkmobiles.com/blog/popular-types-of-apps/>

Figure 95 <https://blog.poscope.com/usb-oscilloscope-performance-properties/what-is-analog-verse-digital-explanation/>

Figure 96 <http://gate-exam.in/EE/Syllabus/Electrical-Engineering/Analog-Digital-Electronics/AD-DA-Converters>

Figure 97 <https://eandt.theiet.org/content/articles/2014/06/teardown-samsung-galaxy-s5/>

Figure 99 https://www.researchgate.net/figure/oltage-conversion-drift-of-the-ADC-and-the-environmental-temperature-variation-as-a_fig23_295243459

