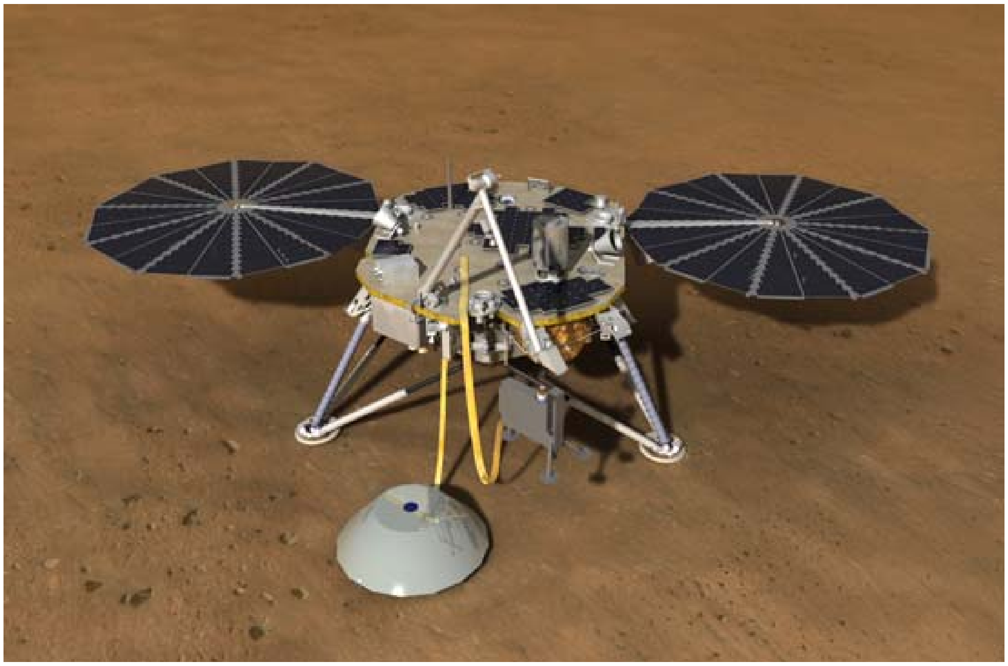


NASA's new mission to Mars called InSight will be launched in March, 2016. It will land in a region of Mars located near the equator and deploy a seismographic station and a heat probe to study the interior of Mars.

Once the InSight lander touches down on September 20, 2016, during the next 60 days it will slowly and carefully deploy the seismometer station called SEIS (the conical package in the image above) and the heat flow experiment called HP^3 (the rectangular instrument package). Both are attached to the lander by yellow cables that carry instrument power and data. A single maneuverable arm will do this work, but it cannot reach closer than 1.5 meter to the Lander, or farther than 2.5 meters from the lander. It can swing left to right, sweeping out an arc of 90 degrees.

Problem 1 – What is the total surface area on Mars near the lander where the maneuverable instrument arm can place its two payloads? (Use $\pi = 3.141$)

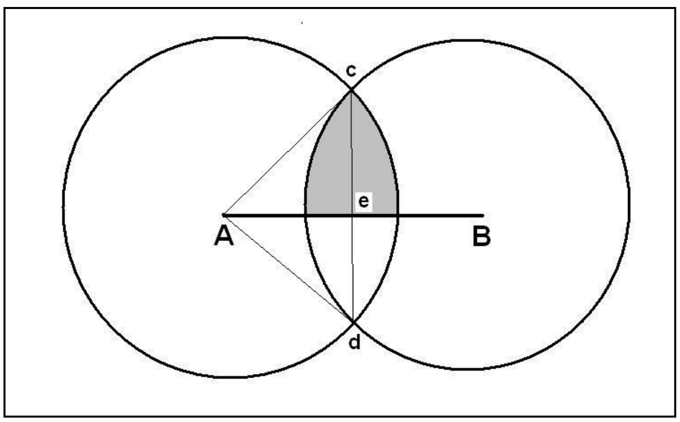
Problem 2 – Suppose the Lander had two identical arms, A and B, whose pivot points were separated by 2.0 meters on the Lander's body. If the arms could reach a maximum of 1.0 meters from their pivot points, what is the area on the martian surface that is common to both arms but above the line joining their pivot points? (Note: For safety reasons, only this common area would be where instruments might be placed so that one or the other arms always have access to them.) Use any method to determine the area.

Problem 1 – What is the total surface area on Mars near the lander where the maneuverable instrument arm can place its two payloads? (Use $\pi = 3.141$)

Answer: The area will be ¼ the area of a circular ring with inner radius of 1.5 meter and outer radius of 2.5 meters. This area is just ¼ the area of the larger circle $A = \pi (2.5)^2 = 19.6\ m^2$ minus the area of the inner circular 'hole' $A = \pi (1.5)^2 = 7.1\ m^2$ which leaves an area of ¼ (12.5) **= 3.1 m²**.

Problem 2 – Suppose the Lander had two identical arms, A and B, whose pivot points were separated by $2/2^{1/2} = 1.41$ meters on the Lander's body. If the arms could reach a maximum of 1.0 meters from their pivot points, what is the area on the martian surface that is common to both arms but above the line joining their pivot points? (Note: For safety reasons, only this common area would be where instruments might be placed so that one or the other arms always have access to them.) Use any method to determine the area.

Answer: It always helps to draw a diagram of the area in question given the information in the problem. This visual information helps you formulate a mathematical approach to the problem and can suggest solutions.



Method 1 – Students can reproduce the figure on gridded graph paper and count the number of whole squares in the overlap region.

Method 2 – Students can use Method 1 but add into their sum an estimate for the total area in the fractional squares.

Method 3 – An exact solution to this problem requires a bit of geometry.
The distance $Ae = Be = 1/2^{1/2}$ and the radius of the circle is 1.0, so the triangle **cAe** is a 45-45-90 right triangle with side lengths $R/2^{1/2}$ and an area A=1/2base X height = $1/4\ R^2$ where R=1 for InSight.
The full arc **cAd** is 90^o, so its area is ¼ of the circle whose radius is R=1 meter so $A_{arc} = \pi/4R^2$.
But we only want the area above the line **AB** so $A_{arc} = \pi/8R^2$.
The shaded area to the right of the line **ce** has an area A = Aarc – Atriangle = $\pi/8\ R^2 - ¼\ R^2$.
A similar solution by symmetry works for the shaded area to the left of the line **ce**, so the total area of the shaded region is $A = 2 \times (\pi/8\ R^2 - 1/4R^2)$ or $A = R^2/4\ (\pi - 2)$
For the specific case of R=1 meter, $A = (\pi-2)/4 =$ **0.28 meters²**. This should be close to the answers determined by Method 1 and 2.