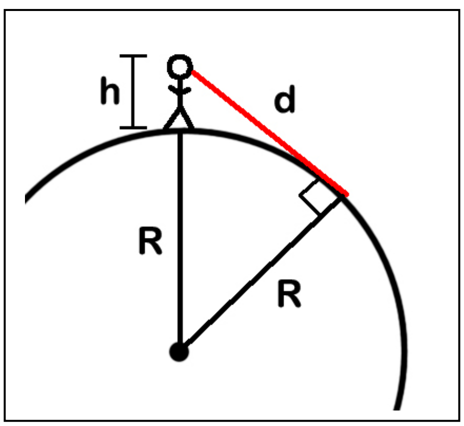
The Distance to the Martian Horizon

Imagine you and your friend standing on the surface of a perfectly flat planet. Your friend starts walking away from you, and you see her size get smaller and smaller until at a distance of 100 kilometers you can't even see her at all.

Now imagine the same experiment on a spherical planet. As many sea-farers discovered 1000 years ago, because Earth is curved, you will see the ships hull disappear from the bottom upwards, then the last thing that vanishes is the top of the main mast.

The image above comes from Johannes de Sacrobosco's Tractatus de Sphaera (*On the Sphere of the World*) written in 1230 AD. It showcases the knowledge that the appearance of ships on the horizon testified to a curved earth. A bit of simple geometry, and some help from the Pythagorean Theorem, will let you calculate the distance to the horizon on Mars as viewed from the InSight Lander!

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.



Problem 2 – R is the radius of Mars, which is 3,378 kilometers. If h is the height of an observer in meters, write a simplified equation for d when h << R. What is the distance to the martian horizon as viewed from the IDS camera located 1 meters above the ground?

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Problem 1 – Use the Pythagorean Theorem to solve for the distance, d, in terms of h and R.

Answer: $d^2 = (R+h)^2 - R^2$

So $d^2 = 2Rh + h^2$

And so $d = (2Rh + h^2)^{1/2}$

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Answer: For the formula to work, R and h must be in the same units of meters (or kilometers!). When h is much less then R, the quantity h^2 is always much, much smaller than 2Rh, so the formula simplifies to $d = (2Rh)^{1/2}$.

For Mars, h = 1 meter and R = 3378000 meters and so **d = 2599 meters or about 2.6 kilometers.**

Problem 3 - Mars has no ionosphere, so radio signals cannot be 'bounced' around Mars to distant locations. Instead, tall 'cell towers' have to be used. If the cell tower is 100 meters tall, how many cell towers will you need to cover the entire surface of Mars?

Answer: h = 100 meters or 0.1 km, so the horizon distance for one cell tower has a radius of $d = (2 \times 3378 \text{ km} \times 0.1 \text{ km})^{1/2}$ = 26 kilometers. The area of this reception circle is $A = \pi R^2 = 2123 \text{ km}^2$. The total surface area of Mars is $A = 4 \times \pi \times (3378)^2 = 1.43\times10^8 \text{ km}^2$.

Then dividing the surface area of Mars by the cell tower reception area we get N = $1.43\times10^8/2123$ = **67,530 cell towers**.

As of 2013, there are over 200,000 cell towers in the Unites States alone!