



Once astronomers have measured the diameter and mass of a planet, they can determine the average density of the planet by dividing its mass by its volume. This is a valuable 'first look' into the interior of a planet because if the average density is close to 1000 kg/m^3 , then most of the planet consists of light materials and gas or even water and ice like Saturn and Uranus. If the value is large and near 4000 kg/m^3 , then the planet may consist mostly of rocky materials like Mercury and Earth.

Problem 1 - The mass of Mars is known to be 6.39×10^{23} kilograms, and the outer radius of the planet is 3400 kilometers. What is the average density of Mars in kilograms/meter³? What would you estimate as the composition of the martian interior if ice has a density of 917 kg/m^3 , granite has a density of 2700 kg/m^3 and iron ore has a density of 7000 kg/m^3 ?

Problem 2 – The interior of Mars can be represented by three main geologic regions: The core is a spherical region with a radius of about 1800 km; the mantle is a spherical shell with an outer radius of 3300 km, and the crust is a 100 km spherical shell located above the mantle. The crust of Mars has been sampled by several NASA landers including Viking, Spirit, Opportunity, Phoenix and Curiosity. The density of the surface rocks appears to be about 2000 kg/m^3 . If models of the core of Mars suggest a density of 6400 kg/m^3 , what is the average density of the rocks in the martian mantle zone to two significant figures?

Problem 1 - The mass of Mars is known to be 6.39×10^{23} kilograms, and the outer radius of the planet is 3400 kilometers. What is the average density of Mars in kilograms/meter³? What would you estimate as the composition of the martian interior if ice has a density of 917 kg/m^3 , granite has a density of 2700 kg/m^3 and iron ore has a density of 7000 kg/m^3 ?

Answer: The volume of Mars as a sphere is given by $V = \frac{4}{3} \pi R^3$ so
 $V = 1.333 \times 3.141 \times (3400000)^3 = 1.65 \times 10^{20} \text{ m}^3$, then the density is just
 $D = \frac{6.39 \times 10^{23} \text{ kg}}{1.65 \times 10^{20} \text{ m}^3} = \mathbf{3872 \text{ kg/m}^3}$. This is between the density of granite and iron, but closer to granite, so on average there is probably very little iron in the interior of Mars.

Problem 2 – The interior of Mars can be represented by three main geologic regions: The core is a spherical region with a radius of about 1800 km; the mantle is a spherical shell with an outer radius of 3300 km, and the crust is a 100 km spherical shell located above the mantle. The crust of Mars has been sampled by several NASA landers including Viking, Spirit, Opportunity, Phoenix and Curiosity. The density of the surface rocks appears to be about 2000 kg/m^3 . If models of the core of Mars suggest a density of 6400 kg/m^3 , what is the average density of the rocks in the martian mantle zone to two significant figures?

Answer: We know:

- The total mass of Mars is $6.39 \times 10^{23} \text{ kg}$.
- The radius of the core is 1800 km.
- The inner and outer radius of the mantle shell as 1800 km and 3300 km.
- The inner and outer radius of the crust shell as 3300 km and 3400 km.
- The density of the core as 6400 kg/m^3
- The density of the crust as 2000 kg/m^3 .

So we subtract from the mass of Mars the mass of the core and the crust to get the mass of the mantle. From the mantle shell volume we can then determine its density:

$$M_{\text{core}} = 6400 \times \frac{4}{3} \pi (1800000)^3 = 1.56 \times 10^{23} \text{ kg.}$$

$$M_{\text{crust}} = 2000 \times \frac{4}{3} \pi (3400000^3 - 3300000^3) = 2.82 \times 10^{22} \text{ kg}$$

$$M_{\text{mantle}} = 6.39 \times 10^{23} \text{ kg} - 1.56 \times 10^{23} \text{ kg} - 2.82 \times 10^{22} \text{ kg} = 4.55 \times 10^{23} \text{ kg.}$$

$$\text{Volume(mantle)} = \frac{4}{3} \pi (3300000^3 - 1800000^3) = 1.26 \times 10^{20} \text{ m}^3.$$

$$\text{So Density} = \frac{4.55 \times 10^{23} \text{ kg}}{1.26 \times 10^{20} \text{ m}^3} = \mathbf{3600 \text{ kg/m}^3}.$$