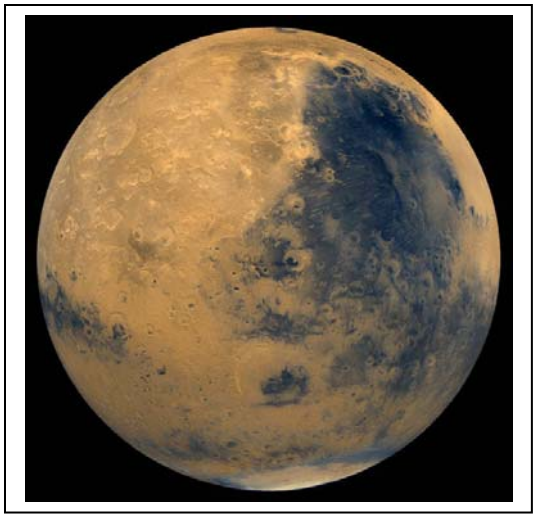

Astronomers can determine the mass of a planet by using Kepler's Third Law, which is written in algebraic form as follows:

$$p^2 = \frac{4\pi^2}{G\,M}\,a^3$$

where G is Newton's constant of gravity equal to 6.67×10^{-11} $m^3/kg\ sec^2$, M is the mass of the planet in kilograms, a is the average orbit radius in meters and T is the orbit period in seconds.

Here are some problems that let you work with this very important formula in astronomy.

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of 9380 km and a period of 0.32 days, what is the mass of Mars?

Problem 2 – The martian moon Deimos has an orbit period of 1.26 days. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of 35.5 hours and an average radius of 25,889 km. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of 3700 km. What is the final orbit period of MRO in the new circular orbit?

Problem 1 – The martian moon Phobos was observed from Earth through telescopes to have an orbit radius of 9380 km and a period of 0.32 days, what is the mass of Mars?

Answer: First solve the algebraic equation for M to get $M = (4\pi^2/G)(a^3/p^2)$. Then use a = 9380 km x 1000m/km = 9.38x106 m, and p = 0.32days x 24h/d x 3600s/hr = 27648 seconds, to get

$M = [4 \times (3.141)2/6.67\times10^{-11}] (9.38\times10^{6})^3/(27648)^2 =$ **6.39×10^{23} kg**.

Problem 2 – The martian moon Diemos has an orbit period of 1.26 days. What is its orbit radius from the center of Mars in kilometers to 3 significant figures?

Answer: In this case we know M and p and need to solve for a:

$a^3 = GMp^2/4\pi^2$ so p = 1.26d x 24h/d x 3600 s/h = 108864 seconds, and so $a^3 = (6.67\times10^{-11})(6.39\times10^{23})(108864)^2/(4 \times 3.141^2) = 1.27\times10^{22}$ so a = 23352 kilometers and so a = **23,400 kilometers**.

Problem 3 – On March 10, 2006, the Mars Reconnaissance Orbiter was originally placed into a very elliptical orbit with a period of 35.5 hours and an average radius of 25,889 km. What is the mass of Mars based on this orbit? Its final orbit was achieved in September 2006 with a circular radius of 3700 km. What is the final orbit period in minutes of MRO in the new circular orbit?

Answer: Solving for p we get $p^2 = (4\pi^2/GM)\, a^3$, and so for a = 3,700,000 meters and $M = 6.39\times10^{23}$ kg we get $p^2 = 4.69\times10^{7}$ and so p = 6848 seconds or **114 minutes**.